



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

20 November 2024

This marking guideline consists of 18 pages.

QUESTION 1

$$1.1 \quad 1.1.1 \quad \frac{\partial z}{\partial y} = f_y(x, y) = 2(y + \cos x)^{-1} - 2y(y + \cos x)^{-2}$$

✓✓ ✓✓

ALTERNATIVE

$$\frac{\partial z}{\partial y} = f_y(x, y) = \frac{2(y + \cos x)✓✓ - 2y✓}{(y + \cos x)^2 ✓} \quad (4)$$

$$1.1.2 \quad \left. \frac{\partial z}{\partial y} \right|_{\substack{x=1 \\ y=2}} = f_y(1, 2) = 2(2 + \cos 1)^{-1} - 2(2)(2 + \cos 1)^{-2} = 0.167 ✓✓$$

ALTERNATIVE

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=1 \\ y=2}} = f_y(1, 2) = \frac{2(2 + \cos 1) - 2(2)}{(2 + \cos 1)^2} = 0.167 ✓✓ \quad (2)$$

$$1.2 \quad \text{Given: } x = t - t^2 \text{ and } y = t - t^3$$

$$\frac{dy}{dt} = 1 - 3t^2 \quad ✓ \quad \frac{dx}{dt} = 1 - 2t \quad ✓$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1 - 3t^2}{1 - 2t} \quad ✓$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \times \frac{1}{\frac{dx}{dt}}$$

$$= \frac{d}{dx} \left(\frac{1 - 3t^2}{1 - 2t} \right) \times \frac{1}{1 - 2t} \quad ✓$$

$$= \left(\frac{-6t(1 - 2t) + 2(1 - 3t^2)}{(1 - 2t)^2} \right) \times \frac{1}{1 - 2t} \quad ✓$$

$$= \frac{6t^2 - 6t + 2}{(1 - 2t)^3} \quad ✓$$

(6)

QUESTION 2

2.1 $f(x) = x^2 - 2x + 1 \Rightarrow f'(x) = 2x - 2 \checkmark$

$$g'(x) = e^{2x} \Rightarrow g(x) = \frac{e^{2x}}{2} \checkmark$$

$$\int (x^2 - 2x + 1)e^{2x} dx = \frac{(x^2 - 2x + 1)e^{2x}}{2} - \frac{2}{2} \int (x - 1)e^{2x} \checkmark \checkmark$$

$$f(x) = x - 1 \Rightarrow f'(x) = 1$$

$$g'(x) = e^{2x} \Rightarrow g(x) = \frac{e^{2x}}{2}$$

$$\int (x^2 - 2x + 1)e^{2x} dx = \frac{(x^2 - 2x + 1)e^{2x}}{2} - \left[\frac{(x - 1)e^{2x}}{2} - \frac{1}{2} \int e^{2x} dx \right] \checkmark \checkmark$$

$$= \frac{(x^2 - 2x + 1)e^{2x}}{2} - \left[\frac{(x - 1)e^{2x}}{2} - \frac{1}{4} e^{2x} \right] + C \checkmark$$

$$= \frac{(x^2 - 2x + 1)e^{2x}}{2} - \frac{(x - 1)e^{2x}}{2} + \frac{1}{4} e^{2x} + C \checkmark$$

ALTERNATIVE

D	I
$x^2 - 2x + 1$	e^{2x}
$2x - 2 \checkmark$	$\frac{e^{2x}}{2} \checkmark$
$2 \checkmark$	$\frac{e^{2x}}{4} \checkmark$
0	$\frac{e^{2x}}{8} \checkmark$

$$\int (x^2 - 2x + 1)e^{2x} dx = \frac{(x^2 - 2x + 1)e^{2x}}{2} - \frac{(2x - 2)e^{2x}}{4} + \frac{2e^{2x}}{8} + C \checkmark \checkmark \checkmark$$

(8)

$$\begin{aligned}
 2.2 \quad & \int \sin^3 4x \cos^2 4x dx \\
 &= \int \sin^2 4x \sin 4x \cos^2 4x dx \checkmark \\
 &= \int (1 - \cos^2 4x) \sin 4x \cos^2 4x dx \checkmark \\
 &= \int \sin 4x \cos^2 4x dx - \int \sin 4x \cos^4 4x dx \checkmark \checkmark \\
 u = \cos 4x &\Rightarrow \frac{du}{dx} = -4 \sin 4x \\
 &= -\frac{1}{4} \int u^2 du + \frac{1}{4} \int u^4 du \checkmark \\
 &= -\frac{u^3}{12} + \frac{u^5}{20} + C \checkmark \\
 &= -\frac{\cos^3 4x}{12} + \frac{\cos^5 4x}{20} + C \checkmark \checkmark
 \end{aligned}$$

ALTERNATIVE

$$\begin{aligned}
 & \int \sin^3 4x \cos^2 4x dx \\
 &= \int \sin^2 4x \sin 4x \cos^2 4x dx \checkmark \\
 &= \int (1 - \cos^2 4x) \sin 4x \cos^2 4x dx \checkmark \\
 &= \int \sin 4x \cos^2 4x dx - \int \sin 4x \cos^4 4x dx \checkmark \checkmark \\
 &= -\frac{1}{4} \int [\cos 4x]^2 (-4 \sin 4x) dx + \frac{1}{4} \int [\cos 4x]^4 (-4 \sin 4x) dx \checkmark \checkmark \\
 &= -\frac{\cos^3 4x}{12} + \frac{\cos^5 4x}{20} + C \checkmark \checkmark
 \end{aligned}$$

(8)

2.3

$$\int \frac{dx}{\sqrt{3+2x-3x^2}}$$

$$3+2x-3x^2 = -3x^2 + 2x + 3 \checkmark$$

$$= -3 \left[x^2 - \frac{2}{3}x - 1 \right] \checkmark$$

$$= -3 \left[x^2 - \frac{2}{3}x + \frac{1}{9} - \frac{1}{9} - 1 \right]$$

$$= -3 \left[\left(x - \frac{1}{3} \right)^2 - \frac{10}{9} \right] \checkmark$$

$$= \frac{10}{3} - 3 \left(x - \frac{1}{3} \right)^2 \checkmark$$

ALTERNATIVE

$$\frac{4ac - b^2}{4a} + a \left(x + \frac{b}{2a} \right)^2 \checkmark$$

$$= \frac{4(-3)(3) - (2)^2}{4(-3)} - 3 \left(x + \frac{2}{2(-3)} \right)^2 \checkmark \checkmark$$

$$= \frac{10}{3} - 3 \left(x - \frac{1}{3} \right)^2 \checkmark$$

$$\int \frac{dx}{\sqrt{\frac{10}{3} - 3 \left(x - \frac{1}{3} \right)^2}} \checkmark = \frac{1}{\sqrt{3}} \sin^{-1} \left[\frac{\sqrt{3} \left(x - \frac{1}{3} \right)}{\sqrt{\frac{10}{3}}} \right] \checkmark \checkmark \checkmark$$

(8)

2.4

$$\int \cot^5 3x dx = \int \cot^3 3x \cot^2 3x dx \checkmark$$

$$= \int \cot^3 3x (\cos ec^2 3x - 1) dx \checkmark$$

$$= \int \cot^3 3x \cos ec^2 3x dx - \int \cot^3 3x dx \checkmark$$

$$= \int \cot^3 3x \cos ec^2 3x dx - \int \cot^2 3x \cot 3x dx \checkmark$$

$$= \int \cot^3 3x \cos ec^2 3x dx - \int \cot 3x (\cos ec^2 3x - 1) dx \checkmark$$

$$= \int \cot^3 3x \cos ec^2 3x dx - \int \cot 3x \cos ec^2 3x dx + \int \cot 3x dx \checkmark$$

$$u = \cot 3x \Rightarrow \frac{-1}{3} du = \cos ec^2 3x dx$$

$$= \frac{-1}{3} \int u^3 du + \frac{1}{3} \int u du + \int \cot 3x dx \checkmark$$

$$= \frac{-1}{12} u^4 + \frac{1}{6} u^2 + \frac{1}{3} \ln(\sin 3x) + C \checkmark \checkmark$$

$$= \frac{-1}{12} \cot^4 3x + \frac{1}{6} \cot^2 3x + \frac{1}{3} \ln(\sin 3x) + C \checkmark$$

ALTERNATIVE 1

$$\begin{aligned}
 \int \cot^5 3x dx &= \int \cot^3 3x \cot^2 3x dx \checkmark \\
 &= \int \cot^3 3x (\operatorname{cosec}^2 3x - 1) dx \checkmark \\
 &= \int \cot^3 3x \operatorname{cosec}^2 3x dx - \int \cot^3 3x dx \checkmark \\
 &= \int \cot^3 3x \operatorname{cosec}^2 3x dx - \int \cot^2 3x \cot 3x dx \checkmark \\
 &= \int \cot^3 3x \operatorname{cosec}^2 3x dx - \int \cot 3x (\operatorname{cosec}^2 3x - 1) dx \checkmark \\
 &= \int \cot^3 3x \operatorname{cosec}^2 3x dx - \int \cot 3x \operatorname{cosec}^2 3x dx + \int \cot 3x dx \checkmark \\
 &= -\frac{1}{3} \int [\cot 3x]^3 (-3 \operatorname{cosec}^2 3x) dx + \frac{1}{3} \int \cot 3x (-3 \operatorname{cosec}^2 3x) dx + \int \cot 3x dx \\
 &= -\frac{1}{12} \cot^4 3x + \frac{1}{6} \cot^2 3x + \frac{1}{3} \ln(\sin 3x) + C \checkmark \checkmark \checkmark
 \end{aligned}$$

ALTERNATIVE 2

$$\begin{aligned}
 \int \cot^5 3x dx &= \int (\cot^2 3x)^2 \cot 3x dx \checkmark \\
 &= \int (\operatorname{cosec}^2 3x - 1)^2 \cot 3x dx \checkmark \\
 &= \int (\operatorname{cosec}^4 3x - 2 \operatorname{cosec}^2 3x + 1) \cot 3x dx \checkmark \\
 &= \int \operatorname{cosec}^4 3x \cot 3x dx - 2 \int \operatorname{cosec}^2 3x \cot 3x dx + \int \cot 3x dx \checkmark \\
 u = \operatorname{cosec} 3x &\Rightarrow \frac{-1}{3} du = \operatorname{cosec} 3x \cot 3x dx \\
 &= \frac{-1}{3} \int u^3 du + \frac{2}{3} \int u du + \int \cot 3x dx \checkmark \checkmark \\
 &= \frac{-1}{12} u^4 + \frac{1}{3} u^2 + \frac{1}{3} \ln(\sin 3x) + C \checkmark \checkmark \checkmark \\
 &= \frac{-1}{12} \operatorname{cosec}^4 3x + \frac{1}{3} \operatorname{cosec}^2 3x + \frac{1}{3} \ln(\sin 3x) + C \checkmark
 \end{aligned}$$

ALTERNATIVE 3

$$\begin{aligned}
 \int \cot^5 3x dx &= \int \frac{\cos^5 3x}{\sin^5 3x} dx \checkmark \\
 &= \int (\cos^2 3x)^2 \cos 3x (\sin 3x)^{-5} dx \checkmark \\
 &= \int (1 - \sin^2 3x)^2 \cos 3x (\sin 3x)^{-5} dx \checkmark \\
 &= \int (1 - 2\sin^2 3x + \sin^4 3x) \cos 3x (\sin 3x)^{-5} dx \checkmark \\
 &= \int (\sin 3x)^{-5} \cos 3x dx - 2 \int (\sin 3x)^{-3} \cos 3x dx + \int \cot 3x dx \checkmark \checkmark \checkmark \\
 &= \frac{1}{3} \int (\sin 3x)^{-5} 3 \cos 3x dx - \frac{2}{3} \int (\sin 3x)^{-3} 3 \cos 3x dx + \int \cot 3x dx \checkmark \\
 &= \frac{(\sin 3x)^{-4}}{-12} + \frac{(\sin 3x)^{-2}}{3} + \frac{1}{3} \ln(\sin 3x) + C \checkmark \checkmark \\
 &= \frac{-1}{12} \operatorname{cosec}^4 3x + \frac{1}{3} \operatorname{cosec}^2 3x + \frac{1}{3} \ln(\sin 3x) + C
 \end{aligned}$$

ALTERNATIVE 4

$$\begin{aligned}
 \int \cot^5 3x dx &= \int \frac{\cos^5 3x}{\sin^5 3x} dx \checkmark \\
 &= \int \frac{(\cos^2 3x)^2 \cos 3x}{(\sin 3x)^5} dx \checkmark \\
 &= \int \frac{(1 - \sin^2 3x)^2 \cos 3x}{(\sin 3x)^5} dx \checkmark \\
 \text{let } u = \sin 3x &\Rightarrow \frac{du}{3 \cos 3x} = dx \\
 &= \int \frac{(1 - u^2)^2 \cos 3x}{u^5} \frac{du}{3 \cos 3x} \checkmark \\
 &= \frac{1}{3} \int \frac{(1 - 2u^2 + u^4)}{u^5} du \checkmark \\
 &= \frac{1}{3} \int \left(u^{-5} - 2u^{-3} + \frac{1}{u} \right) du \checkmark \\
 &= \frac{1}{3} \left[\frac{u^{-4}}{-4} - \frac{2u^{-2}}{-2} + \ln u \right] + C \checkmark \checkmark \checkmark \\
 &= \frac{(\sin 3x)^{-4}}{-12} + \frac{(\sin 3x)^{-2}}{3} + \frac{1}{3} \ln(\sin 3x) + C \checkmark \\
 &= \frac{-1}{12} \operatorname{cosec}^4 3x + \frac{1}{3} \operatorname{cosec}^2 3x + \frac{1}{3} \ln(\sin 3x) + C
 \end{aligned}$$

2.5

$$\int \cos^2\left(\frac{5x}{4}\right) dx = \frac{x}{2} + \frac{\sin\left(\frac{5x}{2}\right)}{5} + C \quad \checkmark\checkmark$$

ALTERNATIVE

$$\begin{aligned} \int \cos^2\left(\frac{5x}{4}\right) dx &= \int \left[\frac{1}{2} + \frac{1}{2} \cos\left(\frac{5x}{2}\right) \right] dx \\ &= \frac{x}{2} + \frac{\sin\left(\frac{5x}{2}\right)}{5} + C \quad \checkmark\checkmark \end{aligned}$$

(2)
[36]

QUESTION 3

3.1

$$\text{Let } \frac{x^2+1}{x^2(x+1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x+1} \quad \checkmark\checkmark\checkmark$$

$$x^2 + 1 = A(x+1) + Bx(x+1) + Cx^2 \quad \checkmark$$

$$x = 0 \Rightarrow A = 1 \quad \checkmark$$

$$x = -1 \Rightarrow C = 2 \quad \checkmark$$

$$x^2 + 1 = Ax + A + Bx^2 + Bx + Cx^2 \quad \checkmark$$

$$x^2: 1 = B + C \Rightarrow B = -1 \quad \checkmark$$

$$\int \frac{x^2+1}{x^3+x^2} dx = \int \frac{dx}{x^2} - \int \frac{dx}{x} + 2 \int \frac{dx}{x+1} \quad \checkmark$$

$$= -\frac{1}{x} - \ln x + 2 \ln(x+1) + C \quad \checkmark\checkmark\checkmark\checkmark \quad [\text{Two ticks for the first term}]$$

(12)

3.2

$$3+5x-2x^2 \over 2x^2+4x+1} -1 \checkmark$$

$$\frac{2x^2-5x-3}{9x+4} \checkmark$$

$$\therefore \frac{2x^2+4x+1}{3+5x-2x^2} = -1 + \frac{9x+4}{3+5x-2x^2} \checkmark$$

$$\text{Let } \frac{9x+4}{(1+2x)(3-x)} = \frac{A}{1+2x} + \frac{B}{3-x} \checkmark \checkmark$$

$$\Rightarrow 9+4x = A(3-x) + B(1+2x) \checkmark$$

$$x=3 \Rightarrow B=3 \checkmark$$

$$x=-\frac{1}{2} \Rightarrow A=2 \checkmark$$

$$\int \frac{2x^2+4x+1}{3+5x-2x^2} dx = -\int dx + 2 \int \frac{1}{1+2x} dx + 3 \int \frac{1}{3-x} dx \checkmark$$

$$= -x + \ln(1+2x) - 3 \ln(3-x) + C \checkmark \checkmark \checkmark$$

(12)

[24]

QUESTION 4

4.1

$$\frac{dy}{dx} - \frac{2}{x}y = x^2 \sec x \tan x \checkmark \checkmark$$

$$\text{IF} = e^{-2 \int \frac{dx}{x}} = e^{-2 \ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2} \checkmark \checkmark \checkmark$$

$$y \left(\frac{1}{x^2} \right) = \int \left(\frac{1}{x^2} \right) x^2 \sec x \tan x dx \checkmark \checkmark$$

$$\Rightarrow \frac{y}{x^2} = \int \sec x \tan x dx \checkmark$$

$$= \sec x + C \checkmark$$

$$\frac{2}{\left(\frac{\pi}{3}\right)^2} = \frac{1}{\cos\left(\frac{\pi}{3}\right)} + C \checkmark$$

$$C = -0.176 \checkmark$$

$$\therefore \frac{y}{x^2} = \sec x - 0.176 \checkmark$$

4.2 $m^2 + 2m + 2 = 0$ ✓

$m = -1 \pm i$ ✓

$y_c = e^{-x} (A \cos x + B \sin x)$ ✓

$y_p = Cx^2 + Dx + E$ ✓; $y'_p = 2Cx + D$ ✓ and $y''_p = 2C$ ✓

$\frac{1}{2}(2C) + 2Cx + D + Cx^2 + Dx + E = 2x^2 + 5x + 1$ ✓

x^2 : $C = 2$ ✓

x : $2C + D = 5 \Rightarrow D = 1$ ✓

Constant: $C + D + E = 1 \Rightarrow E = -2$ ✓

$y_p = 2x^2 + x - 2$ ✓

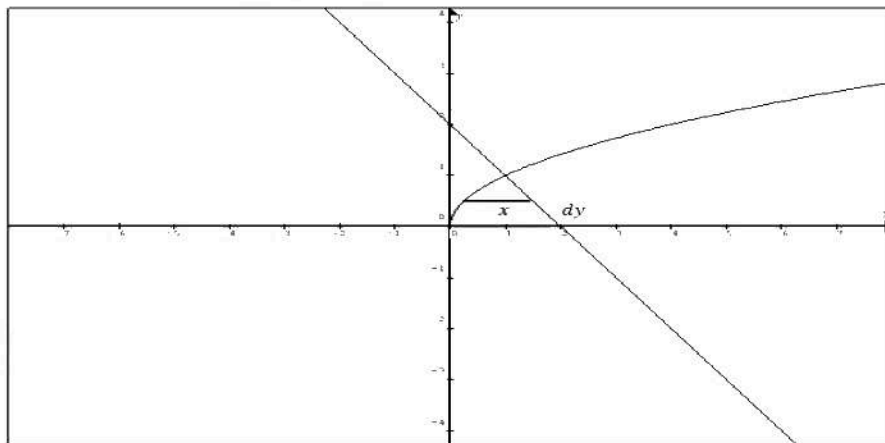
$y = y_c + y_p$

$y = e^{-x} (A \cos x + B \sin x) + 2x^2 + x - 2$ ✓

(12)
[24]

QUESTION 5

5.1 5.1.1



Graphs; ✓✓ Strip; ✓ Shaded area ✓

$\sqrt{x} = 2 - x$

$\Rightarrow x^2 = (2 - x)^2$

$\Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow x = 1$ or $x = 4$ ✓

$\Rightarrow y = 1$ or $y = -2$ ✓

$\therefore (1; 1)$ and $(4; -2)$

5.1.2

$$V_x = 2\pi \int_a^b y(x_1 - x_2) dy \quad \text{or} \quad V_x = 2\pi \int_a^b xy dy \quad \checkmark$$

$$= 2\pi \int_0^1 y(2 - y - y^2) dy \quad \checkmark \checkmark$$

$$= 2\pi \int_0^1 (2y - y^2 - y^3) dy \quad \checkmark$$

$$= 2\pi \left[y^2 - \frac{1}{3} y^3 - \frac{1}{4} y^4 \right]_0^1 \quad \checkmark \checkmark$$

$$= 2\pi \left[1 - \frac{1}{3} - \frac{1}{4} \right] \quad \checkmark$$

$$= \frac{5}{6} \pi \quad \checkmark$$

$$= 2.618 (\text{units})^3$$

(8)

5.2

5.2.1

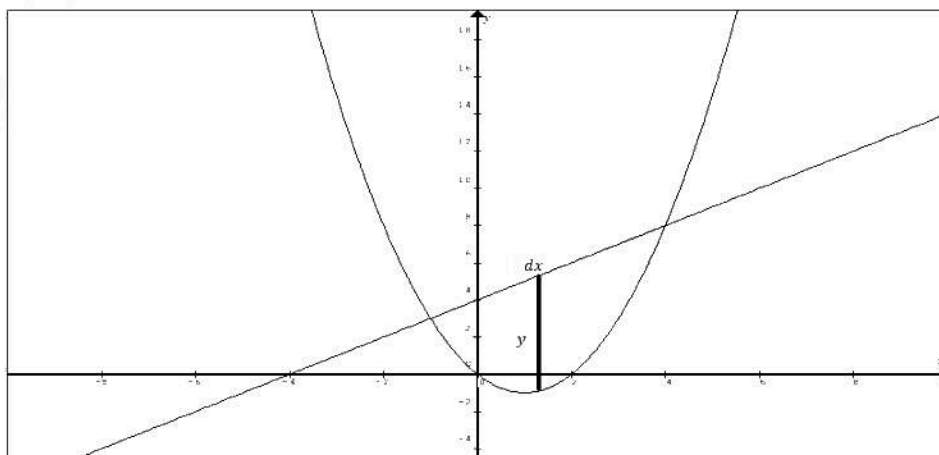
$$x^2 - 2x = x + 4$$

$$x^2 - 3x - 4 = 0$$

$$x = -1 \quad \text{or} \quad x = 4$$

$$y = 3 \quad \text{or} \quad y = 8$$

$$(-1; 3) \quad \checkmark \quad \text{and} \quad (4; 8) \quad \checkmark$$



Graphs; $\checkmark \checkmark$ Strip; $\checkmark$ Shaded area $\checkmark$

(6)

5.2.2

$$A_x = \int_a^b (y_1 - y_2) dx \quad \checkmark$$

$$A = \int_{-1}^4 (x + 4 - x^2 + 2x) dx \quad \checkmark \checkmark$$

$$= \int_{-1}^4 (-x^2 + 3x + 4) dx$$

$$= \left[-\frac{x^3}{3} + \frac{3x^2}{2} + 4x \right]_{-1}^4 \quad \checkmark$$

$$= -\frac{(4)^3}{3} + \frac{3(4)^2}{2} + 4(4) + \frac{(-1)^3}{3} - \frac{3(-1)^2}{2} - 4(-1) \quad \checkmark$$

$$= 20.833 (\text{units})^2 \quad \checkmark$$

$$= \frac{125}{6} (\text{units})^2$$

(6)

5.2.3

$$A_{m-y} = \int_a^b r \, dA \quad \checkmark$$

$$= \int_a^b x(y_1 - y_2) dx \quad \checkmark$$

$$= \int_{-1}^4 x(-x^2 + 3x + 4) dx \quad \checkmark \checkmark$$

$$= \int_{-1}^4 (-x^3 + 3x^2 + 4x) dx \quad \checkmark$$

$$= \left[-\frac{x^4}{4} + x^3 + 2x^2 \right]_{-1}^4 \quad \checkmark$$

$$= \left[-\frac{(4)^4}{4} + (4)^3 + 2(4)^2 + \frac{(-1)^4}{4} - (-1)^3 - 2(-1)^2 \right] \quad \checkmark$$

$$= 31.25 \quad \checkmark$$

$$= \frac{125}{4}$$

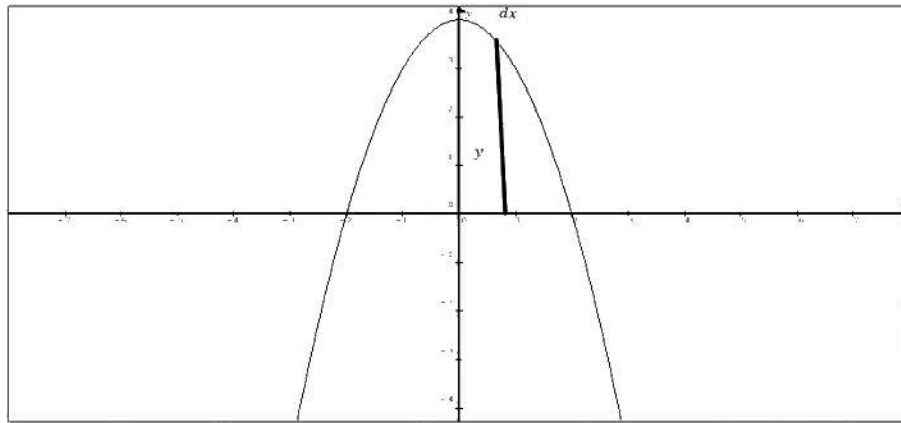
$$\bar{x} = \frac{31.25}{20.833} \quad \checkmark$$

$$= 1.500 \quad \checkmark$$

$$= \frac{3}{2}$$

5.3

5.3.1



Graph;✓ Strip;✓ Shaded area;✓ Intercepts✓

(4)

5.3.2

$$A_x = \int_a^b y dx \quad \checkmark$$

$$A = \int_{-2}^2 (4 - x^2) dx \quad \checkmark \checkmark$$

$$= 2 \int_0^2 (4 - x^2) dx$$

$$= 2 \left[4x - \frac{x^3}{3} \right]_0^2 \quad \checkmark$$

$$= 2 \left[4(2) - \frac{(2)^3}{3} \right] \quad \checkmark$$

$$= 10.667 (\text{units})^2 \quad \checkmark$$

$$= \frac{32}{3} (\text{units})^2$$

ALTERNATIVE

$$A_x = \int_a^b (y_1 - y_2) dx \quad \checkmark$$

$$= \int_{-2}^2 (4 - x^2) dx \quad \checkmark \checkmark$$

$$= \left[4x - \frac{x^3}{3} \right]_{-2}^2 \quad \checkmark$$

$$= \left[4(2) - \frac{(2)^3}{3} \right] - \left[4(-2) - \frac{(-2)^3}{3} \right] \quad \checkmark$$

$$= \frac{32}{3} (\text{units})^2 \quad \checkmark$$

$$= 10.667 (\text{units})^2$$

(6)

5.3.3

$$\begin{aligned}
 I_y &= \int_a^b r^2 dA \checkmark \\
 &= \int_a^b x^2(y_1 - y_2) dx \checkmark \\
 &= \int_{-2}^2 x^2(4 - x^2) dx \checkmark \checkmark \\
 &= \int_{-2}^2 (4x^2 - x^4) dx \checkmark \\
 &= \left[\frac{4x^3}{3} - \frac{x^5}{5} \right]_{-2}^2 \checkmark \\
 &= \left[\frac{4(2)^3}{3} - \frac{(2)^5}{5} \right] - \left[\frac{4(-2)^3}{3} - \frac{(-2)^5}{5} \right] \checkmark \\
 &= \frac{128}{15} (\text{units})^4 \checkmark \\
 &= 8.533(\text{units})^4
 \end{aligned}$$

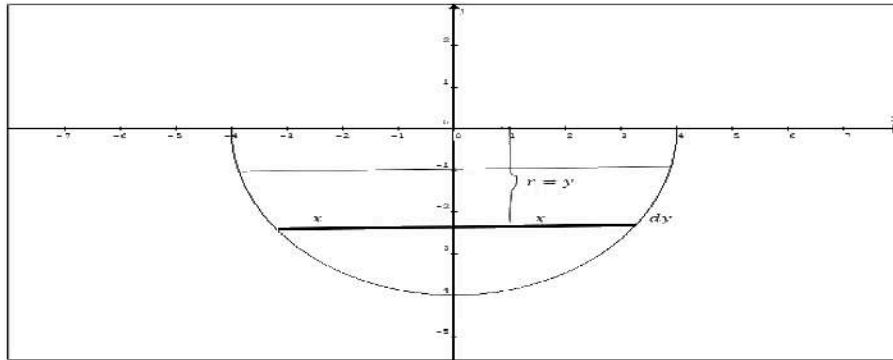
ALTERNATIVE

$$\begin{aligned}
 I_y &= \int_a^b r^2 dA \checkmark \\
 &= \int_a^b x^2(y_1 - y_2) dx \checkmark \\
 &= 2 \int_0^2 x^2(4 - x^2) dx \checkmark \checkmark \\
 &= 2 \int_0^2 (4x^2 - x^4) dx \checkmark \\
 &= 2 \left[\frac{4x^3}{3} - \frac{x^5}{5} \right]_0^2 \checkmark \\
 &= 2 \left[\frac{4(2)^3}{3} - \frac{(2)^5}{5} \right] - 2 \left[\frac{4(0)^3}{3} - \frac{(0)^5}{5} \right] \checkmark \\
 &= \frac{128}{15} (\text{units})^4 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 5.3.4 \quad I_y &= \frac{128}{15} \div \frac{32}{3} A \quad \checkmark \\
 &= \frac{4}{5} A \quad \checkmark \\
 &= 0.8A
 \end{aligned}$$

(2)

5.4 5.4.1



$$y = ax^2 - 4$$

$$0 = a(4)^2 - 4$$

$$a = \frac{4}{16} = \frac{1}{4} \quad \checkmark$$

$$y = \frac{1}{4}x^2 - 4 \quad \checkmark$$

$$x = 2\sqrt{y+4}$$

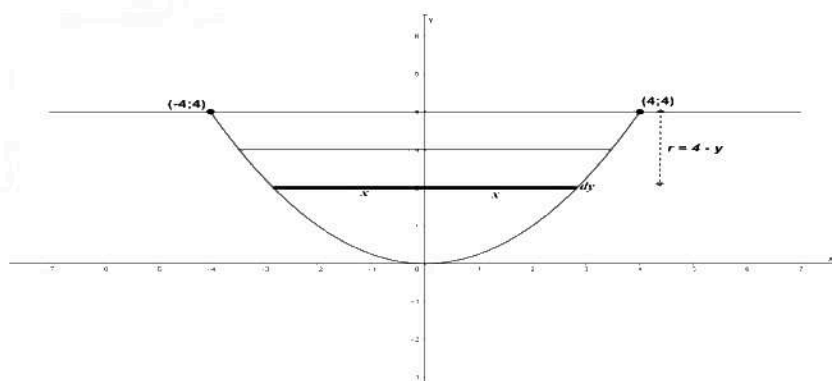
Strip $\checkmark$

Retaining wall at (0; -1) $\checkmark$

x - intercepts at (-4; 0) and (4; 0) $\checkmark$

Bottom at (0; -4) $\checkmark$

ALTERNATIVE



$$y = ax^2$$

$$4 = 16a$$

$$\Rightarrow a = \frac{4}{16} = \frac{1}{4} \quad \checkmark$$

$$\therefore y = \frac{1}{4}x^2 \quad \checkmark$$

Strip $\checkmark$

Retaining wall at (0; 3) $\checkmark$

Coordinates (-4; 4) and (4; 4) $\checkmark$

water level at (0; 4) $\checkmark$

5.4.2 First moment = $\int r dA$ ✓

$$= 4 \int_{-4}^{-1} y \sqrt{y+4} dy \quad \checkmark \checkmark \quad \text{let } u = y+4 \quad y = u-4$$

$$= 4 \int_{-4}^{-1} (u-4) u^{\frac{1}{2}} du \quad \checkmark$$

$$= 4 \int_{-4}^{-1} \left(u^{\frac{3}{2}} - 4u^{\frac{1}{2}} \right) du \quad \checkmark$$

Note
 $u = 0$ and $u = 3$ can also
 be the limits

$$= 4 \left[\frac{2}{5} u^{\frac{5}{2}} - \frac{8}{3} u^{\frac{3}{2}} \right]_{-4}^{-1} \quad \checkmark$$

$$= 4 \left[\frac{2}{5} (y+4)^{\frac{5}{2}} - \frac{8}{3} (y+4)^{\frac{3}{2}} \right]_{-4}^{-1}$$

$$= 4 \left[\frac{2}{5} (3)^{\frac{5}{2}} - \frac{8}{3} (3)^{\frac{3}{2}} \right] \quad \checkmark$$

$$= -30.484 \quad \checkmark$$

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5.4.3 Second moment = $\int r^2 dA$ ✓

$$= 4 \int_{-4}^{-1} y^2 \sqrt{y+4} dy \quad \checkmark$$

$$= 4 \int_{-4}^{-1} (u-4)^2 u^{\frac{1}{2}} du \quad \checkmark$$

Note
 $u = 0$ and $u = 3$ can also
 be the limits

$$= 4 \int_{-4}^{-1} (u^2 - 8u + 16) u^{\frac{1}{2}} du \quad \checkmark$$

$$= 4 \int_{-4}^{-1} \left(u^{\frac{5}{2}} - 8u^{\frac{3}{2}} + 16u^{\frac{1}{2}} \right) du \quad \checkmark$$

$$= 4 \left[\frac{2}{7} u^{\frac{7}{2}} - \frac{16}{5} u^{\frac{5}{2}} + \frac{32}{3} u^{\frac{3}{2}} \right]_{-4}^{-1} \quad \checkmark$$

$$= 4 \left[\frac{2}{7} (y+4)^{\frac{7}{2}} - \frac{16}{5} (y+4)^{\frac{5}{2}} + \frac{32}{3} (y+4)^{\frac{3}{2}} \right]_{-4}^{-1}$$

$$= 4 \left[\frac{2}{7} (3)^{\frac{7}{2}} - \frac{16}{5} (3)^{\frac{5}{2}} + \frac{32}{3} (3)^{\frac{3}{2}} \right] \quad \checkmark$$

$$= 75.616 \quad \checkmark$$

$$\frac{y}{x} = \frac{75.616}{-30.484} = -2.481 \quad \checkmark \checkmark$$

QUESTION 6

$$6.1 \quad \frac{dy}{dx} = \frac{1}{\cos x} \times -\sin x = -\tan x \checkmark \checkmark$$

$$\left(\frac{dy}{dx}\right)^2 + 1 = \tan^2 x + 1 = \sec^2 x \checkmark$$

$$S = \int_a^b \sqrt{\left(\frac{dy}{dx}\right)^2 + 1} dx \checkmark$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{\sec^2 x} dx \checkmark \checkmark$$

$$= \int_0^{\frac{\pi}{3}} \sec x dx \checkmark$$

$$= \ln(\sec x + \tan x) \Big|_0^{\frac{\pi}{3}} \checkmark$$

$$= \ln\left(\frac{1}{\cos \frac{\pi}{3}} + \tan \frac{\pi}{3}\right) - \ln\left(\frac{1}{\cos 0} + \tan 0\right) \checkmark$$

$$= 1.317 \checkmark$$

(10)

$$6.2 \quad \frac{dy}{dt} = 2t \quad \checkmark \quad \text{and} \quad \frac{dx}{dt} = 3t^2 \quad \checkmark$$

$$\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2 = 9t^4 + 4t^2 \quad \checkmark$$

$$A_x = 2\pi \int_a^b y \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt \quad \checkmark$$

$$= 2\pi \int_0^1 t^2 \sqrt{9t^4 + 4t^2} dt \quad \checkmark \checkmark \checkmark$$

$$= 2\pi \int_0^1 t^2 \sqrt{t^2(9t^2 + 4)} dt \quad \checkmark$$

$$= 2\pi \int_0^1 t^2 t \sqrt{9t^2 + 4} dt \quad \checkmark$$

$$\text{Let } u = 9t^2 + 4 \Rightarrow t^2 = \frac{u-4}{9} \quad \text{and} \quad dt = \frac{du}{18t}$$

$$= \frac{2\pi}{18} \int_0^1 \frac{u-4}{9} u^{\frac{1}{2}} du \quad \checkmark$$

$$= \frac{\pi}{81} \int_0^1 \left(u^{\frac{3}{2}} - 4u^{\frac{1}{2}}\right) du \quad \checkmark$$

$$= \frac{\pi}{81} \left[\frac{2}{5} u^{\frac{5}{2}} - \frac{8}{3} u^{\frac{3}{2}} \right]_0^1 \quad \checkmark$$

$$= \frac{\pi}{81} \left[\frac{2}{5} (9t^2 + 4)^{\frac{5}{2}} - \frac{8}{3} (9t^2 + 4)^{\frac{3}{2}} \right]_0^1$$

$$= \frac{\pi}{81} \left[\frac{2}{5} (13)^{\frac{5}{2}} - \frac{8}{3} (13)^{\frac{3}{2}} \right] - \frac{\pi}{81} \left[\frac{2}{5} (4)^{\frac{5}{2}} - \frac{8}{3} (4)^{\frac{3}{2}} \right] \quad \checkmark$$

$$= 4.936 \quad \checkmark$$

Note

$u = 4$ and $u = 13$ can also

be the limits

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TOTAL: 100