



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

30 March 2023

This marking guideline consists of 20 pages.

QUESTION 1

1.1 1.1.1 $\frac{\partial z}{\partial x} = f_x(x, y) = \frac{1}{e^{-x} + e^{-y}} \times -e^{-x}$ ✓
 $\left. \frac{\partial z}{\partial x} \right|_{\substack{x=0 \\ y=0}} = f_x(0, 0) = \frac{1}{e^0 + e^0} \times -e^0 = -\frac{1}{2}$ ✓ (2)

1.1.2 $\frac{\partial z}{\partial y} = f_y(x, y) = \frac{1}{e^{-x} + e^{-y}} \times -e^{-y}$ ✓ (1)

1.2 1.2.1 $x = a \cos 2\theta$ and $y = a \sin \theta$
 $\Rightarrow \frac{dx}{d\theta} = -2a \sin 2\theta$ and $\frac{dy}{d\theta} = a \cos \theta$ ✓

$$\therefore \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a \cos \theta}{-2a \sin 2\theta}$$

$$\therefore \frac{\cos \theta}{-4 \cos \theta \sin \theta}$$

$$\therefore \frac{\operatorname{cosec} \theta}{-4}$$

(2)

1.2.2 $\frac{d^2 y}{dx^2} = \frac{\frac{d}{d\theta} \left(\frac{dy}{dx} \right)}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta} \left(\frac{-1}{4} \operatorname{cosec} \theta \right)}{\frac{dx}{d\theta}} \left(\frac{1}{-2a \sin 2\theta} \right)$
 $= \frac{-1}{4} (-\operatorname{cosec} \theta \cot \theta) \left(\frac{1}{-2a \sin 2\theta} \right)$

$$\therefore \left(\frac{\cos \theta}{4 \sin^2 \theta} \right) \left(\frac{1}{-4a \sin \theta \cos \theta} \right)$$

$$= \frac{-1}{16a \sin^3 \theta}$$

(1)
[6]

QUESTION 2

2.1 $3x^2 + 6x + 7$

$$\hookrightarrow 3 \left[x^2 + 2x + \frac{7}{3} \right]$$

$$\hookrightarrow 3 \left[x^2 + 2x + (1)^2 - (1)^2 + \frac{7}{3} \right]$$

$$\hookrightarrow 3 \left[(x+1)^2 + \frac{4}{3} \right]$$

$$\hookrightarrow 3(x+1)^2 + 4$$

$$\therefore \int \frac{2}{\sqrt{7+6x+3x^2}} dx = \int \frac{2}{\sqrt{3(x+1)^2+4}} dx$$

$$\hookrightarrow \frac{2}{\sqrt{3}} \ln \left[\sqrt{3}(x+1) + \sqrt{3(x+1)^2+4} \right] + C$$

(4)

2.2 $f(x) = \tan^{-1} x \wedge g'(x) = x$

$$f'(x) = \frac{1}{x^2+1} \wedge g(x) = \frac{x^2}{2}$$

$$\therefore \int x \tan^{-1} x dx = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{x^2+1} dx$$

$$\text{but } \frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1} \hookrightarrow \text{the long division}$$

$$\int x \tan^{-1} x dx = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left(1 - \frac{1}{x^2+1} \right) dx$$

$$\hookrightarrow \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} [x - \tan^{-1} x] + C \hookrightarrow$$

ALTERNATIVE for completing the square method

$$ax^2 + bx + c = \frac{4ac - b^2}{4a} + a \left(x + \frac{b}{2a} \right)^2$$

$$\therefore 3x^2 + 6x + 7 = \frac{4(3)(7) - (6)^2}{4(3)} + 3 \left(x + \frac{6}{2 \times 3} \right)^2$$

$$\hookrightarrow 4 + 3(x+1)^2$$

ALTERNATIVE

D	I
$\tan^{-1} x$ +	x
$\frac{1}{x^2+1}$ -	$\frac{x^2}{2}$ ✓

$$\int x \tan^{-1} x \, dx = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{x^2+1} \, dx$$

$$\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$$

$$\int x \tan^{-1} x \, dx = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \left(1 - \frac{1}{x^2+1} \right) dx$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} [x - \tan^{-1} x] + C$$

(4)

2.3

$$\int \cot^5 7x \, dx = \int \cot^3 7x \cot^2 7x \, dx \quad \checkmark$$

$$= \int \cot^3 7x (\cos^2 7x - 1) \, dx$$

$$= \int \cot^3 7x \cos^2 7x \, dx - \int \cot^3 7x \, dx$$

$$= \int \cot^3 7x \cos^2 7x \, dx - \int \cot^2 7x \cot 7x \, dx \quad \checkmark$$

$$= \int \cot^3 7x \cos^2 7x \, dx - \int \cot 7x (\cos^2 7x - 1) \, dx$$

$$= \int \cot^3 7x \cos^2 7x \, dx - \int \cot 7x \cos^2 7x \, dx + \int \cot 7x \, dx \quad \checkmark$$

$$u = \cot 7x \Rightarrow du = -7 \cos^2 7x \, dx$$

$$= \frac{-1}{7} \int u^3 \, du + \frac{1}{7} \int u \, du + \int \cot 7x \, dx \quad \checkmark \quad [\text{For u-substitution}]$$

$$= \frac{-1}{28} u^4 + \frac{1}{14} u^2 + \frac{1}{7} \ln(\sin 7x) + C$$

$$= \frac{-1}{28} \cot^4 7x + \frac{1}{14} \cot^2 7x + \frac{1}{7} \ln(\sin 7x) + C \quad \checkmark \text{ [Tick is for } u = \cot 7x \text{]}$$

ALTERNATIVE 1

$$\int \cot^5 7x dx = \int (\cot^2 7x)^2 \cot 7x dx \quad \checkmark$$

$$= \int (\operatorname{cosec}^2 7x - 1)^2 \cot 7x dx$$

$$= \int (\operatorname{cosec}^4 7x - 2 \operatorname{cosec}^2 7x + 1) \cot 7x dx \quad \checkmark$$

$$= \int \operatorname{cosec}^4 7x \cot 7x dx - 2 \int \operatorname{cosec}^2 7x \cot 7x dx + \int \cot 7x dx \quad \checkmark$$

$$u = \operatorname{cosec} 7x \quad du = -7 \operatorname{cosec} 7x \cot 7x$$

$$= \frac{-1}{7} \int u^3 du + \frac{2}{7} \int u du + \int \cot 7x dx \quad \checkmark \text{ [For correct u-substitution]}$$

$$= \frac{-1}{28} u^4 + \frac{1}{7} u^2 + \frac{1}{7} \ln(\sin 7x) + C$$

$$= \frac{-1}{28} \operatorname{cosec}^4 7x + \frac{1}{7} \operatorname{cosec}^2 7x + \frac{1}{7} \ln(\sin 7x) + C \quad [\checkmark \text{ For } u = \operatorname{cosec} 7x]$$

ALTERNATIVE 2

$$\int \cot^5 7x dx = \int \frac{\cos^5 7x}{\sin^5 7x} dx$$

$$\int \frac{(\cos^2 7x)^2 \cos 7x}{(\sin 7x)^5} dx$$

$$\int \frac{(1 - \sin^2 7x)^2 \cos 7x}{(\sin 7x)^5} dx$$

$$\text{Let } u = \sin 7x \quad \frac{du}{7 \cos 7x} = dx$$

$$\int \frac{(1 - u^2)^2 \cos 7x}{u^5} \frac{du}{7 \cos 7x}$$

$$\textcircled{1} \frac{1}{7} \int \frac{(1-2u^2+u^4)}{u^5} du$$

$$\textcircled{1} \frac{1}{7} \int \left(u^{-5} - 2u^{-3} + \frac{1}{u} \right) du$$

$$\textcircled{1} \frac{1}{7} \left[\frac{u^{-4}}{-4} - \frac{2u^{-2}}{-2} + \ln u \right] + C$$

$$\textcircled{1} \frac{(\sin 7x)^{-4}}{-28} + \frac{(\sin 7x)^{-2}}{7} + \frac{1}{7} \ln(\sin 7x) + C$$

$$= \frac{-1}{28} \operatorname{cosec}^4 7x + \frac{1}{7} \operatorname{cosec}^2 7x + \frac{1}{7} \ln(\sin 7x) + C \quad (5)$$

2.4

$$\int \sin^4 4x dx = \int (\sin^2 4x)^2 dx \quad \checkmark \quad (4)$$

$$= \int \left[\frac{1}{2}(1 - \cos 8x) \right]^2 dx$$

$$= \frac{1}{4} \int (1 - \cos 8x)^2 dx$$

$$= \frac{1}{4} \int (1 - 2\cos 8x + \cos^2 8x) dx \quad \checkmark$$

$$= \frac{1}{4} \int \left(1 - 2\cos 8x + \frac{1}{2} + \frac{1}{2} \cos 16x \right) dx \quad \checkmark$$

$$= \frac{1}{4} \int \left(\frac{3}{2} - 2\cos 8x + \frac{1}{2} \cos 16x \right) dx$$

$$= \frac{1}{4} \left[\frac{3}{2}x - \frac{\sin 8x}{4} + \frac{\sin 16x}{32} \right] + C \quad \checkmark$$

$$= \frac{3}{8}x - \frac{\sin 8x}{16} + \frac{\sin 16x}{128} + C$$

ALTERNATIVE

$$\int \sin^4 4x dx = \int (\sin^2 4x)^2 dx$$

$$\textcircled{1} \int \left(\frac{1}{2} - \frac{1}{2} \cos 8x \right)^2 dx$$

$$\textcircled{1} \int \left(\frac{1}{4} - \frac{1}{2} \cos 8x + \frac{1}{4} \cos^2 4x \right) dx$$

$$\frac{1}{4}x - \frac{\sin 8x}{16} + \frac{1}{4} \left[\frac{x}{2} + \frac{\sin 16x}{32} \right] + C$$

2.5 $\int \tan\left(\frac{x}{2}\right) dx$

Let $u = \frac{x}{2}$ then $2du = dx$
 $\int \tan\left(\frac{x}{2}\right) dx = 2 \int \tan u du$

$$2 \ln \sec u + C$$

$$2 \ln \sec\left(\frac{x}{2}\right) + C$$

ALTERNATIVE

$$\int \tan \frac{x}{2} dx = \frac{\ln \sec\left(\frac{x}{2}\right)}{\frac{1}{2}} + C$$

$$2 \ln \sec\left(\frac{x}{2}\right) + C$$

(1)
[18]

QUESTION 3

3.1 $\int \frac{x^2 + 2x + 1}{x^3 + 4x} dx$

(6)

$$x^3 + 4x = x(x^2 + 4) \checkmark$$

$$\text{Let } \frac{x^2 + 2x + 1}{x(x^2 + 4)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

$$x^2 + 2x + 1 = A(x^2 + 4) + x(Bx + C) \checkmark$$

$$\text{Let } A = \frac{1}{4}$$

$$x^2 + 2x + 1 = A x^2 + 4A + B x^2 + C x$$

$$x^2: 1 = A + B \Rightarrow B = \frac{3}{4} \checkmark \text{ and } x: 2 = C$$

$$\begin{aligned}
 \int \frac{x^2 + 2x + 1}{x^3 + 4x} dx &= \int \left(\frac{1}{\frac{4}{x}} + \frac{\frac{3}{4}x + 2}{x^2 + 4} \right) dx \\
 &= \frac{1}{4} \int \frac{dx}{x} + \frac{3}{4} \int \frac{x dx}{x^2 + 4} + 2 \int \frac{dx}{x^2 + 4} \checkmark \\
 &= \frac{1}{4} \ln x + \frac{3}{8} \ln(x^2 + 4) + \arctan\left(\frac{x}{2}\right) + C \checkmark
 \end{aligned}$$

3.2

$$\int \frac{x^2 + 5x - 1}{x^3 + 3x^2} dx$$

$$x^3 + 3x^2 = x^2(x + 3)$$

$$\text{Let } \frac{x^2 + 5x - 1}{x^2(x + 3)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x + 3} \checkmark$$

$$x^2 + 5x - 1 = A(x + 3) + Bx(x + 3) + Cx^2$$

$$\text{Let } A = \frac{-1}{3}$$

$$\text{Let } x = -3 \quad C = \frac{-7}{9}$$

$$x^2 + 5x - 1 = Ax + 3A + Bx^2 + 3Bx + Cx^2$$

$$x^2 : 1 = B + C \Rightarrow B = \frac{16}{9}$$

$$\begin{aligned}
 \int \frac{x^2 + 5x - 1}{x^3 + 3x^2} dx &= \int \left(\frac{-\frac{1}{3}}{x^2} + \frac{\frac{16}{9}x}{x} - \frac{\frac{7}{9}}{x + 3} \right) dx \\
 &= -\frac{1}{3} \int \frac{dx}{x^2} + \frac{16}{9} \int \frac{dx}{x} - \frac{7}{9} \int \frac{dx}{x + 3} \checkmark \\
 &= \frac{1}{3x} + \frac{16}{9} \ln x - \frac{7}{9} \ln(x + 3) + C \checkmark
 \end{aligned}$$

ALTERNATIVE

$$\int \frac{x^2 + 5x - 1}{x^3 + 3x^2} dx$$

$$x^3 + 3x^2 = x^2(x + 3) \checkmark$$

$$\frac{x^2 + 5x - 1}{x^2(x + 3)} = \frac{Ax + B}{x^2} + \frac{C}{x + 3}$$

$$x^2 + 5x - 1 = (Ax + B)(x + 3) + Cx^2$$

$$\text{Let } x = 0 \quad B = \frac{-1}{3}$$

$$\text{Let } x = -3 \quad C = \frac{-7}{9}$$

$$x^2 + 5x - 1 = Ax^2 + 3Ax + Bx + 3B + Cx^2$$

$$x^2 : 1 = A + C \quad A = \frac{16}{9}$$

$$\therefore \int \frac{x^2 + 5x - 1}{x^2(x + 3)} dx = \int \left(\frac{\frac{16}{9}x - \frac{1}{3}}{x^2} + \frac{\frac{-7}{9}}{x + 3} \right) dx$$

$$\int \frac{16}{9} \frac{1}{x} dx + \int \frac{-1}{3} \frac{1}{x^2} dx + \int \frac{-7}{9} \frac{1}{x + 3} dx$$

$$\frac{16}{9} \ln x + \frac{1}{3x} - \frac{7}{9} \ln(x + 3) + C$$

(6)
[12]

QUESTION 4

$$4.1 \quad \frac{dy}{dx} + \frac{2}{x}y = \frac{\cos^2 x}{x^2} \quad \checkmark$$

$$\text{If } = e^{2 \int \frac{dx}{x}} = e^{2 \ln x} = e^{\ln x^2} = x^2 \quad \checkmark \quad \checkmark$$

$$y x^2 = \int \frac{\cos^2 x}{x^2} \times x^2 dx$$

$$x^2 y = \int \cos^2 x dx$$

$$= \frac{1}{2}x + \frac{\sin 2x}{4} + C \quad \checkmark$$

(5)

$$4.2 \quad r^2 + r - 2 = 0$$

(7)

$$r = 1 \text{ or } r = -2 \quad \checkmark$$

$$y_c = Ae^x + Be^{-2x}$$

$$y_p = Cx + D \quad \checkmark \quad y_p' = C \wedge y_p'' = 0$$

$$\therefore 0 + C - 2Cx - 2D = \frac{1}{2}x$$

$$x: -2C = \frac{1}{2}C = \frac{-1}{4}$$

$$\text{Constant: } C - 2D = 0 \quad D = \frac{-1}{8}$$

$$y_p = \frac{-1}{4}x - \frac{1}{8}$$

$$y = A e^x + B e^{-2x} - \frac{1}{4}x - \frac{1}{8}$$

But $y=1$ when $x=0$ $\frac{9}{8} = A+B$ Eqn 1

$$\frac{dy}{dx} = A e^x - 2B e^{-2x} - \frac{1}{4}$$

But $\frac{dy}{dx}=0$ when $x=0$ $\frac{1}{4} = A-2B$ Eqn 2

$$\text{Eqn 1} - \text{Eqn 2 } B = \frac{7}{24}$$

$$\text{Subst. } B = \frac{7}{24} \in \text{Eqn 1 } A = \frac{5}{6}$$

$$y = \frac{5}{6}e^x + \frac{7}{24}e^{-2x} - \frac{1}{4}x - \frac{1}{8}$$

[12]

QUESTION 5

5.1 5.1.1

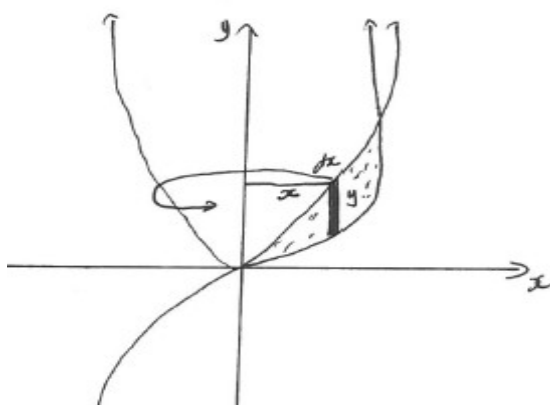
$$x^3 - 2x^2 = 0$$

$$x^2(x-2) = 0$$

$$x = 0 \text{ or } x = 2$$

$$y = 0 \quad y = 8$$

$$(0;0) \checkmark \text{ and } (2;8) \checkmark$$



Vertical strip

Enclosed area

Graph of $y = x^3$ (1/2✓)Graph of $y = 2x^2$ (1/2✓)

(3)

5.1.2

$$dV_y = 2\pi xy dx \quad V_y = 2\pi \int_a^b xy dx \quad V_y = 2\pi \int_a^b x(y_1 - y_2) dx$$

(3)

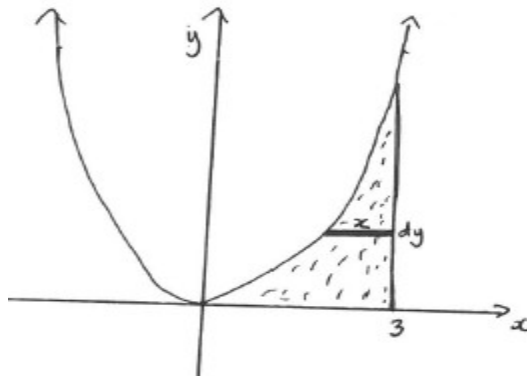
$$V_y = 2\pi \int_0^2 x(2x^2 - x^3) dx \checkmark$$

$$\begin{aligned}
 &= 2\pi \int_0^2 (2x^3 - x^4) dx \\
 &= 2\pi \left[\frac{1}{2}x^4 - \frac{1}{5}x^5 \right]_0^2 \checkmark \\
 &= 2\pi \left[\frac{1}{2}(2)^4 - \frac{1}{5}(2)^5 - 0 \right] \\
 &= 10.053(\text{units})^2 \checkmark
 \end{aligned}$$

$$\therefore \frac{16}{5}\pi(\text{units})^2$$

5.2

5.2.1



Horizontal strip

Enclosed area

Graph of $x=3$ ✓Graph of $y=3x^2$ ✓

(2)

5.2.2

$$\begin{aligned}
 A_y &= \int_a^b (x_1 - x_2) dy \\
 A &= \int_0^{27} \left(3 - \frac{1}{\sqrt{3}} y^{\frac{1}{2}} \right) dy \checkmark \\
 &= \left[3y - \frac{2}{3\sqrt{3}} y^{\frac{3}{2}} \right]_0^{27} \\
 &= \left[3(27) - \frac{2}{3\sqrt{3}} (27)^{\frac{3}{2}} - 0 \right]
 \end{aligned}$$

$$\therefore 27(\text{units})^2$$

ALTERNATIVE for vertical strip

$$A_x = \int_a^b (y_1 - y_2) dx$$

$$\therefore \int_0^3 3x^2 dx$$

$$\therefore [x^3]_0^3$$

$$\therefore (3)^3 - (0)^3$$

$$\therefore 27(\text{units})^2$$

(3)

5.2.3

$$dA_{\text{max}} = r dA$$

$$\therefore A_{\text{max}} = \int_a^b y(x_1 - x_2) dy$$

$$A_{\text{max}} = \int_0^{27} y \left(3 - \frac{1}{\sqrt{3}} y^{\frac{1}{2}} \right) dy \quad \checkmark$$

$$= \int_0^{27} \left(3y - \frac{1}{\sqrt{3}} y^{\frac{3}{2}} \right) dy \quad \checkmark$$

$$= \left[\frac{3}{2} y^2 - \frac{2}{5\sqrt{3}} y^{\frac{5}{2}} \right]_0^{27} \quad \checkmark$$

$$= \left[\frac{3}{2} (27)^2 - \frac{2}{5\sqrt{3}} (27)^{\frac{5}{2}} - 0 \right] \quad \checkmark$$

$$= 218.7 (\text{units})^3$$

$$\bar{y} = \frac{218.7}{27} = 8.1 \text{ units} \quad \checkmark$$

(5)

5.3

5.3.1

$$y^2 - y - 2 = 0$$

$$(y+1)(y-2) = 0$$

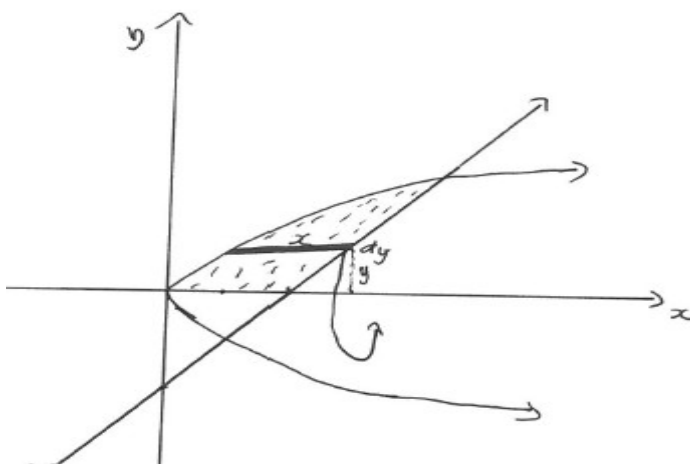
$$y = -1 \text{ or } y = 2$$

$$x = 1 \quad x = 4$$

$$(1; -1) \quad (1/2 \checkmark) \text{ and } (4; 2) \quad (1/2 \checkmark)$$

Horizontal strip (1/2✓)

Enclosed area (1/2✓)

Graph of $x = y + 2$ (1/2✓)Graph of $x = y^2$ (1/2✓)

(3)

5.3.2

$$dV_x = 2\pi xy dy \quad \vee \quad V_x = 2\pi \int_a^b xy dy \quad \vee \quad V_x = 2\pi \int_a^b y(x_1 - x_2) dy$$

$$\begin{aligned} V_x &= 2\pi \int_0^2 y(y + 2 - y^2) dy \\ &= 2\pi \int_0^2 (y^2 + 2y - y^3) dy \quad \checkmark \\ &= 2\pi \left[\frac{y^3}{3} + y^2 - \frac{y^4}{4} \right]_0^2 \\ &= 2\pi \left[\frac{(2)^3}{3} + (2)^2 - \frac{(2)^4}{4} - 0 \right] \quad \checkmark \\ &= 16.755 (\text{units})^3 \quad \checkmark \end{aligned}$$

$$\therefore \frac{16}{3} \pi (\text{units})^3 \quad (3)$$

5.3.3

$$\begin{aligned} I_x &= \int_a^b r^2 dm = \int_a^b r^2 dV \\ &\therefore 2\pi \int_0^2 y^2 \times y(x_1 - x_2) dy \\ &\therefore 2\pi \int_0^2 y^3(y + 2 - y^2) dy \\ &\therefore 2\pi \int_0^2 (y^4 + 2y^3 - y^5) dy \\ &\therefore 2\pi \left[\frac{y^5}{5} + \frac{y^4}{2} - \frac{y^6}{6} \right]_0^2 \\ &\therefore 2\pi \left[\frac{32}{5} + 8 - \frac{64}{6} \right] - 2\pi [0 + 0 - 0] \\ &\therefore \frac{112}{15} \pi \rho \\ &\therefore 23,457 \rho \quad (5) \end{aligned}$$

$$5.3.4 \quad I_x = 23,457 \times \frac{m}{V}$$

$$= \frac{23,457}{16,755} m$$

$$= 1,4 m$$

$$= \frac{7}{5} m$$

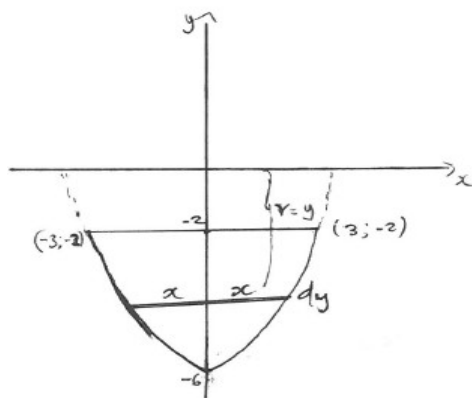
(1)

5.4 Two alternatives are given for this question.

ALTERNATIVE 1

5.4.1

(3)



Strip (1/2✓)

Top of wall at (0;-2) (1/2✓)

Bottom of wall at (0;-6) (1/2✓)

(-3;-2) and (3;-2) (1/2✓)

$$y = ax^2 - 6$$

$$-2 = a(3)^2 - 6$$

$$\Rightarrow a = \frac{4}{9} \quad (1/2✓)$$

$$\therefore y = \frac{4}{9}x^2 - 6$$

$$x = \frac{3}{2}\sqrt{y+6} \quad (1/2\checkmark)$$

5.4.2

$$\begin{aligned} \text{First moment} &= \int_a^b r dA \\ &= 3 \int_{-6}^{-2} y \sqrt{y+6} \, dy \checkmark \end{aligned}$$

$$\checkmark 3 \int_0^4 (u-6) u^{\frac{1}{2}} du$$

$$\checkmark 3 \int_0^4 \left(u^{\frac{3}{2}} - 6u^{\frac{1}{2}} \right) du$$

$$\begin{aligned} \checkmark 3 \left[\frac{2u^{\frac{5}{2}}}{5} - 4u^{\frac{3}{2}} \right]_0^4 \\ = 3 \left[\frac{2}{5}(4)^{\frac{5}{2}} - 4(4)^{\frac{3}{2}} \right] \checkmark \end{aligned}$$

$$= -57.6 \checkmark$$

$$\text{Let } u = y+6 \quad y = u-6$$

$$y = -2 \quad u = 4$$

$$y = -6 \quad u = 0$$

$$\checkmark -\frac{288}{5} \quad (4)$$

5.4.3

$$\text{Second moment} = \int r^2 dA \checkmark \quad (5)$$

$$= 3 \int_{-6}^{-2} y^2 \sqrt{y+6} \, dy \checkmark$$

$$\checkmark 3 \int_0^4 (u-6)^2 u^{\frac{1}{2}} du$$

$$\checkmark 3 \int_0^4 (u^2 - 12u + 36) u^{\frac{1}{2}} du$$

$$\checkmark 3 \int_0^4 \left(u^{\frac{5}{2}} - 12u^{\frac{3}{2}} + 36u^{\frac{1}{2}} \right) du$$

$$\checkmark 3 \left[\frac{2u^{\frac{7}{2}}}{7} - \frac{24u^{\frac{5}{2}}}{5} + 24u^{\frac{3}{2}} \right]_0^4$$

$$= 3 \left[\frac{2}{7} (4)^{\frac{7}{2}} - \frac{24}{5} (4)^{\frac{5}{2}} + 24 (4)^{\frac{3}{2}} \right]$$

$$= 224.914$$

$$\therefore \frac{7872}{35}$$

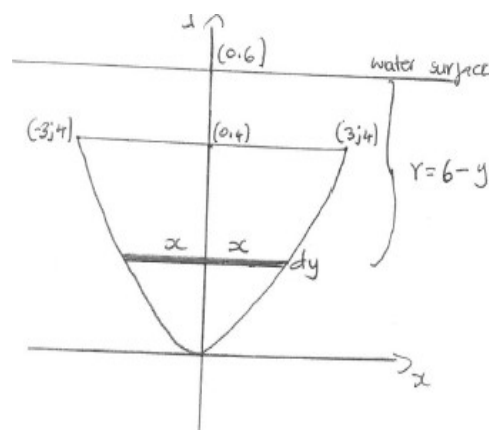
$$\therefore \dot{y} = \frac{224,914}{-57,6}$$

$$\therefore -\frac{82}{21}$$

$$\therefore -3,905$$

ALTERNATIVE 2

5.4.1



Strip (1/2✓)

Top of wall at (0; 4) (1/2✓)

Water level at (0; 6) (1/2✓)

(-3; 4) and (3; 4) (1/2✓)

$$y = ax^2$$

$$4 = a(3)^2$$

$$a = \frac{4}{9} \quad (1/2✓)$$

(3)

$$y = \frac{4}{9}x^2$$

$$x = \frac{3}{2}y^{\frac{1}{2}} \quad (1/2\checkmark)$$

5.4.2

$$\text{First moment} = \int r dA$$

$$= 3 \int_0^4 (6-y) y^{\frac{1}{2}} dy \quad \checkmark$$

$$= 3 \int_0^4 \left(6y^{\frac{1}{2}} - y^{\frac{3}{2}} \right) dy \quad \checkmark$$

$$= 3 \left[4y^{\frac{3}{2}} - \frac{2}{5}y^{\frac{5}{2}} \right]_0^4 \quad \checkmark$$

$$= 3 \left[4(4)^{\frac{3}{2}} - \frac{2}{5}(4)^{\frac{5}{2}} \right]$$

$$= \frac{288}{5}$$

$$= 57,6\checkmark$$

(4)

5.4.3

$$\text{Second moment} = \int r^2 dA$$

(5)

$$= 3 \int_0^4 (6-y)^2 y^{\frac{1}{2}} dy \quad \checkmark$$

$$= 3 \int_0^4 (36 - 12y + y^2) y^{\frac{1}{2}} dy$$

$$= 3 \int_0^4 \left(36y^{\frac{1}{2}} - 12y^{\frac{3}{2}} + y^{\frac{5}{2}} \right) dy$$

$$= 3 \left[24y^{\frac{3}{2}} - \frac{24}{5}y^{\frac{5}{2}} + \frac{2}{7}y^{\frac{7}{2}} \right]_0^4 \quad \checkmark$$

$$= 3 \left[24(4)^{\frac{3}{2}} - \frac{24}{5}(4)^{\frac{5}{2}} + \frac{2}{7}(4)^{\frac{7}{2}} \right]$$

$$\therefore \frac{7872}{35}$$

$$\therefore 224,914$$

$$\therefore \dot{y} = \frac{224,914}{57,6}$$

$$\therefore \frac{82}{21}$$

$$\therefore 3,905$$

[40]

QUESTION 6

6.1

$$y = x^2 - \frac{1}{8} \ln x$$

(6)

$$\frac{dy}{dx} = 2x - \frac{1}{8x} \quad \checkmark$$

$$\left(\frac{dy}{dx} \right)^2 = \left(2x - \frac{1}{8x} \right)^2$$

$$\therefore 4x^2 - \frac{1}{2} + \frac{1}{64x^2}$$

$$1 + \left(\frac{dy}{dx} \right)^2 = 1 + 4x^2 - \frac{1}{2} + \frac{1}{64x^2}$$

ALTERNATIVE

$$\left(\frac{dy}{dx} \right)^2 = \left(2x - \frac{1}{8x} \right)^2$$

$$\therefore 4x^2 - \frac{1}{2} + \frac{1}{64x^2}$$

$$\therefore \frac{256x^4 - 32x^2 + 1}{64x^2}$$

$$1 + \left(\frac{dy}{dx} \right)^2 = 1 + \frac{256x^4 - 32x^2 + 1}{64x^2}$$

$$\therefore \frac{256x^4 + 32x^2 + 1}{64x^2}$$

$$4x^2 + \frac{1}{2} + \frac{1}{64x^2}$$

$$\left(2x + \frac{1}{8x}\right)^2$$

$$\therefore S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= \int_1^2 \sqrt{\left(2x + \frac{1}{8x}\right)^2} dx$$

$$= \int_1^2 \left(2x + \frac{1}{8x}\right) dx \quad \checkmark$$

$$= x^2 + \frac{1}{8} \ln x \Big|_1^2 \quad \checkmark$$

$$= 2^2 + \frac{1}{8} \ln(2) - 1 - 0 \quad \checkmark$$

$$4,087 \quad \checkmark$$

6.2 $x=1+t$ and $y = \ln(1+t)$

$$\frac{dx}{dt} = 1 \checkmark \quad \text{and} \quad \frac{dy}{dt} = \frac{1}{1+t}$$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 1 + \frac{1}{(1+t)^2}$$

$$\therefore \frac{(1+t)^2 + 1}{(1+t)^2}$$

$$\therefore A_y = \int_{u=1}^{u=2} 2\pi x \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2} du \quad \checkmark$$

$$\therefore \int_0^2 2\pi(1+t) \sqrt{\frac{(1+t)^2 + 1}{(1+t)^2}} dt$$

$$\therefore 2\pi \int_0^2 \sqrt{(1+t)^2 + 1} dt$$

$$\therefore 2\pi \left[\frac{1+t}{2} \sqrt{(1+t)^2 + 1} + \frac{1}{2} \ln(1+t + \sqrt{(1+t)^2 + 1}) \right]_0^2$$

$$\therefore 2\pi \left[\frac{3}{2} \sqrt{10} + \frac{1}{2} \ln(3 + \sqrt{10}) \right] - 2\pi \left[\frac{1}{2} \sqrt{2} + \frac{1}{2} \ln(1 + \sqrt{2}) \right]$$

$$\therefore 28,305 (\text{units})^2$$

(6)
[12]

TOTAL: 100