



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N6

29 Julie 2024

Hierdie nasienriglyn bestaan uit 23 bladsye.

VRAAG 1

1.1 1.1.1 $z = f(x, y) = 2x^3 - 4x^2y^2 + y^3$

$$\frac{\partial z}{\partial x} = f_x(x, y) = 6x^2 - 8xy^2 \quad \checkmark \checkmark$$

$$\frac{\partial^2 z}{\partial x^2} = f_{xx}(x, y) = 12x - 8y^2 \quad \checkmark \checkmark \quad (4)$$

1.1.2 $\left. \frac{\partial^2 z}{\partial x^2} \right|_{\substack{x=2 \\ y=-2}} = f_{xx}(2, -2) = 12(2) - 8(-2) = 40 \quad \checkmark \checkmark \quad (2)$

1.2 $V = \pi r^2 h \quad \checkmark$

Gegee: $\delta r = 0.2 \text{ cm}$ en $\delta h = -0.1 \text{ cm}$

Indien $r = 5 \text{ cm}$ en $h = 10 \text{ cm}$

$$\frac{\partial V}{\partial r} = 2\pi r h \quad \checkmark \quad \frac{\partial V}{\partial h} = \pi r^2 \quad \checkmark$$

$$= 2\pi(5)(10) \quad = \pi(5)^2$$

$$= 100\pi \text{ cm}^2 \quad = 25\pi \text{ cm}^2$$

$$\delta V = \frac{\partial V}{\partial r} \delta r + \frac{\partial V}{\partial h} \delta h \quad \checkmark$$

$$= (100\pi)(0.2) + (25\pi)(-0.1) \quad \checkmark$$

$$= 17.5\pi \text{ cm}^3 \quad \checkmark$$

$$= 54,978 \text{ cm}^3 \quad (6)$$

[12]

VRAAG 2

$$\begin{aligned}
 2.1 \quad \int \sin^2(2x-1)dx &= \frac{1}{2} \int \sin^2 u du \\
 &= \frac{1}{2} \left[\frac{u}{2} - \frac{\sin(2u)}{4} \right] + C \quad \checkmark \\
 &= \frac{1}{2} \left[\frac{2x-1}{2} - \frac{\sin(2(2x-1))}{4} \right] + C \quad \checkmark
 \end{aligned}$$

Laat $u = 2x-1$
 $du = 2dx$
 $\frac{1}{2} du = dx$

(2)

$$\begin{aligned}
 2.2 \quad &\int x^3 \cos 3x dx \\
 &f(x) = x^3 \text{ en } g'(x) = \cos 3x \\
 &f'(x) = 3x^2 \text{ en } g(x) = \frac{\sin 3x}{3} \\
 &\therefore \int x^3 \cos 3x dx = \frac{x^3 \sin 3x}{3} - \int x^2 \sin 3x dx \quad \checkmark \checkmark \\
 &f(x) = x^2 \text{ en } g'(x) = \sin 3x \\
 &f'(x) = 2x \text{ en } g(x) = \frac{\cos 3x}{-3} \\
 &\therefore \int x^3 \cos 3x dx = \frac{x^3 \sin 3x}{3} - \left[\frac{x^2 \cos 3x}{-3} - \int 2x \times \frac{\cos 3x}{-3} dx \quad \checkmark \checkmark \right] \\
 &= \frac{x^3 \sin 3x}{3} - \left[\frac{x^2 \cos 3x}{-3} + \frac{2}{3} \int x \cos 3x dx \quad \checkmark \right] \\
 &f(x) = x \text{ en } g'(x) = \cos 3x \\
 &f'(x) = 1 \text{ en } g(x) = \frac{\sin 3x}{3} \\
 &\therefore \int x^3 \cos 3x dx = \frac{x^3 \sin 3x}{3} + \frac{x^2 \cos 3x}{3} - \frac{2}{3} \left[\frac{x \sin 3x}{3} - \frac{1}{3} \int \sin 3x dx \quad \checkmark \checkmark \right] \\
 &= \frac{x^3 \sin 3x}{3} + \frac{x^2 \cos 3x}{3} - \frac{2x \sin 3x}{9} + \frac{2}{9} \left[\frac{\cos 3x}{-3} \right] + C \quad \checkmark \\
 &= \frac{x^3 \sin 3x}{3} + \frac{x^2 \cos 3x}{3} - \frac{2x \sin 3x}{9} - \frac{2 \cos 3x}{27} + C
 \end{aligned}$$

Alternatief

D	I
x^3	$\cos 3x$
$3x^2$	$\frac{\sin 3x}{3} \checkmark$
$6x$	$\frac{-\cos 3x}{9} \checkmark$
6	$\frac{-\sin 3x}{27} \checkmark$
0	$\frac{\cos 3x}{81} \checkmark$

$$\int x^3 \cos 3x dx = \frac{x^3 \sin 3x}{3} + \frac{3x^2 \cos 3x}{9} - \frac{6x \sin 3x}{27} - \frac{6 \cos 3x}{81} + C \checkmark \checkmark \checkmark \checkmark \quad (8)$$

2.3

$$\int \sqrt{3x^2 + 2x - 1} dx$$

$$3x^2 + 2x - 1$$

$$= 3 \left[x^2 + \frac{2}{3}x - \frac{1}{3} \right] \checkmark$$

$$= 3 \left[x^2 + \frac{2}{3}x + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2 - \frac{1}{3} \right] \checkmark \checkmark$$

$$= 3 \left[\left(x + \frac{1}{3}\right)^2 - \frac{4}{9} \right] \checkmark \checkmark$$

$$\begin{aligned} ax^2 + bx + c &= \frac{4ac - b^2}{4a} + a \left(x + \frac{b}{2a}\right)^2 \checkmark \\ \therefore 3x^2 + 2x - 1 &= \frac{4(3)(-1) - (2)^2}{4(3)} + 3 \left(x + \frac{2}{2 \times 3}\right)^2 \checkmark \\ &= -\frac{4}{3} + 3 \left(x + \frac{1}{3}\right)^2 \checkmark \\ &= 3 \left[\left(x + \frac{1}{3}\right)^2 - \frac{4}{9} \right] \checkmark \checkmark \end{aligned}$$

$$\int \sqrt{3x^2 + 2x - 1} dx = \sqrt{3} \left[\int \sqrt{\left(x + \frac{1}{3}\right)^2 - \frac{4}{9}} dx \right] \checkmark$$

$$= \sqrt{3} \left[\frac{x + \frac{1}{3}}{2} \sqrt{\left(x + \frac{1}{3}\right)^2 - \frac{4}{9}} - \frac{2}{9} \ln \left\{ \left(x + \frac{1}{3}\right) + \sqrt{\left(x + \frac{1}{3}\right)^2 - \frac{4}{9}} \right\} \right] + C \checkmark \checkmark \quad (8)$$

$$2.4 \quad \int \tan^5 x dx = \int \tan^3 x \tan^2 x dx \checkmark$$

$$= \int \tan^3 x (\sec^2 x - 1) dx \checkmark$$

$$= \int \tan^3 x \sec^2 x dx - \int \tan^3 x dx \checkmark$$

$$= \int \tan^3 x \sec^2 x dx - \int \tan x \tan^2 x dx \checkmark$$

$$= \int \tan^3 x \sec^2 x dx - \int \tan x (\sec^2 x - 1) dx \checkmark$$

$$= \int \tan^3 x \sec^2 x dx - \int \tan x \sec^2 x dx + \int \tan x dx \checkmark$$

$$= \int u^3 du - \int u du + \int \tan x dx \checkmark \text{ [For u-substitution]}$$

$$= \frac{u^4}{4} - \frac{u^2}{2} + \ln(\sec x) + C \checkmark \checkmark$$

$$= \frac{(\tan x)^4}{4} - \frac{(\tan x)^2}{2} + \ln(\sec x) + C \checkmark \text{ [Regmerk is vir } u = \tan x \text{]}$$

Laat $u = \tan x$
 $du = \sec^2 x dx$

Alternatief

$$\int \tan^5 x dx = \int (\tan^2 x)^2 \tan x dx \checkmark$$

$$= \int (\sec^2 x - 1)^2 \tan x dx \checkmark$$

$$= \int (\sec^4 x - 2\sec^2 x + 1) \tan x dx \checkmark$$

$$= \int \sec^4 x \tan x dx - 2 \int \sec^2 x \tan x dx + \int \tan x dx \checkmark$$

$$= \int u^3 du - 2 \int u du + \int \tan x dx \checkmark \checkmark \text{ [Vir korrekte u-vervanging]}$$

$$= \frac{u^4}{4} - u^2 + \ln(\sec x) + C \checkmark \checkmark \checkmark$$

$$= \frac{\sec^4 x}{4} - \sec^2 x + \ln(\sec x) + C \checkmark$$

Laat $u = \sec x$
 $du = \sec x \tan x dx$

Alternatief

$$\begin{aligned}
 \int \tan^5 x dx &= \int \frac{\sin^5 x}{\cos^5 x} dx \checkmark \\
 &= \int (\sin^2 x)^2 \sin x (\cos x)^{-5} dx \checkmark \\
 &= \int (1 - \cos^2 x)^2 \sin x (\cos x)^{-5} dx \checkmark \\
 &= \int (1 - 2 \cos^2 x + \cos^4 x) \sin x (\cos x)^{-5} dx \checkmark \\
 &= \int (\cos x)^{-5} \sin x dx - 2 \int (\cos x)^{-3} \sin x dx + \int \tan x dx \checkmark \checkmark \checkmark \\
 &= -\frac{1}{3} \int (\cos x)^{-5} (-3 \sin x) dx + \frac{2}{3} \int (\cos x)^{-3} (-3 \sin x) dx + \int \tan x dx \checkmark \\
 &= \frac{(\cos x)^{-4}}{12} - \frac{(\cos x)^{-2}}{3} + \frac{1}{3} \ln(\sec x) + C \checkmark \checkmark \\
 &= \frac{\sec^4 x}{12} - \frac{1}{3} \sec^2 x + \frac{1}{3} \ln(\sec x) + C
 \end{aligned} \tag{10}$$

$$\begin{aligned}
 2.5 \quad &\int \sin^4 3x \cos^5 3x dx \\
 &= \int \sin^4 3x (\cos^2 3x)^2 \cos 3x dx \checkmark \\
 &= \int \sin^4 3x (1 - \sin^2 3x)^2 \cos 3x dx \checkmark \\
 &= \int \sin^4 3x (1 - 2 \sin^2 3x + \sin^4 3x) \cos 3x dx \checkmark \\
 &= \int \sin^4 3x \cos 3x dx - 2 \int \sin^6 3x \cos 3x dx + \int \sin^8 3x \cos 3x dx \checkmark \\
 &= \frac{1}{3} \int \sin^4 3x \cdot 3 \cos 3x dx - \frac{2}{3} \int \sin^6 3x \cdot 3 \cos 3x dx + \frac{1}{3} \int \sin^8 3x \cdot 3 \cos 3x dx \checkmark \\
 &= \frac{1}{3} \left[\frac{\sin^5 3x}{5} \right] - \frac{2}{3} \left[\frac{\sin^7 3x}{7} \right] + \frac{1}{3} \left[\frac{\sin^9 3x}{9} \right] + C \checkmark \checkmark \checkmark
 \end{aligned}$$

Alternatief vir die laaste 4 regmerkies

$$\text{laat } u = \sin 3x \Rightarrow \cos 3x dx = \frac{du}{3}$$

VRAAG 3

3.1

$$\int \frac{x^2}{(x-1)(x^2+x+1)} dx$$

$$\text{Let } \frac{x^2}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} \checkmark$$

$$x^2 = A(x^2+x+1) + (x-1)(Bx+C) \checkmark$$

$$x=1 : 1=3A \Rightarrow A=\frac{1}{3} \checkmark$$

$$x^2 = Ax^2 + Ax + A + Bx^2 + Cx - Bx - C \checkmark$$

$$x^2 : 1 = A + B \Rightarrow B = \frac{2}{3} \checkmark$$

$$\text{Konstante: } 0 = A - C \Rightarrow C = \frac{1}{3} \checkmark$$

$$\int \frac{x^2}{(x-1)(x^2+x+1)} dx = \int \left(\frac{\frac{1}{3}}{x-1} + \frac{\frac{2}{3}x + \frac{1}{3}}{x^2+x+1} \right) dx \checkmark$$

$$= \frac{1}{3} \int \frac{dx}{x-1} + \frac{1}{3} \int \frac{2x+1}{x^2+x+1} dx \checkmark$$

$$= \frac{1}{3} \ln(x-1) + \frac{1}{3} \ln(x^2+x+1) + C \checkmark \checkmark$$

Laat
 $u = x^2 + x + 1$
 $du = (2x+1) dx$

(10)

3.2

$$\int \frac{x^3}{x^3 - 16x} dx$$

$$\frac{x^3 - 16x}{x^3} \checkmark$$

$$\frac{x^3 - 16x}{16x} \checkmark$$

$$\int \frac{x^3}{x^3 - 16x} dx = \int \left(1 + \frac{16x}{x^3 - 16x} \right) dx \checkmark$$

$$x^3 - 16x = x(x^2 - 16) = x(x - 4)(x + 4) \checkmark$$

$$\text{Let } \frac{16x}{x(x-4)(x+4)} = \frac{A}{x} + \frac{B}{x-4} + \frac{C}{x+4} \checkmark \checkmark$$

$$16x = A(x-4)(x+4) + Bx(x+4) + Cx(x-4) \checkmark$$

$$x = 0 : 0 = -16A \Rightarrow A = 0 \checkmark$$

$$x = 4 : 64 = 32B \Rightarrow B = 2 \checkmark$$

$$x = -4 : -64 = 32C \Rightarrow C = -2 \checkmark$$

$$\int \frac{x^3}{x^3 - 16x} dx = \int \left(1 + \frac{2}{x-4} - \frac{2}{x+4} \right) dx \checkmark$$

$$= \int dx + 2 \int \frac{dx}{x-4} - 2 \int \frac{dx}{x+4}$$

$$= x + 2 \ln(x-4) - 2 \ln(x+4) + C \checkmark \checkmark \checkmark$$

Alternatief

$$\int \frac{x^3}{x^3 - 16x} dx$$

$$\frac{x^3 - 16x}{x^3 - 16x} \cdot \frac{1}{x^3} \checkmark$$

$$\frac{x^3 - 16x}{16x} \checkmark$$

$$\int \frac{x^3}{x^3 - 16x} dx = \int \left(1 + \frac{16x}{x^3 - 16x} \right) dx \checkmark$$

$$x^3 - 16x = x(x^2 - 16) \checkmark$$

$$\frac{16x}{x(x^2 - 16)} = \frac{A}{x} + \frac{Bx + C}{x^2 - 16} \checkmark$$

$$\Rightarrow 16x = A(x^2 - 16) + x(Bx + C) \checkmark$$

$$x = 0 : 0 = -16A \Rightarrow A = 0 \checkmark$$

$$16x = Ax^2 - 16A + Bx^2 + Cx \checkmark$$

$$x^2 : 0 = A + B \Rightarrow B = 0 \checkmark$$

$$x : 16 = C \checkmark$$

$$\therefore \int \frac{x^3}{x^3 - 16x} dx = \int \left(1 + \frac{16}{x^2 - 16} \right) dx \checkmark$$

$$= \int \left(1 - \frac{16}{4^2 - x^2} \right) dx \checkmark$$

$$= x - \frac{16}{2(4)(1)} \ln \left[\frac{4+x}{4-x} \right] + C \checkmark \checkmark$$

$$= x - 2 \ln(4+x) + 2 \ln(4-x) + C$$

(14)

[24]

VRAAG 4

$$4.1 \quad \frac{dy}{dx} - 3y = 3x^2 e^x$$

$$IF = e^{-3 \int dx} = e^{-3x} \checkmark \checkmark$$

$$ye^{-3x} = \int 3x^2 e^x \times e^{-3x} dx \checkmark \checkmark$$

$$ye^{-3x} = 3 \int x^2 e^{-2x} dx \checkmark$$

$$\begin{array}{ll} f(x) = x^2 & g'(x) = e^{-2x} \\ f'(x) = 2x & \checkmark \quad g(x) = -\frac{1}{2} e^{-2x} \end{array}$$

$$\therefore ye^{-3x} = 3 \left[-\frac{x^2 e^{-2x}}{2} + \int x e^{-2x} dx \right] \checkmark \checkmark$$

$$\begin{array}{ll} f(x) = x & g'(x) = e^{-2x} \\ f'(x) = 1 & \checkmark \quad g(x) = -\frac{1}{2} e^{-2x} \end{array}$$

$$\therefore ye^{-3x} = 3 \left[-\frac{x^2 e^{-2x}}{2} + \left(-\frac{x e^{-2x}}{2} + \frac{1}{2} \int e^{-2x} dx \right) \right] \checkmark \checkmark$$

$$= 3 \left[-\frac{x^2 e^{-2x}}{2} + \left(-\frac{x e^{-2x}}{2} - \frac{e^{-2x}}{4} \right) \right] + C \checkmark$$

$$= \frac{-3x^2 e^{-2x}}{2} - \frac{3x e^{-2x}}{2} - \frac{3}{4} e^{-2x} + C$$

(12)

4.2

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = x^2$$

$$r^2 + 6r + 9 = 0 \checkmark$$

$$r_1 = -3 \text{ en } r_2 = -3 \quad \checkmark$$

$$y_c = Ae^{-3x} + Bxe^{-3x} \text{ OF } y_c = e^{-3x}(A + Bx) \quad \checkmark$$

$$y_p = Cx^2 + Dx + E \quad \checkmark$$

$$y'_p = 2Cx + D \quad \checkmark$$

$$y''_p = 2C \quad \checkmark$$

$$2C + 12Cx + 6D + 9Cx^2 + 9Dx + 9E = x^2 \quad \checkmark$$

$$x^2: \quad 9C = 1 \Rightarrow C = \frac{1}{9} \quad \checkmark$$

$$x: \quad 12C + 9D = 0 \Rightarrow D = -\frac{4}{27} \quad \checkmark$$

$$\textbf{Konstante: } 2C + 6D + 9E = 0 \Rightarrow E = \frac{2}{27} \quad \checkmark$$

$$y_p = \frac{1}{9}x^2 - \frac{4}{27}x + \frac{2}{27} \quad \checkmark$$

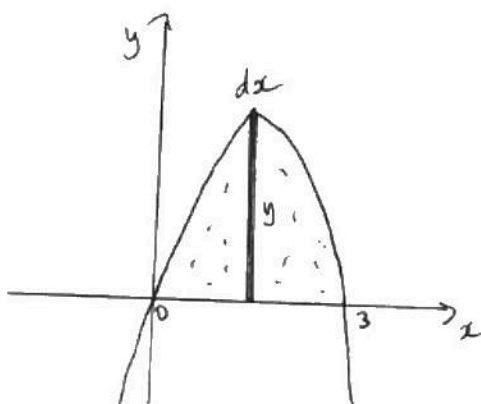
$$y = y_c + y_p$$

$$\therefore y = e^{-3x}(A + Bx) + \frac{1}{9}x^2 - \frac{4}{27}x + \frac{2}{27} \quad \checkmark$$

(12)
[24]

VRAAG 5

5.1 5.1.1



Vertikale strook✓

Verskadude gebied✓

Grafiek van of $y = 3x - x^2$ ✓

(0;0) en (0;3) ✓

(4)

5.1.2

$$\begin{aligned}
 V_x &= \pi \int_a^b y^2 dx \quad \text{OF} \quad V_x = \pi \int_a^b (y_1^2 - y_2^2) dx \quad \checkmark \\
 &= \pi \int_0^3 (3x - x^2)^2 dx \quad \checkmark \checkmark \\
 &= \pi \int_0^3 (9x^2 - 6x^3 + x^4) dx \quad \checkmark \\
 &= \pi \left[3x^3 - \frac{3}{2}x^4 + \frac{x^5}{5} \right]_0^3 \quad \checkmark \checkmark \\
 &= \pi \left[3(3)^3 - \frac{3}{2}(3)^4 + \frac{(3)^5}{5} \right] - \pi \left[3(0)^3 - \frac{3}{2}(0)^4 + \frac{(0)^5}{5} \right] \quad \checkmark \\
 &= \frac{81}{10} \pi (\text{eenhede})^3 \quad \checkmark
 \end{aligned}$$

$$25,447(\text{eenhede})^3$$

(6)

5.1.3

$$V_{m-y} = \int_a^b r dV \checkmark$$

$$= \pi \int_0^3 x(9x^2 - 6x^3 + x^4) dx \checkmark \checkmark \checkmark$$

$$= \pi \int_0^3 (9x^3 - 6x^4 + x^5) dx \checkmark$$

$$= \pi \left[\frac{9}{4}x^4 - \frac{6}{5}x^5 + \frac{x^6}{6} \right]_0^3 \checkmark$$

$$= \pi \left[\frac{9}{4}(3)^4 - \frac{6}{5}(3)^5 + \frac{(3)^6}{6} \right] - \pi \left[\frac{9}{4}(0)^4 - \frac{6}{5}(0)^5 + \frac{(0)^6}{6} \right] \checkmark$$

$$= \frac{243}{20} \pi (\text{eenhede})^4$$

$$= 38,170 (\text{eenhede})^4 \checkmark$$

$$\therefore \bar{y} = \frac{38,170}{25,447} \checkmark$$

$$= 1,500 \text{ eenhede} \checkmark$$

$$= \frac{3}{2} \text{ eenhede}$$

(10)

5.2

5.2.1

$$x^2 - x - 6 = 0$$

$$x = -2 \neq \text{ of } \neq x = 3$$

$$y = 4 \quad y = 9$$

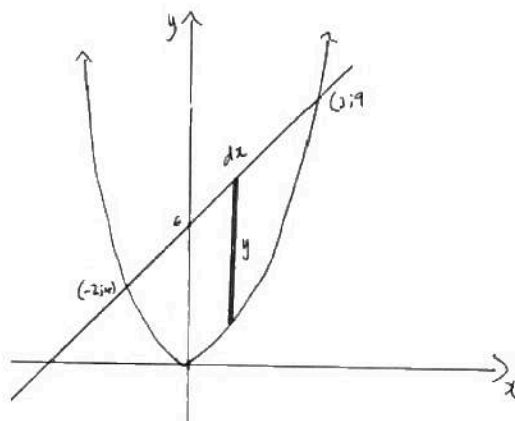
$$(-2; 4) \checkmark \text{ en } (3; 9) \checkmark$$

Vertikale strook $\checkmark$

Verskadude gebied $\checkmark$

Grafiek van $y = x^2$ $\checkmark$

Grafiek van $y = x + 6$ $\checkmark$



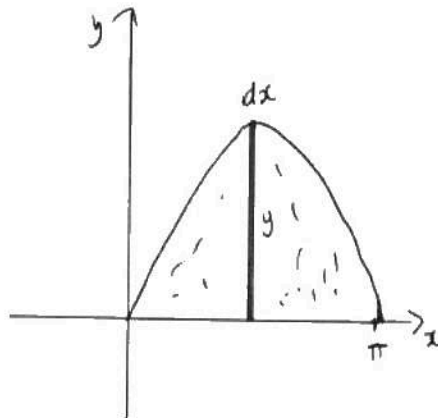
$$\begin{aligned}
 5.2.2 \quad A_x &= \int_a^b (y_1 - y_2) dx \quad \text{OF} \quad dA_x = (y_1 - y_2) dx \quad \checkmark \\
 &= \int_0^3 (x + 6 - x^2) dx \quad \checkmark \checkmark \\
 &= \left[\frac{1}{2} x^2 + 6x - \frac{1}{3} x^3 \right]_0^3 \quad \checkmark \\
 &= \left[\frac{1}{2} (3)^2 + 6(3) - \frac{1}{3} (3)^3 \right] - \left[\frac{1}{2} (0)^2 + 6(0) - \frac{1}{3} (0)^3 \right] \quad \checkmark \\
 &= 13.5 \text{ (eenhede)}^1 \checkmark \\
 &= \frac{27}{2} \text{ (eenhede)}^2 \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 5.2.3 \quad I_y &= \int_a^b r^2 dA \quad \text{OF} \quad dI_y = r^2 dA \quad \checkmark \\
 I_y &= \int_0^3 x^2 (x + 6 - x^2) dx \quad \checkmark \checkmark \checkmark \\
 &= \int_0^3 (x^3 + 6x^2 - x^4) dx \quad \checkmark \\
 &= \left[\frac{1}{4} x^4 + 2x^3 - \frac{1}{5} x^5 \right]_0^3 \quad \checkmark \\
 &= \left[\frac{1}{4} (3)^4 + 2(3)^3 - \frac{1}{5} (3)^5 - 0 \right] \checkmark \\
 &= 25.65 \text{ (eenhede)}^3 \checkmark \\
 &= \frac{513}{20} \text{ (eenheid)}^3 \quad (8)
 \end{aligned}$$

$$\begin{aligned}
 5.2.4 \quad I_y &= 25.65 \frac{A}{A} \\
 &= \frac{25.65}{13.5} A \quad \checkmark \\
 &= 1.9 A \quad \checkmark \\
 &= \frac{19}{10} A \quad (9)
 \end{aligned}$$

5.3

5.3.1



Vertikale strook ✓

Verskadude gebied ✓

Grafiek van $y = \sin x$ ✓

$(0; 0)$ en $(0; \pi)$ ✓

(4)

5.3.2

$$V_x = \pi \int_a^b y^2 dx \quad \checkmark$$

$$= \pi \int_0^\pi \sin^2 x dx \quad \checkmark \checkmark$$

$$= \frac{\pi}{2} \int_0^\pi (1 - \cos 2x) dx$$

$$= \frac{\pi}{2} \left[x - \frac{\sin 2x}{2} \right]_0^\pi \quad \checkmark$$

$$= \frac{\pi}{2} \left[\pi - \frac{\sin 2\pi}{2} - 0 \right]_0^\pi \quad \checkmark$$

$$= 4.935 (\text{eenhede})^3 \quad \checkmark$$

Alternatief

$$V_x = \pi \int_a^b (y_1^2 - y_2^2) dx \quad \checkmark$$

$$= \pi \int_0^\pi \sin^2 x dx \quad \checkmark \checkmark$$

$$= \pi \left[\frac{x}{2} - \frac{\sin 2x}{4} \right]_0^\pi \quad \checkmark$$

$$= \pi \left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} \right] - 0 \quad \checkmark$$

$$= 4.935 (\text{eenhede})^3 \quad \checkmark$$

(6)

5.3.3

$$\begin{aligned}
 I_x &= \int_a^b r^2 dm = \rho \int_a^b r^2 dV = \frac{\rho\pi}{2} \int_0^\pi y^4 dx \quad \checkmark \\
 &= \frac{\rho\pi}{2} \int_0^\pi \sin^4 x dx \quad \checkmark \checkmark \\
 &= \frac{\rho\pi}{2} \int_0^\pi (\sin^2 x)^2 dx \quad \checkmark \\
 &= \frac{\rho\pi}{2} \int_0^\pi \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right)^2 dx \quad \checkmark \\
 &= \frac{\rho\pi}{2} \int_0^\pi \left(\frac{1}{4} - \frac{1}{2} \cos 2x + \frac{1}{4} \cos^2 2x \right) dx \quad \checkmark \\
 &= \frac{\rho\pi}{2} \left[\frac{1}{4} x - \frac{\sin 2x}{4} + \frac{1}{4} \left(\frac{x}{2} + \frac{\sin 4x}{8} \right) \right]_0^\pi + C \quad \checkmark \checkmark \\
 &= \frac{\rho\pi}{2} \left[\frac{1}{4} \pi - \frac{\sin 2\pi}{4} + \frac{1}{4} \left(\frac{\pi}{2} + \frac{\sin 4\pi}{8} \right) \right] - 0 \quad \checkmark \\
 &= 1,851\rho(\text{eenhede})^5 \quad \checkmark
 \end{aligned}$$

Alternatief

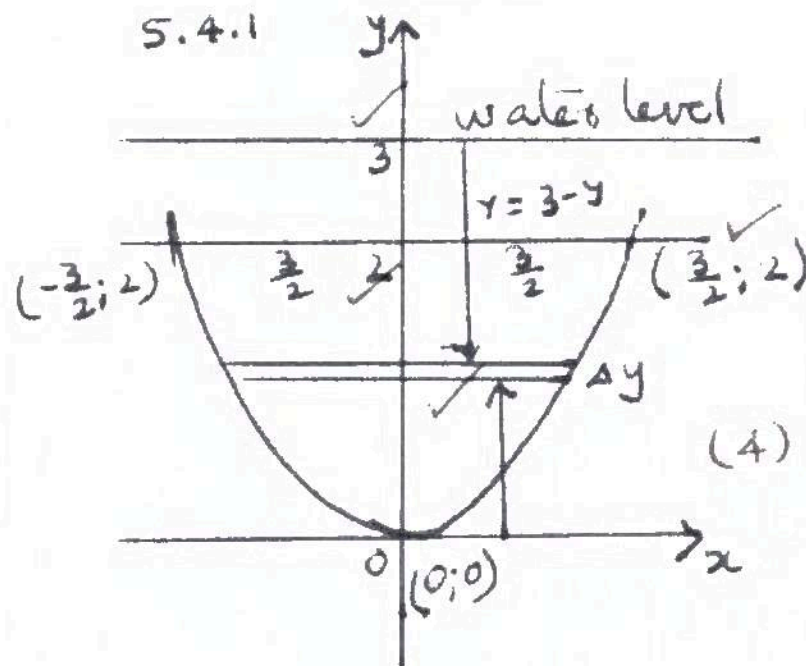
$$\begin{aligned}
 I_x &= \int_a^b r^2 dm = \rho \int_a^b r^2 dV = \frac{\rho\pi}{2} \int_0^\pi y^4 dx \quad \checkmark \\
 &= \frac{\rho\pi}{2} \int_0^\pi \sin^4 x dx \quad \checkmark \checkmark \\
 &= \frac{\rho\pi}{2} \int_0^\pi (\sin^2 x)^2 dx \quad \checkmark \\
 &= \frac{\rho\pi}{8} \int_0^\pi (1 - \cos 2x)^2 dx \quad \checkmark \\
 &= \frac{\rho\pi}{8} \int_0^\pi (1 - 2\cos 2x + \cos^2 2x) dx \quad \checkmark \\
 &= \frac{\rho\pi}{8} \int_0^\pi \left(1 - 2\cos 2x + \frac{1}{2} + \frac{1}{2} \cos 4x \right) dx \quad \checkmark \\
 &= \frac{\rho\pi}{8} \int_0^\pi \left(\frac{3}{2} - 2\cos 2x + \frac{1}{2} \cos 4x \right) dx
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\rho\pi}{8} \left[\frac{3}{2}x - \sin 2x + \frac{1}{8} \sin 4x \right]_0^{\pi} \checkmark \\
 &= \frac{\rho\pi}{8} \left[\frac{3}{2}\pi - \sin 2\pi + \frac{1}{8} \sin 4\pi - 0 \right] \checkmark \\
 &= 1,851\rho(\text{eenhede})^5 \quad \checkmark \quad (10)
 \end{aligned}$$

5.3.4 $I_x = 1,851 \times \frac{m}{V} \quad \checkmark$

$$\begin{aligned}
 &= \frac{1,851}{4,935} m \\
 &= 0,375 m \checkmark \\
 &= \frac{617}{1645} m \quad (2)
 \end{aligned}$$

5.4 5.4.1



$$y = ax^2$$

$$2 = a \left(\frac{3}{2} \right)^2 \text{ teen } \left(\frac{3}{2}; 2 \right) \Rightarrow a = \frac{8}{9} \quad \checkmark$$

$$\therefore y = \frac{8}{9}x^2 \quad \text{OF} \quad x = \frac{3}{2\sqrt{2}}y^{\frac{1}{2}} \quad \checkmark \quad (6)$$

5.4.2 Eerste moment = $\int r dA$ ✓

$$= \int_0^2 (3-y) \frac{3}{\sqrt{2}} y^{\frac{1}{2}} dy \checkmark \checkmark \checkmark$$

$$= \frac{3}{\sqrt{2}} \int_0^2 \left(3y^{\frac{1}{2}} - y^{\frac{3}{2}} \right) dy \checkmark$$

$$= \frac{3}{\sqrt{2}} \left[\frac{3y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{y^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^2 \checkmark$$

$$= \frac{3}{\sqrt{2}} \left[\frac{3(2)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(2)^{\frac{5}{2}}}{\frac{5}{2}} \right] - 0 \checkmark$$

$$= 7,2(\text{eenhede})^3 \checkmark$$

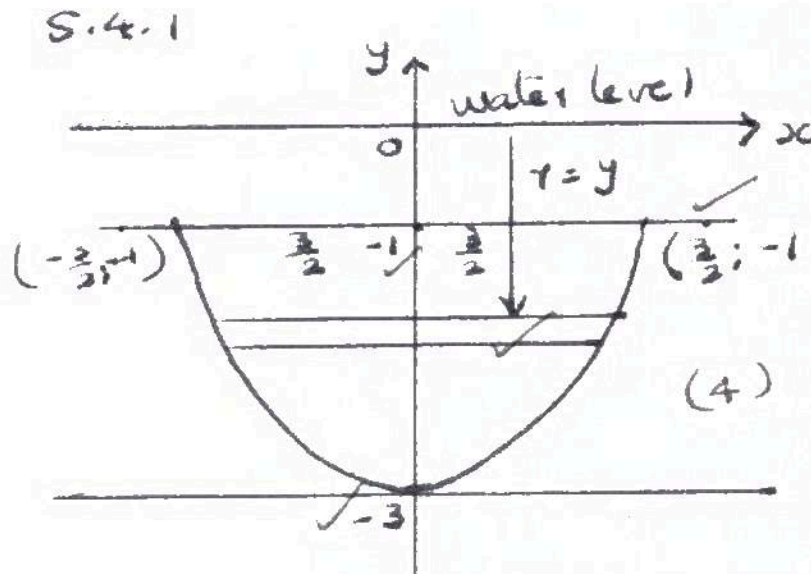
$$= \frac{36}{5}(\text{eenhede})^3 \quad (8)$$

5.4.3 $\bar{y} = \frac{14,057}{7,2} \checkmark$

$$= 1,952 \text{ eenhede} \checkmark \quad (2)$$

Alternatief

5.4 5.4.1



$$y = ax^2 - 3$$

$$-1 = a\left(\frac{3}{2}\right)^2 - 3 \text{ teen } \left(\frac{3}{2}; -1\right) \Rightarrow a = \frac{8}{9} \quad \checkmark$$

$$\therefore y = \frac{8}{9}x^2 - 3 \quad \text{OF} \quad x = \frac{3}{2\sqrt{2}}(y + 3)^{\frac{1}{2}} \quad \checkmark \quad (6)$$

5.4.2 Eerste moment = $\int r dA \quad \checkmark$

$$= \int_{-3}^{-1} y \cdot \frac{3}{\sqrt{2}} (y + 3)^{\frac{1}{2}} dy \quad \checkmark \checkmark$$

$$= \frac{3}{\sqrt{2}} \int_0^2 (u - 3) u^{\frac{1}{2}} du \quad \checkmark$$

$$= \frac{3}{\sqrt{2}} \int_0^2 \left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}} \right) du \quad \checkmark$$

$$= \frac{3}{\sqrt{2}} \left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{3u^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^2 \quad \checkmark$$

$$= \frac{3}{\sqrt{2}} \left[\frac{(2)^{\frac{5}{2}}}{\frac{5}{2}} - \frac{3(2)^{\frac{3}{2}}}{\frac{3}{2}} \right] - 0 \quad \checkmark$$

$$= -7,2(\text{eenhede})^3 \quad \checkmark$$

$$= -\frac{36}{5}(\text{eenhede})^3$$

laat $y + 3 = u \Rightarrow y = u - 3$

en $dy = du$

$y = -1 \Rightarrow u = 2$ en

$y = -3 \Rightarrow u = 0$

$$5.4.3 \quad \bar{y} = \frac{14,057}{-7,2} \checkmark$$

$$= -1,952 \text{ eenhede } \checkmark$$

(2)
[80]

VRAAG 6

6.1

$$x = e^t - t$$

$$y = 4e^{\frac{t}{2}}$$

$$\frac{dx}{dt} = e^t - 1 \checkmark$$

$$\frac{dy}{dt} = 4e^{\frac{t}{2}} \times \frac{1}{2} = 2e^{\frac{t}{2}} \checkmark$$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 4e^t + e^{2t} - 2e^t + 1 \checkmark$$

$$= e^{2t} + 2e^t + 1 \checkmark$$

$$= (e^t + 1)^2 \checkmark$$

$$S = \int_{u1}^{u2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} du \checkmark$$

$$= \int_{-8}^3 \sqrt{(e^t + 1)^2} dt \checkmark$$

$$= \int_{-8}^3 (e^t + 1) dt \checkmark$$

$$= e^t + t \Big|_{-8}^3 \checkmark \checkmark$$

$$= e^3 + 3 - e^{-8} + 8 \checkmark$$

$$= 31.085 \checkmark$$

(12)

$$6.2 \quad x = \sqrt{a^2 - y^2}$$

$$\frac{dx}{dy} = \frac{1}{2}(a^2 - y^2)^{-\frac{1}{2}} \times -2y \checkmark \checkmark$$

$$= \frac{-y}{\sqrt{a^2 - y^2}} \checkmark$$

$$1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{y^2}{a^2 - y^2} = \frac{a^2}{a^2 - y^2} \checkmark \checkmark$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy \checkmark$$

$$= 2\pi \int_0^{\frac{a}{2}} \sqrt{a^2 - y^2} \sqrt{\frac{a^2}{a^2 - y^2}} dy \checkmark \checkmark$$

$$= 2a\pi \int_0^{\frac{a}{2}} dy \checkmark$$

$$= 2a\pi [y]_0^{\frac{a}{2}} \checkmark$$

$$= 2\pi a \left[\frac{a}{2} - 0 \right] \checkmark$$

$$= a^2 \pi \checkmark$$

6.2 Alternatief 1

$$x = \sqrt{a^2 - y^2}$$

$$\Rightarrow x^2 = a^2 - y^2 \checkmark$$

$$\Rightarrow 2x \frac{dx}{dy} = -2y \checkmark$$

$$\Rightarrow \frac{dx}{dy} = -\frac{y}{x} \checkmark$$

$$\Rightarrow \left(\frac{dx}{dy}\right)^2 = \frac{y^2}{x^2} \checkmark$$

$$\Rightarrow 1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{y^2}{x^2} = \frac{x^2 + y^2}{x^2} = \frac{a^2}{x^2} \checkmark$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy \checkmark$$

$$= \int_0^{\frac{a}{2}} 2\pi x \sqrt{\frac{a^2}{x^2}} dy \checkmark \checkmark$$

$$= 2\pi a \int_0^{\frac{a}{2}} dy \checkmark$$

$$= 2\pi a [y]_0^{\frac{a}{2}} \checkmark$$

$$= 2\pi a \left[\frac{a}{2} - 0\right] \checkmark$$

$$= a^2 \pi (\text{eenhede})^2 \checkmark$$

6.2 Alternatief 2

$$x = \sqrt{a^2 - y^2}$$

$$\Rightarrow y = \sqrt{a^2 - x^2} = (a^2 - x^2)^{\frac{1}{2}} \quad \checkmark$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(a^2 - x^2)^{-\frac{1}{2}} \times -2x \quad \checkmark$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{x^2}{a^2 - x^2} \quad \checkmark$$

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{x^2}{a^2 - x^2} = \frac{a^2}{a^2 - x^2} \quad \checkmark$$

$$A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad \checkmark$$

$$= \int_{\frac{a\sqrt{3}}{2}}^a 2\pi x \sqrt{\frac{a^2}{a^2 - x^2}} dx \quad \checkmark$$

$$= 2\pi a \int_{\frac{a\sqrt{3}}{2}}^a x(a^2 - x^2)^{-\frac{1}{2}} dx \quad \checkmark$$

$$= -\pi a \int_{\frac{a\sqrt{3}}{2}}^a -2x(a^2 - x^2)^{-\frac{1}{2}} dx \quad \checkmark$$

$$= -\pi a \left[\frac{(a^2 - x^2)^{\frac{1}{2}}}{1/2} \right]_{\frac{a\sqrt{3}}{2}}^a \quad \checkmark$$

$$= -2\pi a \left[0 - \frac{a}{2} \right] \quad \checkmark$$

$$= a^2 \pi (\text{eenhede})^2 \quad \checkmark$$

$$x = \sqrt{a^2 - y^2}$$

$$y = 0 \Rightarrow x = a$$

$$y = \frac{a}{2} \Rightarrow x = \frac{a\sqrt{3}}{2} \quad \checkmark$$

(12)

[24]

TOTAAL: 200