



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

1 April 2020

This marking guideline consists of 15 pages.

NOTE: This marking guideline adds up to 200, and the total must be divided by 2 to get a mark out of 100.

QUESTION 1

1.1 $z = \ln(\sqrt{x} + \sqrt{y})$

$$\frac{\partial z}{\partial x} = \frac{1}{\sqrt{x} + \sqrt{y}} \cdot \frac{1}{2\sqrt{x}} \quad \checkmark \checkmark$$

$$\frac{\partial z}{\partial y} = \frac{1}{\sqrt{x} + \sqrt{y}} \cdot \frac{1}{2\sqrt{y}} \quad \checkmark \checkmark$$

$$\sqrt{x} \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} \sqrt{y} = \sqrt{x} \frac{1}{\sqrt{x} + \sqrt{y}} \frac{1}{2\sqrt{x}} + \sqrt{y} \frac{1}{\sqrt{x} + \sqrt{y}} \frac{1}{2\sqrt{y}} \quad \checkmark$$

$$= \frac{1}{2\sqrt{x} + \sqrt{y}} + \frac{1}{2\sqrt{x} + \sqrt{y}}$$

$$\frac{1+1}{2\sqrt{x} + \sqrt{y}}$$

$$= \frac{1}{\sqrt{x} + \sqrt{y}} \quad \checkmark$$

(6)

1.2 $V = \pi r^2 h$

$$\Delta V = \frac{\partial V}{\partial r} \Delta r + \frac{\partial V}{\partial h} \Delta h \quad \checkmark$$

$$= 2\pi r h \Delta r + \pi r^2 \Delta h \quad \checkmark \checkmark$$

$$= 2\pi(4)(20)(0,1) + \pi(4)^2(0,5) \quad \checkmark \checkmark$$

$$= 75,398 \text{ cm}^3 \quad \checkmark$$

(6)
[12]

QUESTION 2

$$\begin{aligned}
2.1 \quad & (x+3)^2 - 8x \\
&= x^2 + 6x + 9 - 8x \checkmark \\
&= x^2 - 2x + 9 \checkmark \\
&= x^2 - 2x + 1 + 9 - 1 \checkmark \\
&= (x-1)^2 + 8 \checkmark \checkmark \\
&\int \frac{1}{(x+3)^2 - 8x} dx \\
&= \int \frac{1}{8 + (x-1)^2} dx \checkmark \\
&= \frac{1}{\sqrt{8.1}} \tan^{-1} \frac{x-1}{\sqrt{8}} + C \checkmark \checkmark
\end{aligned}$$

$$\begin{aligned}
\text{or } ax^2 + bx + c &= \frac{4ac - b^2}{4a} + a \left(x + \frac{b}{2a} \right)^2 \\
x^2 - 2x + 9 &= \frac{4.1.9 - (-2)^2}{4.1} + 1 \left(x + \frac{-2}{2.1} \right)^2 \\
&= 8 + (x-1)^2
\end{aligned}$$

(8)

$$\begin{aligned}
2.2 \quad & \int \ln 2x \ln x dx = \ln 2x(x \ln x - x) - \int \frac{1}{x}(x \ln x - x) dx \checkmark \checkmark \checkmark \\
&= \ln 2x(x \ln x - x) - \int (\ln x - 1) dx \checkmark \\
&= \ln 2x(x \ln x - x) - (x \ln x - x - x) + C \checkmark \checkmark \checkmark \\
&= \ln 2x(x \ln x - x) - x \ln x + 2x + C \checkmark
\end{aligned}$$

$$\begin{array}{ll}
f(x) = \ln 2x & g'(x) = \ln x \\
f'(x) = \frac{1}{x} & g(x) = x \ln x - x
\end{array}$$

Alternative

$$\begin{aligned}
& \int \ln 2x \ln x dx \\
&= \ln x(x \ln 2x - x) - \int \frac{1}{x}(x \ln 2x - x) dx \checkmark \checkmark \checkmark \\
&= \ln x(x \ln 2x - x) - \int (\ln 2x - 1) dx \checkmark \\
&= \ln x(x \ln 2x - x) - (x \ln 2x - x - x) + C \checkmark \checkmark \checkmark \\
&= \ln x(x \ln 2x - x) - x \ln 2x + 2x + C \checkmark \\
&= x \ln x \ln 2x - x \ln x - x \ln 2x + 2x + C \\
&= \ln 2x(x \ln x - x) - x \ln x + 2x + C
\end{aligned}$$

$$\begin{array}{ll}
f(x) = \ln x & g'(x) = \ln 2x \\
f'(x) = \frac{1}{x} & g(x) = x \ln 2x - x
\end{array}$$

(8)

2.3

$$\begin{aligned}
 & \int \frac{1 + \tan^2 x}{\tan^3 x} dx \\
 & \int (\tan x)^{-3} \sec^2 x dx \checkmark \checkmark \\
 & = \frac{(\tan x)^{-2}}{-2} + C \checkmark \checkmark \\
 & = -\frac{1}{2} \frac{1}{\tan^2 x} + C \\
 & = -\frac{1}{2} \cot^2 x + C
 \end{aligned}$$

Alternative 1

$$\begin{aligned}
 & \int \frac{1 + \tan^2 x}{\tan^3 x} dx \\
 & = \int \left(\frac{1}{\tan^3 x} + \frac{\tan^2 x}{\tan^3 x} \right) dx \\
 & = \int (\cot^3 x + \cot x) dx \checkmark \\
 & = \int (\cot x \cot^2 x + \cot x) dx \\
 & = \int (\cot x (\cos ec^2 x - 1) + \cot x) dx \checkmark \\
 & = \int \cot x \cos ec^2 x - \cot x + \cot x dx \\
 & = \int \cot x \cos ec^2 x dx \checkmark \\
 & = -\frac{1}{2} \cot^2 x + C \checkmark
 \end{aligned}$$

(4)

Alternative 2

$$\begin{aligned}
 & \int \frac{1 + \tan^2 x}{\tan^3 x} dx \\
 & = \int \frac{\sec^2 x}{\tan^3 x} dx \\
 & = \int \frac{1}{\cos^2 x} \frac{\cos^3 x}{\sin^3 x} dx \checkmark \\
 & = \int \frac{\cos x}{\sin^3 x} dx \checkmark \\
 & = \int (\sin x)^{-3} \cos x dx \checkmark \\
 & = \frac{(\sin x)^{-2}}{-2} + C \checkmark \\
 & = -\frac{1}{2} (\cos ec^2 x) + C \\
 & = -\frac{1}{2} (1 + \cot^2 x) + C \\
 & = -\frac{1}{2} \cot^2 x - \frac{1}{2} + C \\
 & = -\frac{1}{2} \cot^2 x + C \quad \text{Because } -\frac{1}{2} + C \text{ is yet another constant}
 \end{aligned}$$

2.4

$$\begin{aligned}
 & \int (\sin^3 x + \cos^3 x) dx \\
 & = \int (\sin^2 x \sin x + \cos^2 x \cos x) dx \checkmark \checkmark \\
 & = \int (1 - \cos^2 x) \sin x + (1 - \sin^2 x) \cos x dx \checkmark \checkmark \\
 & = \int \sin x - \cos^2 x \sin x + \cos x - \sin^2 x \cos x dx \checkmark \checkmark \\
 & = -\cos x + \frac{1}{3} \cos^3 x + \sin x - \frac{1}{3} \sin^3 x + C \checkmark \checkmark \checkmark \checkmark
 \end{aligned}$$

(10)

Alternative

$$\begin{aligned}
 & \int (\sin^3 x + \cos^3 x) dx \\
 &= \int (\sin x + \cos x) (\sin^2 x - \sin x \cos x + \cos^2 x) dx \checkmark \checkmark \checkmark \\
 &= \int (\sin x + \cos x) (1 - \sin x \cos x) dx \checkmark \\
 &= \int (\sin x + \cos x - \sin^2 x \cos x - \cos^2 x \sin x) dx \checkmark \checkmark \\
 &= -\cos x + \sin x - \frac{1}{3} \sin^3 x + \frac{1}{3} \cos^3 x + C \checkmark \checkmark \checkmark \checkmark
 \end{aligned}$$

2.5

$$\begin{aligned}
 & \int 3 \tan^{-1} \frac{x}{3} dx \\
 &= 3x \tan^{-1} \frac{x}{3} - \int \frac{3}{x^2 + 9} 3x dx \checkmark \checkmark \\
 &= 3x \tan^{-1} \frac{x}{3} - \frac{9}{2} \ln(x^2 + 9) + C \checkmark
 \end{aligned}$$

$$f(x) = \tan^{-1} \frac{x}{3}$$

$$g'(x) = 3$$

$$\begin{aligned}
 f'(x) &= \frac{\frac{1}{3}}{1 + \frac{x^2}{9}} \checkmark \\
 &= \frac{3}{x^2 + 9} \checkmark
 \end{aligned}$$

$$g(x) = 3x \checkmark$$

(6)

Alternative

$$\int 3 \tan^{-1} \frac{x}{3} dx = 3x \tan^{-1} \frac{x}{3} - \int \frac{x}{1 + \frac{x^2}{9}} dx \checkmark \checkmark \checkmark$$

$$= 3x \tan^{-1} \frac{x}{3} - \frac{9}{2} \int \frac{\frac{2}{9}x}{1 + \frac{x^2}{9}} dx \checkmark$$

$$= 3x \tan^{-1} \frac{x}{3} - \frac{9}{2} \ln \left(1 + \frac{x^2}{9} \right) + C \checkmark \checkmark$$

$$f(x) = \tan^{-1} \frac{x}{3} \Rightarrow f'(x) = \frac{\frac{1}{3}}{1 + \frac{x^2}{9}}$$

$$g'(x) = 3 \Rightarrow g(x) = 3x$$

[36]

QUESTION 3

3.1 $6x^2 + x - 1 = (3x - 1)(2x + 1) \checkmark$

$$\frac{5 - 5x}{6x^2 + x - 1} = \frac{A}{3x - 1} + \frac{B}{2x + 1} \checkmark$$

$$5 - 5x = A(2x + 1) + B(3x - 1) \checkmark$$

$$\text{when } x = -\frac{1}{2} \quad B = -3 \checkmark$$

$$\text{when } x = \frac{1}{3} \quad A = 2 \checkmark$$

$$\int x + \frac{5 - 5x}{6x^2 + x - 1} dx = \int x + \frac{2}{3x - 1} + \frac{-3}{2x + 1} dx \checkmark \checkmark \therefore A = 2 \quad B = -3$$

$$= \frac{x^2}{2} + \frac{2}{3} \ln(3x - 1) - \frac{3}{2} \ln(2x + 1) + C \checkmark \checkmark \checkmark \quad (10)$$

3.2
$$\frac{2x^3 + 6x^2 - 12}{x(x+3)(x^2 + 3x + 4)} = \frac{A}{x} + \frac{B}{x+3} + \frac{Cx+D}{x^2 + 3x + 4} \quad \checkmark$$

$$2x^3 + 6x^2 - 12 = A(x+3)(x^2 + 3x + 4) + Bx(x^2 + 3x + 4) + (Cx+D)x(x+3) \quad \checkmark$$

If $x = -3$ $-54 + 54 - 12 = B(-3)(9 - 9 + 4) \quad \checkmark \quad -12 = -12B \quad \therefore B = 1 \quad \checkmark$

If $x = 0$ $-12 = A.3.4 \quad \checkmark \quad \therefore A = -1 \quad \checkmark$

equating x^3 : $A + B + C = 2 \Rightarrow -1 + 1 + C = 2 \quad \checkmark \quad \therefore C = 2 \quad \checkmark$

If $x = 1$ $2 + 6 - 12 = A(1+3)(1+3+4) + B.1.(1+3+4) + (C.1+D).1.(1+3) \quad \checkmark$

$$-4 = 32A + 8B + 4C + 4D \quad \dots(1)$$

substituting $A = -1; B = 1$ and $C = 2$ $-4 = 32(-1) + 8.1 + 4.2 + 4D \quad \therefore D = 3 \quad \checkmark$

$$\int \frac{2x^3 + 6x^2 - 12}{x(x+3)(x^2 + 3x + 4)} = \int \frac{-1}{x} dx + \int \frac{1}{x+3} dx + \int \frac{2x+3}{x^2 + 3x + 4} dx \quad \checkmark$$

$$= -\ln x + \ln(x+3) + \ln(x^2 + 3x + 4) + C \quad \checkmark \quad \checkmark \quad \checkmark \quad (14)$$

Alternative for constants A, B, C and D

$$2x^3 + 6x^2 - 12 = A(x^3 + 3x^2 + 4x + 3x^2 + 9x + 12) + B(x^3 + 3x^2 + 4x) + (Cx^2 + Dx)(x+3)$$

$$= Ax^3 + 6Ax^2 + 13Ax + 12A + Bx^3 + 3Bx^2 + 4Bx + Cx^3 + Dx^2 + 3Cx^2 + 3Dx \quad \checkmark \dots\dots\dots(1)$$

Equating x^3 $A + B + C = 2 \quad \dots\dots(2)$

Equating x^2 $6A + 3B + D + 3C = 6 \quad \dots\dots\dots(3)$

Equating x $13A + 4B + 3D = 0 \quad \dots\dots\dots(4)$

Equating constants $12A = -12 \quad \therefore A = -1 \quad \checkmark$

$A = -1$ in (2) $B + C = 3 \quad \Rightarrow C = 3 - B \quad \checkmark$

$A = -1$ in (4) $-13 + 4B + 3D = 0 \Rightarrow D = \frac{1}{3}(13 - 4B) \quad \checkmark$

$A = -1; C = 3 - B$ and $D = \frac{1}{3}(13 - 4B)$ into (3) $-6 + 3B + \frac{1}{3}(13 - 4B) + 3(3 - B) = 6 \quad \checkmark$

$$-18 + 9B + 13 - 4B + 27 - 9B = 18$$

$$-4B = -4 \quad \therefore B = 1 \quad \checkmark \quad \text{then} \quad C = 2 \quad \checkmark \quad \text{and} \quad D = 3 \quad \checkmark$$

[24]

QUESTION 4

4.1

$$\frac{dx}{dy} - 3y = 2x$$

$$\frac{dx}{dy} - 2x = 3y \quad \checkmark$$

$$e^{\int p dy} = e^{\int -2 dy} = e^{-2y} \quad \checkmark \checkmark$$

$$\int Q e^{\int p dy} dy = \int 3y e^{-2y} dy \quad \checkmark$$

$$= 3y \frac{e^{-2y}}{-2} - \int 3 \frac{e^{-2y}}{-2} dy \quad \checkmark$$

$$= 3y \frac{e^{-2y}}{-2} + \frac{3}{2} \int e^{-2y} dy$$

$$= 3y \frac{e^{-2y}}{-2} + \frac{3}{2} \cdot \frac{e^{-2y}}{-2} + C \quad \checkmark$$

$$= -\frac{3}{2} y e^{-2y} - \frac{3}{4} e^{-2y} + C$$

$$x e^{-2y} = -\frac{3}{2} y e^{-2y} - \frac{3}{4} e^{-2y} + C \quad \checkmark$$

$$x = 1; \quad y = 0 \quad 1 \cdot e^{-2 \cdot 0} = -\frac{3}{2} \cdot 0 \cdot e^{-2 \cdot 0} - \frac{3}{4} e^{-2 \cdot 0} + C \quad \checkmark$$

$$1 = -\frac{3}{4} + C \quad C = \frac{7}{4} \quad \checkmark$$

$$x e^{-2y} = -\frac{3}{2} y e^{-2y} - \frac{3}{4} e^{-2y} + \frac{7}{4} \quad \checkmark$$

(10)

4.2 $\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = 18e^{-3x}$ $y = 1$ $x = 0$ and $\frac{dy}{dx} = 2$; $x = 0$

$$r^2 - 6r + 9 = 0 \quad \checkmark$$

$$(r - 3)(r - 3) = 0 \quad \checkmark$$

$$r = 3 \quad \checkmark$$

$$y_c = e^{3x}(A + Bx) \quad \text{or} \quad y_c = Ae^{3x} + Bxe^{3x} \quad \checkmark$$

$$y = Ce^{-3x} \quad \checkmark \quad \frac{dy}{dx} = -3Ce^{-3x} \quad \checkmark \quad \frac{d^2 y}{dx^2} = 9Ce^{-3x} \quad \checkmark$$

$$\therefore 9Ce^{-3x} - 6(-3Ce^{-3x}) + 9Ce^{-3x} = 18e^{-3x}$$

$$36Ce^{-3x} = 18e^{-3x} \quad \therefore C = \frac{1}{2} \quad \checkmark$$

$$\therefore y_p = \frac{1}{2}e^{-3x} \quad \checkmark$$

$$y = y_c + y_p \quad y = e^{3x}(A + Bx) + \frac{1}{2}e^{-3x} \quad \checkmark$$

$$y = 1 \quad x = 0 \quad 1 = A + \frac{1}{2} \quad \therefore A = \frac{1}{2} \quad \checkmark$$

Differentiating the general solution : $\frac{dy}{dx} = e^{3x}B + 3e^{3x}(A + Bx) - \frac{3}{2}e^{-3x} \quad \checkmark$

$$\frac{dy}{dx} = 2 ; x = 0 \quad 2 = e^{3 \cdot 0}B + 3e^{3 \cdot 0}(A + B \cdot 0) - \frac{3}{2}e^{-3 \cdot 0}$$

$$2 = B + 3A - \frac{3}{2}$$

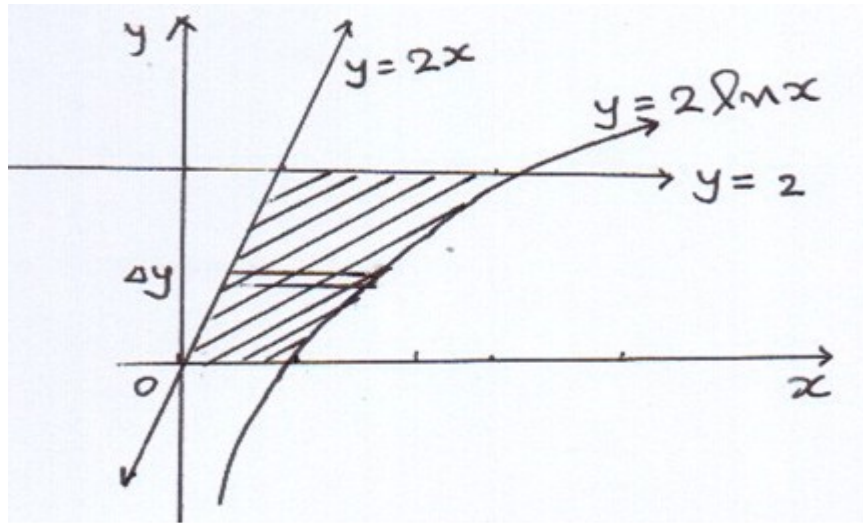
$$2 = B + 3\left(\frac{1}{2}\right) - \frac{3}{2} \quad \therefore B = 2 \quad \checkmark$$

$$y = e^{3x}\left(\frac{1}{2} + 2x\right) + \frac{1}{2}e^{-3x} \quad \checkmark$$

(14)
[24]

QUESTION 5

5.1 5.1.1



- ✓ for $y = 2x$
- ✓ for $y = 2\ln x$
- ✓ strip
- ✓ area

(4)

5.1.2

$$A = \int_a^b (x_1 - x_2) dy \quad \checkmark \quad y = 2\ln x \Rightarrow \ln x = \frac{y}{2} \therefore x = e^{\frac{y}{2}} \quad \checkmark \quad \text{and from } y = 2x \quad x = \frac{y}{2}$$

$$A = \int_0^2 \left(e^{\frac{y}{2}} - \frac{y}{2} \right) dy \quad \checkmark \checkmark$$

$$= \left[2e^{\frac{y}{2}} - \frac{y^2}{4} \right]_0^2 \quad \checkmark \checkmark$$

$$= \left[2e^{\frac{2}{2}} - \frac{2^2}{4} - (2 - 0) \right] \quad \checkmark = 2e - 1 - 2 = 2,437 \text{ units}^2 \quad \checkmark$$

$$A_{m-y} = \int_a^b r dA \quad \checkmark$$

(9)

5.1.3

$$= \int_a^b \frac{x_1 + x_2}{2} (x_1 - x_2) dA \quad \checkmark = \frac{1}{2} \int_a^b x_1^2 - x_2^2 dy \quad \checkmark$$

$$= \frac{1}{2} \int_0^2 \left(e^{\frac{y}{2}} \right)^2 - \left(\frac{y}{2} \right)^2 dy \quad \checkmark \checkmark \checkmark$$

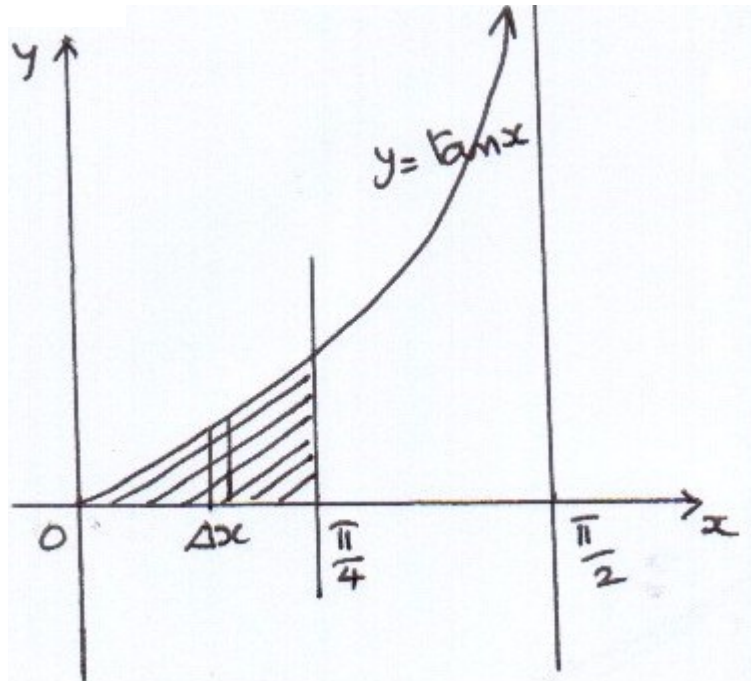
$$= \frac{1}{2} \int_0^2 e^y - \frac{y^2}{4} dy \quad \checkmark$$

$$= \frac{1}{2} \left[e^y - \frac{y^3}{12} \right]_0^2 \quad \checkmark = \frac{1}{2} \left[e^2 - \frac{2^3}{12} - (1 - 0) \right] \quad \checkmark = 2,861 \text{ units}^3 \quad \checkmark$$

$$\bar{x} = \frac{2,861}{2,437} = 1,174 \quad \checkmark \checkmark$$

(11)

5.2 5.2.1



✓✓ each graph
 ✓ strip
 ✓ area

(4)

5.2.2

$$V_x = \pi \int_a^b y_1^2 - y_2^2 dx \quad \text{or} \quad \pi \int_a^b y^2 dx \checkmark$$

$$= \pi \int_0^{\frac{\pi}{4}} \tan^2 x \, dx \checkmark \checkmark$$

$$= \pi \left[\tan x - x \right]_0^{\frac{\pi}{4}} \checkmark$$

$$= \pi \left[\tan \frac{\pi}{4} - \frac{\pi}{4} \right]_0^{\frac{\pi}{4}} \checkmark$$

$$= 0,674 \text{ units}^3 \checkmark$$

$$\pi \int_0^{\frac{\pi}{4}} \tan^2 x \, dx$$

$$= \pi \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) dx$$

(6)

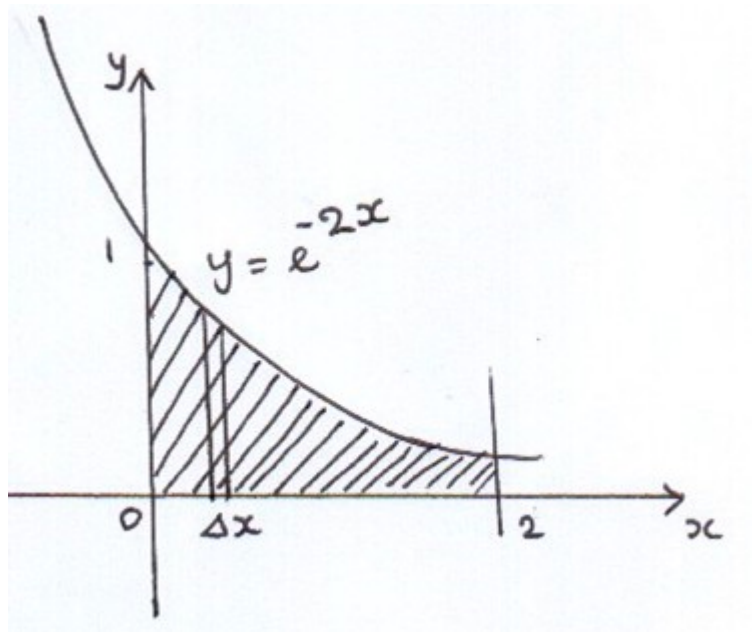
5.2.3

$$\begin{aligned}
 I_x &= \frac{1}{2} \pi \rho \int_a^b y^4 dx \quad \checkmark \\
 &= \frac{1}{2} \pi \rho \int_0^{\frac{\pi}{4}} \tan^4 x dx \quad \checkmark \checkmark \\
 &= \frac{1}{2} \pi \rho \int_0^{\frac{\pi}{4}} \tan^2 x \tan^2 x dx \quad \checkmark \\
 &= \frac{1}{2} \pi \rho \int_0^{\frac{\pi}{4}} \tan^2 x (\sec^2 x - 1) dx \quad \checkmark \\
 &= \frac{1}{2} \pi \rho \int_0^{\frac{\pi}{4}} (\tan^2 x \sec^2 x - \tan^2 x) dx \quad \checkmark \\
 &= \frac{1}{2} \pi \rho \left[\frac{\tan^3 x}{3} - \tan x + x \right]_0^{\frac{\pi}{4}} \quad \checkmark \checkmark = \frac{1}{2} \pi \rho \left[\frac{\tan^3 \frac{\pi}{4}}{3} - \tan \frac{\pi}{4} + \frac{\pi}{4} - (0 - 0 + 0) \right] \checkmark = 0,187 \rho \quad \checkmark
 \end{aligned}$$

(10)

5.3

5.3.1



$\checkmark \checkmark$ graphs
 $\checkmark$ strip
 $\checkmark$ area

(4)

5.3.2

$$A = \int_a^b y dx \quad \text{or} \quad \int_a^b y_1 - y_2 dx \quad \checkmark$$

$$= \int_0^2 e^{-2x} dx \quad \checkmark \checkmark$$

$$= \left[\frac{e^{-2x}}{-2} \right]_0^2 \quad \checkmark$$

$$= \frac{e^{-4}}{-2} - \frac{e^0}{-2} \quad \checkmark$$

$$= 0,491 \text{ units}^2 \quad \checkmark$$

$$I_y = \int_a^b r^2 dA \quad \checkmark$$

$$= \int_0^2 x^2 e^{-2x} dx \quad \checkmark \checkmark$$

(9)

5.3.3

$$= \left[x^2 \frac{e^{-2x}}{-2} - \int 2x \cdot \frac{e^{-2x}}{-2} dx \right]_0^2 \quad \checkmark \checkmark$$

$$\left[x^2 \frac{e^{-2x}}{-2} - \int x e^{-2x} dx \right]_0^2 \quad \checkmark$$

$$\left[x^2 \frac{e^{-2x}}{-2} + x \frac{e^{-2x}}{-2} - \int \frac{e^{-2x}}{-2} dx \right]_0^2 \quad \checkmark$$

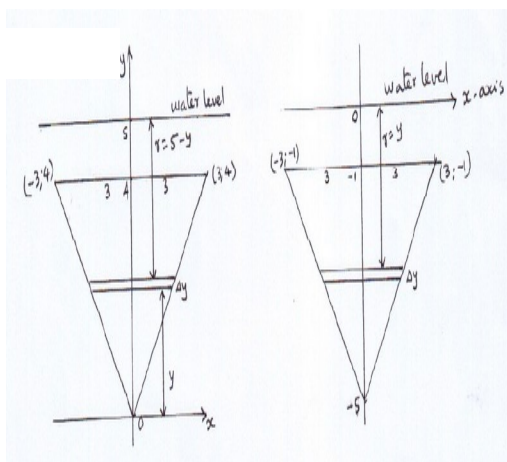
$$\left[x^2 \frac{e^{-2x}}{-2} + x \frac{e^{-2x}}{-2} + \frac{1}{2} \frac{e^{-2x}}{-2} \right]_0^2 \quad \checkmark$$

$$\left[e^{-2x} \left(-\frac{x^2}{2} - \frac{1}{2}x - \frac{1}{4} \right) \right]_0^2 = e^{-4} \left(-\frac{2^2}{2} - \frac{1}{2}(2) - \frac{1}{4} \right) - e^0 \left(0 - 0 - \frac{1}{4} \right) \quad \checkmark$$

$$= 0.190 \text{ units}^4 \quad \checkmark$$

(7)

5.4 5.4.1



- ✓ water level at 5
- ✓ height of plate at 4
- ✓ point (3;4)
- ✓ strip

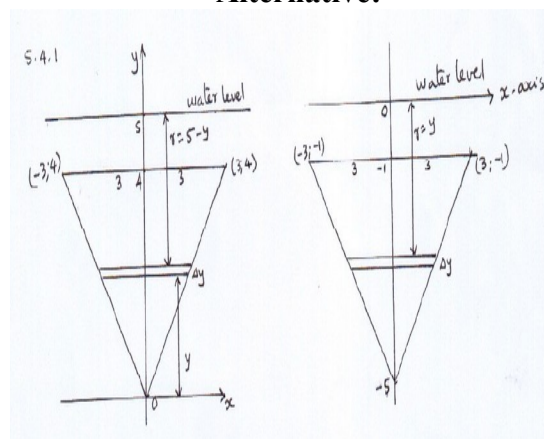
$$m = \frac{4-0}{3-0} = \frac{4}{3} \quad \checkmark$$

$$y - y_1 = m(x - x_1)$$

$$y = \frac{4}{3}x \quad \checkmark \quad x = \frac{3}{4}y$$

$$\text{second moment} = \int_a^b r^2 dA$$

Alternative:



- ✓ Water level at x axis
- ✓ point (3;-1)
- ✓ height of plate at -1
- ✓ strip

$$(3; -1) \quad (0; -5) \quad m = \frac{-1+5}{3-0} = \frac{4}{3} \quad \checkmark$$

$$y + 5 = \frac{4}{3}x \quad \checkmark \quad x = \frac{3}{4}(y + 5)$$

$$\text{second moment} = \int_a^b r^2 dA$$

(6)

5.4.2

$$\begin{aligned} & \int_0^4 (5-y)^2 2 \cdot \frac{3}{4} y dy \quad \checkmark \checkmark \checkmark \\ & \frac{3}{2} \int_0^4 (25 - 10y + y^2) y dy \quad \checkmark \\ & \frac{3}{2} \int_0^4 (25y - 10y^2 + y^3) dy \quad \checkmark \\ & \frac{3}{2} \left[25 \frac{y^2}{2} - 10 \frac{y^3}{3} + \frac{y^4}{4} \right]_0^4 \quad \checkmark \\ & \frac{3}{2} \left[25 \frac{4^2}{2} - 10 \frac{4^3}{3} + \frac{4^4}{4} - 0 \right] \quad \checkmark \\ & = 76m^4 \quad \checkmark \\ & = \frac{76}{28} = 2,714m \quad \checkmark \checkmark \end{aligned}$$

Alternative:

$$\begin{aligned} & = \int_{-5}^{-1} y^2 2 \cdot \frac{3}{4} (y+5) dy \quad \checkmark \checkmark \checkmark \\ & \frac{3}{2} \int_{-5}^{-1} y^3 + 5y^2 dy \quad \checkmark \\ & \frac{3}{2} \left[\frac{y^4}{4} + \frac{5y^3}{3} \right]_{-5}^{-1} \quad \checkmark \\ & \frac{3}{2} \left[\frac{(-1)^4}{4} + \frac{5(-1)^3}{3} - \left\{ \frac{(-5)^4}{4} + \frac{(-5)^3}{3} \right\} \right] \quad \checkmark \checkmark \\ & 76m^4 \quad \checkmark \\ & = \frac{76}{-28} = -2,714m \quad \checkmark \checkmark \end{aligned}$$

(10)
[80]

QUESTION 6

6.1

$$y = 9 - x^2 \quad \frac{dy}{dx} = -2x \quad \left(\frac{dy}{dx}\right)^2 = 4x^2$$

$$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + 4x^2$$

$$= \int_0^3 \sqrt{1 + 4x^2} dx = \int_0^3 \sqrt{4\left(\frac{1}{4} + x^2\right)} dx$$

$$= 2 \int_0^3 \sqrt{\frac{1}{4} + x^2} dx$$

$$= 2 \left[\frac{x}{2} \sqrt{\frac{1}{4} + x^2} + \frac{1}{2} \ln \left(x + \sqrt{\frac{1}{4} + x^2} \right) \right]_0^3$$

$$= 2 \left[\frac{3}{2} \sqrt{\frac{1}{4} + 3^2} + \frac{1}{2} \ln \left(3 + \sqrt{\frac{1}{4} + 3^2} \right) - \left(0 + \frac{1}{2} \ln \left(0 + \frac{1}{2} \right) \right) \right]$$

$$9,747 \text{ units}$$

(12)

6.2

$$A_y = 2\pi \int_a^b x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$x = y^3 \quad \frac{dx}{dy} = 3y^2 \quad \left(\frac{dx}{dy}\right)^2 = 9y^4$$

$$1 + \left(\frac{dx}{dy}\right)^2 = 1 + 9y^4$$

$$A_y = 2\pi \int_0^1 y^3 \sqrt{1 + 9y^4} dy$$

$$A_y = \frac{2\pi}{36} \int_0^1 36y^3 \sqrt{1 + 9y^4} dy \quad \text{or using } u = 1 + 9y^4$$

$$= \frac{2\pi}{36} \frac{2}{3} \left[(1 + 9y^4)^{\frac{3}{2}} \right]_0^1$$

$$= \frac{\pi}{27} \left[(1 + 9 \cdot 1)^{\frac{3}{2}} - (1 + 0)^{\frac{3}{2}} \right]$$

$$= 3,563 \text{ units}^2$$

(12)

ALTERNATIVE

$$= y^3 \text{ between } 0 \leq y \leq 1 \Rightarrow y = x^{\frac{1}{3}} \text{ between } 0 \leq x \leq 1 \checkmark$$

$$\Rightarrow A_y = \int_a^b 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \checkmark$$

$$= \int_0^1 2\pi x \sqrt{1 + \frac{1}{9x^{\frac{4}{3}}}} dx$$

$$= \int_0^1 2\pi x \sqrt{\frac{1+9x^{\frac{4}{3}}}{9x^{\frac{4}{3}}}} dx \checkmark$$

$$= \frac{2}{3} \pi \int_0^1 x \cdot \frac{1}{x^{\frac{2}{3}}} \sqrt{1 + 9x^{\frac{4}{3}}} dx \checkmark$$

$$= \frac{2}{3} \pi \int_0^1 x^{\frac{1}{3}} \sqrt{1 + 9x^{\frac{4}{3}}} dx \checkmark$$

$$= \frac{2}{3} \pi \int_0^1 x^{\frac{1}{3}} \sqrt{u} \frac{du}{12x^{\frac{1}{3}}} \checkmark$$

$$= \frac{\pi}{18} \int_1^{10} u^{\frac{1}{2}} du$$

$$= \frac{\pi}{18} \left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^{10} \checkmark$$

$$= \frac{\pi}{27} \left[10^{\frac{3}{2}} - 1 \right] \checkmark$$

$$= 3,563 \checkmark$$

$$y = x^{\frac{1}{3}} \Rightarrow \frac{dy}{dx} = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}} \checkmark$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{9x^{\frac{4}{3}}}$$

$$\therefore \left(\frac{dy}{dx}\right)^2 = \frac{1}{9x^{\frac{4}{3}}}$$

$$\text{let } u = 1 + 9x^{\frac{4}{3}} \Rightarrow \frac{du}{dx} = 12x^{\frac{1}{3}}$$

$$\Rightarrow dx = \frac{du}{12x^{\frac{1}{3}}} \checkmark$$

$$\text{if } x = 0 \Rightarrow u = 1$$

$$\text{if } x = 1 \Rightarrow u = 10$$

[24]

TOTAL: $200 \div 2 = 100$

