



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

1 August 2023

This marking guideline consists of 18 pages.

QUESTION 1

$$1.1 \quad 1.1.1 \quad Z = \log_y x = \frac{\log_e x}{\log_e y} = \frac{\ln x}{\ln y} = \ln x (\ln y)^{-1}$$

$$\Rightarrow \frac{\partial Z}{\partial x} = \frac{1}{x \ln y} \checkmark$$

$$1.1.2 \quad \frac{\partial Z}{\partial y} = -\ln x (\ln y)^{-2} \cdot \frac{1}{y} = -\frac{\ln x}{y (\ln y)^2}$$

(2 × 2) (4)

$$1.2 \quad y = \tan t \quad x = \sec t$$

$$\frac{dy}{dt} = \sec^2 t \quad \checkmark$$

$$\frac{dx}{dt} = \sec t \tan t \quad \checkmark$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec^2 t}{\sec t \tan t} = \frac{\sec t}{\tan t} = \frac{1}{\cos t} \times \frac{\cos t}{\sin t} = \frac{1}{\sin t} \checkmark$$

$$m = \frac{1}{\sin \frac{\pi}{4}} = \sqrt{2}$$

$$x_1 = \sec \frac{\pi}{4} = \frac{1}{\cos \frac{\pi}{4}} = \sqrt{2}$$

$$y_1 = \tan \frac{\pi}{4} = 1$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \sqrt{2}(x - \sqrt{2})$$

$$y = \sqrt{2}x - 1$$

$$y = 1.414x - 1$$

ALTERNATIVE

$$y = mx + C$$

$$1 = \sqrt{2}(\sqrt{2}) + C$$

$$C = -1$$

$$y = \sqrt{2}x - 1$$

(8)
[12]

QUESTION 2

2.1 $f(x) = \ln x \quad f'(x) = \frac{1}{x} \wedge g'(x) = x^3 \quad g(x) = \frac{x^4}{4}$

$$\int x^3 \ln x dx = \frac{x^4 \ln x}{4} - \frac{1}{4} \int \frac{1}{x} \cdot x^4 dx \quad \checkmark \checkmark$$

$$= \frac{x^4 \ln x}{4} - \frac{1}{4} \int x^3 dx \quad \checkmark$$

$$= \frac{x^4 \ln x}{4} - \frac{1}{16} x^4 + C \quad \checkmark$$

(4)

2.2 $\int \cos^3 2x \sin^5 2x dx$

$$= \int \cos^2 2x \cos 2x \sin^5 2x dx \quad \checkmark$$

$$= \int (1 - \sin^2 2x) \cos 2x \sin^5 2x dx \quad \checkmark$$

$$= \int \sin^5 2x \cos 2x dx - \int \sin^7 2x \cos 2x dx \quad \checkmark \checkmark$$

$u = \sin 2x \Rightarrow \frac{du}{dx} = 2 \cos 2x$

$$= \frac{1}{2} \int u^5 du - \frac{1}{2} \int u^7 du \quad \checkmark$$

$$= \frac{u^6}{12} - \frac{u^8}{16} + C \quad \checkmark$$

$$= \frac{\sin^6 2x}{12} - \frac{\sin^8 2x}{16} + C \quad \checkmark \checkmark$$

(8)

ALTERNATIVE 1

$$\int \cos^3 2x \sin^5 2x dx$$

$$= \int \cos^2 2x \cos 2x \sin^5 2x dx \quad \checkmark$$

$$= \int (1 - \sin^2 2x) \cos 2x \sin^5 2x dx \quad \checkmark$$

$$= \int \sin^5 2x \cos 2x dx - \int \sin^7 2x \cos 2x dx \quad \checkmark \checkmark$$

$$\int \frac{1}{2} \sin^6 2x \, dx$$

$$= \frac{\sin^6 2x}{12} - \frac{\sin^8 2x}{16} + C \quad \checkmark \checkmark$$

(8)

ALTERNATIVE 2

$$\int \cos^3 2x \sin^5 2x \, dx$$

$$\int \cos^3 2x (\sin^2 2x)^2 \sin 2x \, dx$$

$$\int \cos^3 2x (1 - \cos^2 2x)^2 \sin 2x \, dx$$

$$\int \cos^3 2x (1 - 2\cos^2 2x + \cos^4 2x) \sin 2x \, dx$$

$$\int \cos^3 2x \sin 2x \, dx - 2 \int \cos^5 2x \sin 2x \, dx + \int \cos^7 2x \sin 2x \, dx$$

$$-\frac{1}{2} \int [\cos 2x]^3 \sin 2x \, dx$$

$$-\frac{\cos^4 2x}{8} + \frac{\cos^6 2x}{6} - \frac{\cos^8 2x}{16} + C$$

(8)

ALTERNATIVE 3

$$\int \cos^3 2x \sin^5 2x \, dx$$

$$\int \cos^3 2x (\sin^2 2x)^2 \sin 2x \, dx$$

$$\int \cos^3 2x (1 - \cos^2 2x)^2 \sin 2x \, dx$$

$$\int \cos^3 2x (1 - 2\cos^2 2x + \cos^4 2x) \sin 2x \, dx$$

$$\int \cos^3 2x \sin 2x \, dx - 2 \int \cos^5 2x \sin 2x \, dx + \int \cos^7 2x \sin 2x \, dx$$

$$-\frac{1}{2} \int u^3 \, du + \int u^5 \, du - \frac{1}{2} \int u^7 \, du$$

$$-\frac{u^4}{8} + \frac{u^6}{6} - \frac{u^8}{16} + C$$

$$-\frac{\cos^4 2x}{8} + \frac{\cos^6 2x}{6} - \frac{\cos^8 2x}{16} + C$$

$$u = \cos 2x \Rightarrow \frac{du}{dx} = -2 \sin 2x$$

(8)

2.3

$$4x^2 - 4x - 3 = 4 \left[x^2 - x - \frac{3}{4} \right] \checkmark$$

$$= 4 \left[x^2 - x + \frac{1}{4} - \frac{1}{4} - \frac{3}{4} \right] \checkmark$$

$$= 4 \left[\left(x - \frac{1}{2} \right)^2 - 1 \right] \checkmark$$

$$\therefore 4 \left(x - \frac{1}{2} \right)^2 - 4$$

$$\therefore \int \frac{dx}{\sqrt{4x^2 - 4x - 3}} = \int \frac{dx}{\sqrt{4 \left(x - \frac{1}{2} \right)^2 - 4}}$$

$$\therefore \frac{1}{2} \ln \left[2 \left(x - \frac{1}{2} \right) + \sqrt{4 \left(x - \frac{1}{2} \right)^2 - 4} \right] + C$$

(8)

2.4

$$\int \cos^4 4x dx = \int (\cos^2 4x)^2 dx \checkmark$$

$$= \int \left[\frac{1}{2} (1 + \cos 8x) \right]^2 dx \checkmark$$

$$= \frac{1}{4} \int (1 + 2 \cos 8x + \cos^2 8x) dx \checkmark$$

$$= \frac{1}{4} \int \left(1 + 2 \cos 8x + \frac{1}{2} + \frac{1}{2} \cos 16x \right) dx \checkmark$$

$$= \frac{1}{4} \int \left(\frac{3}{2} + 2 \cos 8x + \frac{1}{2} \cos 16x \right) dx \checkmark$$

$$= \frac{1}{4} \left[\frac{3}{2} x + \frac{\sin 8x}{4} + \frac{\sin 16x}{32} \right] + C \checkmark \checkmark \checkmark$$

$$= \frac{3}{8} x + \frac{\sin 8x}{16} + \frac{\sin 16x}{128} + C$$

(8)

ALTERNATIVE

$$\begin{aligned}
\int \cos^4 4x dx &= \int \cos^2 4x \cos^2 4x dx \quad \checkmark \\
&= \int \left(\frac{1}{2} + \frac{1}{2} \cos 8x \right) \left(\frac{1}{2} + \frac{1}{2} \cos 8x \right) dx \quad \checkmark \\
&= \int \left(\frac{1}{4} + \frac{1}{2} \cos 8x + \frac{1}{4} \cos^2 8x \right) dx \quad \checkmark \\
&= \frac{1}{4} \int (1 + 2 \cos 8x + \cos^2 8x) dx \quad \checkmark \\
&= \frac{1}{4} \left[x + \frac{\sin 8x}{4} + \frac{1}{2} x + \frac{\sin 16x}{32} \right] + C \quad \checkmark \checkmark \checkmark \checkmark \\
&= \frac{1}{4} \left[\frac{3}{2} x + \frac{\sin 4x}{2} + \frac{\sin 8x}{16} \right] + C \\
&= \frac{3}{8} x + \frac{\sin 8x}{16} + \frac{\sin 16x}{128} + C
\end{aligned} \tag{8}$$

2.5

$$f(x) = 5x - x^3 \quad f'(x) = 5 - 3x^2 \quad g'(x) = \cos 6x \quad g(x) = \frac{\sin 6x}{6}$$

$$\int (5x - x^3) \cos 6x dx$$

$$\int \frac{1}{6} (5x - x^3) \sin 6x - \frac{1}{6} \int (5 - 3x^2) \sin 6x dx$$

$$f(x) = 5 - 3x^2 \quad f'(x) = -6x \quad g'(x) = \sin 6x \quad g(x) = \frac{-\cos 6x}{6}$$

$$\int \frac{1}{6} (5x - x^3) \sin 6x - \frac{1}{6} \left[\frac{-1}{6} (5 - 3x^2) \cos 6x - \int x \cos 6x dx \right]$$

$$f(x) = x \quad f'(x) = 1 \quad g'(x) = \cos 6x \quad g(x) = \frac{\sin 6x}{6}$$

$$\int \frac{1}{6} (5x - x^3) \sin 6x + \frac{1}{36} (5 - 3x^2) \cos 6x + \frac{1}{6} \left[\frac{1}{6} x \sin 6x - \frac{1}{6} \int \sin 6x dx \right]$$

$$\int \frac{1}{6} (5x - x^3) \sin 6x + \frac{1}{36} (5 - 3x^2) \cos 6x + \frac{1}{36} x \sin 6x + \frac{1}{216} \cos 6x + C$$

(8)
[36]

QUESTION 3

3.1

$$\text{Let } \frac{1-x}{x(x+1)^2} = \frac{A}{x} + \frac{B}{(x+1)^2} + \frac{C}{x+1} \quad \checkmark \checkmark$$

$$1-x = A(x+1)^2 + Bx + Cx(x+1) \quad \checkmark$$

$$x=0 \quad A=1$$

$$x=-1 \quad B=-2$$

$$1-x = Ax^2 + 2Ax + A + Bx + Cx^2 + Cx$$

OR

$$x^2: 0 = A + C \quad C = -1 \quad \checkmark$$

$$x=-2 \quad 3 = A - 2B + 2C$$

$$C = -1$$

$$\int \frac{1-x}{x^3 + 2x^2 + x} dx = \int \frac{dx}{x} - 2 \int \frac{dx}{(x+1)^2} - \int \frac{dx}{x+1}$$

$$\int \frac{dx}{x} - 2 \int (x+1)^{-2} dx - \int \frac{dx}{x+1}$$

$$= \ln x + \frac{2}{x+1} - \ln(x+1) + C \quad \checkmark \checkmark \checkmark$$

(12)

3.2

$$\frac{-x^2+1}{(3-2x)^3} = \frac{A}{3-2x} + \frac{B}{(3-2x)^2} + \frac{C}{(3-2x)^3}$$

$$-x^2+1 = A(3-2x)^2 + B(3-2x) + C$$

$$-x^2+1 = (9-12x+4x^2) + 3B-2Bx + C$$

$$x^2: -1 = 4A \quad A = \frac{-1}{4}$$

OR

$$x: 0 = -12A - 2B \quad B = \frac{3}{2}$$

$$\text{constant: } 1 = 9A + 3B + C \quad C = \frac{-5}{4}$$

$$x = \frac{3}{2} \quad C = \frac{-5}{4}$$

$$x = 0 \quad \frac{3}{4} = 3A + B \dots \text{Eqn 1}$$

$$x = 1 \quad \frac{5}{4} = A + B \dots \text{Eqn 2}$$

$$\text{Eqn 1} - \text{Eqn 2} \quad A = \frac{-1}{4}$$

$$\text{Eqn 2} \quad B = \frac{3}{2}$$

$$\int \frac{-x^2+1}{(3-2x)^3} dx = \frac{-1}{4} \int \frac{1}{3-2x} dx + \frac{3}{2} \int \frac{1}{(3-2x)^2} dx - \frac{5}{4} \int \frac{1}{(3-2x)^3} dx$$

$$\frac{1}{8} \int \frac{-2}{3-2x} dx - \frac{3}{4} \int (-2)(3-2x)^{-2} dx + \frac{5}{8} \int (-2)(3-2x)^{-3} dx$$

$$\frac{1}{8} \ln(3-2x) + \frac{3}{4(3-2x)} - \frac{5}{16(3-2x)^2} + C$$

(12)
[24]

QUESTION 4

4.1

$$\frac{dr}{d\theta} + r \cot \theta = \sin \theta \cos \theta \quad \checkmark \checkmark$$

$$\text{IF} = e^{\int \cot \theta d\theta} = e^{\ln(\sin \theta)} = \sin \theta \quad \checkmark \checkmark \checkmark$$

$$r \sin \theta = \int \sin^2 \theta \cos \theta d\theta \quad \checkmark \quad \checkmark$$

OR

$$= \frac{\sin^3 \theta}{3} + C \quad \checkmark$$

$$3 \sin \frac{\pi}{6} = \frac{\left(\sin \frac{\pi}{6}\right)^3}{3} + C \quad C = \frac{35}{24} = 1.458$$

$$r \sin \theta = \frac{\sin^3 \theta}{3} + \frac{35}{24}$$

(10)

4.2

$$m^2 - m - 2 = 0 \quad \checkmark$$

(14)

$$m = -1 \text{ or } m = 2 \quad \checkmark \checkmark$$

$$y_c = Ae^{-x} + Be^{2x} \quad \checkmark$$

$$y_p = Cx + D \quad \checkmark \Rightarrow y'_p = C \quad y''_p = 0 \quad \checkmark$$

$$-C - 2(Cx + D) = 1 - 2x$$

$$x: \quad -2C = -2 \Rightarrow C = 1 \quad \checkmark$$

$$\text{Constant:} \quad -C - 2D = 1 \Rightarrow D = -1 \quad \checkmark$$

$$y_p = x - 1 \quad \checkmark$$

$$y = y_c + y_p = Ae^{-x} + Be^{2x} + x - 1 \quad \checkmark$$

$$x=0 \wedge y=0 = A+B-1 \dots\dots\dots \text{Eqn.1}$$

$$\frac{dy}{dx} = -Ae^{-x} + 2Be^{2x} + 1 \quad \checkmark$$

$$x = \frac{0 \wedge dy}{dx} = 1 = -A + 2B + 1 \dots\dots\dots \text{Eqn.2}$$

$$\text{Eqn.1} + \text{Eqn.2} \quad B = \frac{1}{3}$$

$$\text{Eqn.1} \quad A = \frac{2}{3}$$

$$y = \frac{2}{3}e^{-x} + \frac{1}{3}e^{2x} + x - 1 \quad \checkmark$$

[24]

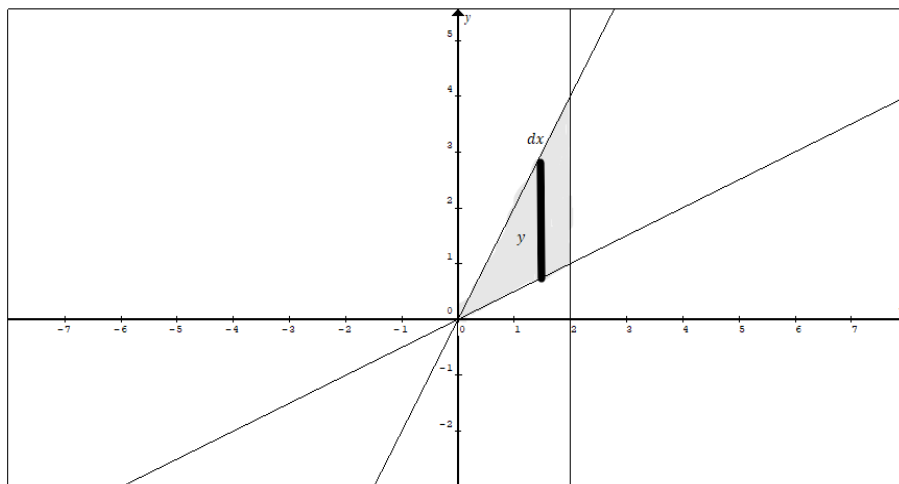
QUESTION 5

5.1

5.1.1

$$2x = \frac{x}{2} \quad x=0 \wedge y=0$$

$$\therefore (0;0)$$



Graphs✓✓✓; Shaded area✓ and Strip✓

(6)

5.1.2

$$V_y = 2\pi \int_a^b xy dx \vee V_y = 2\pi \int_a^b x(y_1 - y_2) dx$$

$$= 2\pi \int_0^2 x \left(2x - \frac{x}{2} \right) dx \quad \checkmark \checkmark$$

$$= 2\pi \int_0^2 x \left(\frac{3x}{2} \right) dx \quad \checkmark$$

$$= 2\pi \int_0^2 \left(\frac{3x^2}{2} \right) dx \quad \checkmark$$

$$= 2\pi \left[\frac{1}{2} x^3 \right]_0^2 \quad \checkmark$$

$$\checkmark 2\pi \left[\frac{1}{2} (2)^3 \right] - 2\pi \left[\frac{1}{2} (0)^3 \right]$$

$$= 8\pi \text{ (units)}^3 \quad \checkmark$$

$$\begin{aligned} & \checkmark 2\pi \int_0^2 \left(2x^2 - \frac{x^2}{2} \right) dx \\ & \checkmark 2\pi \left[\frac{2x^3}{3} - \frac{x^3}{6} \right]_0^2 \\ & \checkmark 2\pi \left[\frac{2(2)^3}{3} - \frac{(2)^3}{6} \right] \\ & = 8\pi \text{ (units)}^3 \quad \checkmark \end{aligned}$$

(8)

5.2

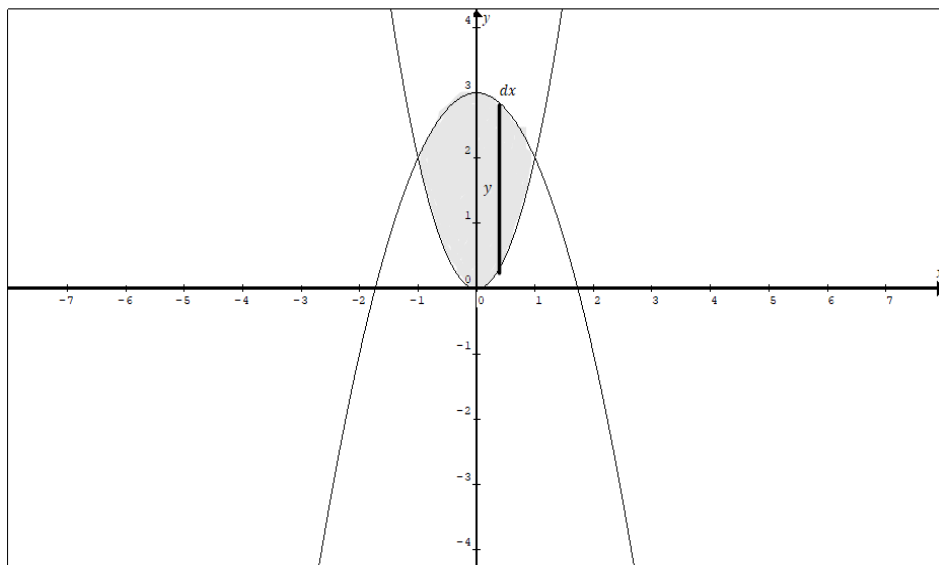
5.2.1

$$3 - x^2 = 2x^2$$

$$x = -1 \text{ or } x = 1$$

$$y = 2 \vee y = 2$$

$$\therefore (-1; 2) \wedge (1; 2)$$



Graphs✓✓; Shaded area✓ and Strip✓

(6)

5.2.2

$$A_x = \int_a^b (y_1 - y_2) dx$$

$$A = \int_{-1}^1 (3 - x^2 - 2x^2) dx \quad \checkmark \checkmark$$

$$= \int_{-1}^1 (3 - 3x^2) dx$$

$$= \left[3x - x^3 \right]_{-1}^1 \checkmark$$

$$= [3 - 1 + 3 - 1] \checkmark$$

$$= 4(\text{units})^2 \checkmark$$

(6)

5.2.3

$$A_{m-x} = \int_a^b r \, dA$$

$$A_{max} = \frac{1}{2} \int_{-1}^1 (y_2 + y_1)(y_2 - y_1) \, dx \quad \checkmark$$

$$= \frac{1}{2} \int_{-1}^1 (y_2^2 - y_1^2) \, dx \quad \checkmark$$

$$= \frac{1}{2} \int_{-1}^1 (9 - 6x^2 + x^4 - 4x^4) \, dx \quad \checkmark$$

$$= \frac{1}{2} \int_{-1}^1 (9 - 6x^2 - 3x^4) \, dx \quad \checkmark$$

$$= \frac{1}{2} \left[9x - 2x^3 - \frac{3x^5}{5} \right]_{-1}^1 \quad \checkmark$$

$$= \frac{1}{2} \left[9 - 2 - \frac{3}{5} + 9 - 2 - \frac{3}{5} \right] \quad \checkmark$$

$$= 6.4 \quad \checkmark$$

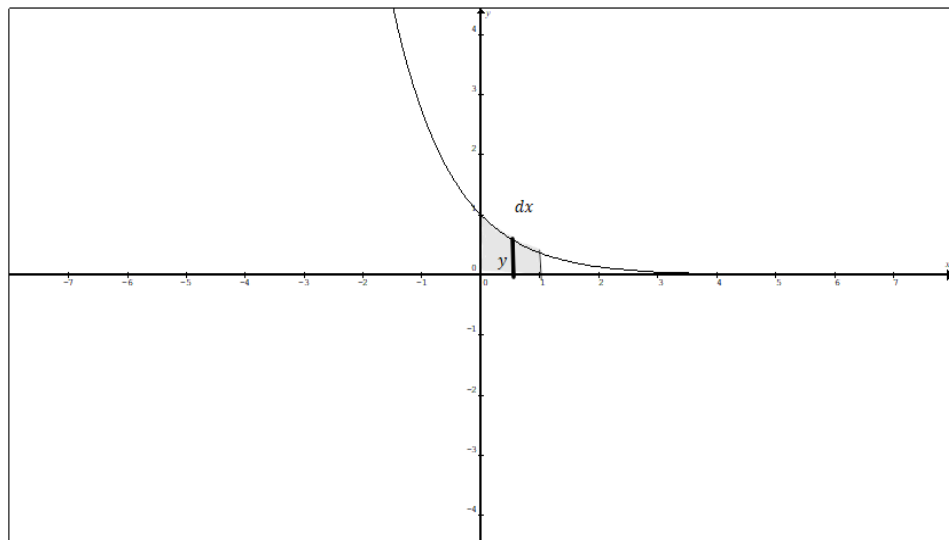
$$\therefore \frac{32}{5}$$

$$\bar{y} = \frac{6.4}{4} = 1.6 = \frac{8}{5}$$

(10)

5.3

5.3.1



Graphs ✓✓; Shaded area✓ and Strip✓

(4)

5.3.2

$$V_y = 2\pi \int_a^b xy dx \vee V_y = 2\pi \int_a^b x(y_1 - y_2) dx$$

$$V_y = 2\pi \int_0^1 xe^{-x} dx \quad \checkmark \checkmark$$

$$f(x) = x \wedge f'(x) = 1 \wedge g'(x) = e^{-x} \wedge g(x) = -e^{-x}$$

$$V_y = 2\pi \left[-xe^{-x} + \int e^{-x} dx \right]_0^1 \checkmark \checkmark$$

$$= 2\pi \left[-xe^{-x} - e^{-x} \right]_0^1 \checkmark$$

$$= 2\pi \left[-e^{-1} - e^{-1} + 1 \right] \checkmark$$

$$= 1.660 \text{ (units)}^3 \checkmark$$

(8)

5.3.3

$$V_{m-y} = \int_a^b r dV$$

$$V_{m-y} = 2\pi \int_0^1 x^2 e^{-x} dx \quad \checkmark \checkmark$$

$$f(x) = x^2 f'(x) = 2x \wedge g'(x) = e^{-x} g(x) = -e^{-x}$$

$$V_{m-y} = 2\pi \left[-x^2 e^{-x} + 2 \int x e^{-x} dx \right]_0^1$$

$$f(x) = x f'(x) = 1 \wedge g'(x) = e^{-x} g(x) = -e^{-x}$$

$$V_{m-y} = 2\pi \left[-x^2 e^{-x} + 2 \left(-x e^{-x} + \int e^{-x} dx \right) \right]_0^1$$

$$V_{m-y} = 2\pi \left[-x^2 e^{-x} - 2x e^{-x} - 2e^{-x} \right]_0^1$$

$$\hookrightarrow 2\pi \left[-e^{-1} - 2e^{-1} - 2e^{-1} \right] - 2\pi \left[0 - 0 - 2 \right]$$

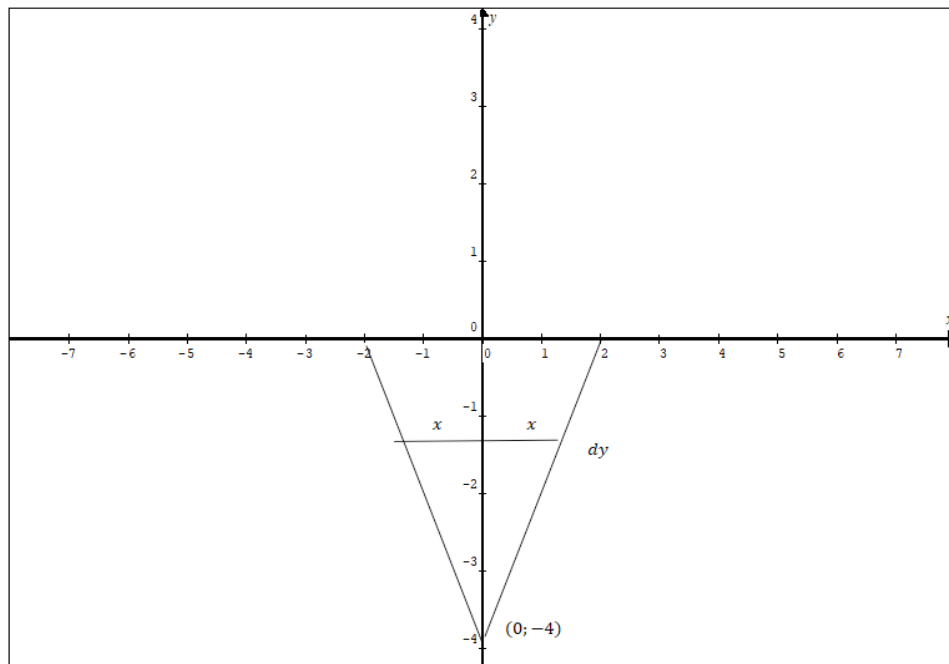
$$\hookrightarrow 1.009(\text{units})^4$$

$$\therefore \bar{y} = \frac{1.009}{1.660} = 0.608 \text{ units}$$

(12)

5.4

5.4.1



$$m = \frac{-4 - 0}{0 - 2} = 2 \quad \checkmark$$

$$y = 2x + C \text{ through } (0; -4)$$

$$-4 = C \quad \checkmark$$

$$y = 2x - 4$$

$$2x = (y + 4) \quad \checkmark$$

*Strip**x-intercepts at $(-2; 0) \wedge (2; 0)$* *Bottom at $(0; -4)$*

(6)

5.4.2

$$\text{first moment} = \int r dA$$

(6)

$$= \int_{-4}^0 y(y + 4) dy \quad \checkmark \quad \checkmark$$

$$= \int_{-4}^0 (y^2 + 4y) dy \quad \checkmark$$

$$= \left[\frac{y^3}{3} + 2y^2 \right]_{-4}^0 \checkmark$$

$$\therefore 0 - \therefore$$

$$= -10.667 \checkmark$$

$$\therefore -\frac{32}{3} \therefore$$

5.4.3

$$\text{Second moment} = \int r^2 dA$$

$$= \int_{-4}^0 y^2 (y+4) dy \checkmark \checkmark$$

$$= \int_{-4}^0 (y^3 + 4y^2) dy \checkmark$$

$$= \left[\frac{y^4}{4} + \frac{4y^3}{3} \right]_{-4}^0 \checkmark$$

$$= \left[-\frac{(-4)^4}{4} - \frac{4(-4)^3}{3} \right] \checkmark$$

$$= 21.333 \checkmark$$

$$\therefore \frac{64}{3} \therefore$$

$$y = \frac{21.333}{-10.667} = -2.000 \checkmark \checkmark$$

(8)

5.4.2

$$\text{first moment} = \int r dA$$

(6)

$$= \int_0^4 y(4-y) dy \checkmark \checkmark$$

$$= \int_0^4 (4y - y^2) dy \checkmark$$

$$= \left[2y^2 - \frac{y^3}{3} \right]_0^4 \checkmark$$

$$= \left[2(4)^2 - \frac{(4)^3}{3} \right] \checkmark$$

$$= 10.667 \checkmark$$

$$\cdot \frac{32}{3} \cdot$$

5.4.3

$$\text{second moment} = \int r^2 dA$$

$$= \int_0^4 y(4-y)^2 dy \checkmark$$

$$= \int_0^4 y(16-8y+y^2) dy \checkmark$$

$$= \int_0^4 (16y-8y^2+y^3) dy \checkmark$$

$$= \left[8y^2 - \frac{8y^3}{3} + \frac{y^4}{4} \right]_0^4 \checkmark$$

$$= \left[8(4)^2 - \frac{8(4)^3}{3} + \frac{(4)^4}{4} \right] \checkmark$$

$$= 21.333 \checkmark$$

$$\cdot \frac{64}{3} \cdot$$

$$y = \frac{21.333}{10.667} = 2.000 \checkmark \checkmark$$

(8)
[80]

QUESTION 6

$$6.1 \quad \frac{dx}{dy} = \frac{y^3}{2} - \frac{1}{2y^3} \quad \checkmark\checkmark \quad (12)$$

$$\left(\frac{dx}{dy}\right)^2 + 1 = \left(\frac{y^3}{2} - \frac{1}{2y^3}\right)^2 + 1$$

$$= \frac{y^6}{4} - \frac{1}{4} - \frac{1}{4} + \frac{1}{4y^6} + 1 \quad \checkmark$$

OR

$$= \frac{y^6}{4} + \frac{1}{2} + \frac{1}{4y^6} \quad \checkmark$$

$$= \left(\frac{y^3}{2} + \frac{1}{2y^3}\right)^2 \quad \checkmark$$

$$\frac{y^6}{4} + \frac{1}{2} + \frac{1}{4y^6} = \frac{y^{12} + 2y^6 + 1}{4y^6}$$

$$\therefore \left(\frac{y^6 + 1}{2y^3}\right)^2$$

$$= \left(\frac{y^3}{2} + \frac{1}{2y^3}\right)^2 \quad \checkmark$$

$$S = \int_a^b \sqrt{\left(\frac{dx}{dy}\right)^2 + 1} dy \quad \checkmark$$

$$= \int_1^2 \sqrt{\left(\frac{y^3}{2} + \frac{1}{2y^3}\right)^2} dy \quad \checkmark\checkmark$$

$$\therefore \int_1^2 \left(\frac{y^3}{2} + \frac{y^{-3}}{2}\right) dy$$

$$\int \left[\frac{y^4}{8} - \frac{1}{4y^2} \right]_1^2$$

$$\int \left[\frac{(2)^4}{8} - \frac{1}{4(2)^2} \right] - \left[\frac{1}{8} - \frac{1}{4} \right]$$

$$= \frac{33}{16}$$

$$= 2.063 \checkmark$$

6.2 $y = 3t^2$ and $x = 3t - t^3$

$$\frac{dx}{dt} = 3 - 3t^2 \quad \checkmark$$

$$\frac{dy}{dt} = 6t \quad \checkmark$$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 36t^2 + 9 - 18t^2 + 9t^4 \quad \checkmark$$

$$= 9t^4 + 18t^2 + 9$$

$$= 9[t^4 + 2t^2 + 1]$$

OR

$$9(t^2+1)^2$$

$$9t^4 + 18t^2 + 9$$

$$9(3t^2+3)^2$$

$$A_x = 2\pi \int_a^b y \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt \quad \checkmark$$

$$= 18\pi \int_a^b t^2 \sqrt{(t^2+1)^2} dt \quad \checkmark \checkmark$$

OR

$$= 18\pi \int_0^1 t^2 (t^2+1) dt \quad \checkmark$$

$$A_x = 2\pi \int_a^b 3t^2 \sqrt{(3t^2+3)^2} dt$$

$$9 \cdot 18\pi \int_0^1 t^2 (t^2+1) dt$$

$$= 18\pi \int_0^1 (t^4 + t^2) dt \quad \checkmark$$

$$= 18\pi \left[\frac{t^5}{5} + \frac{t^3}{3} \right]_0^1 \quad \checkmark$$

$$= 18\pi \left[\frac{1}{5} + \frac{1}{3} \right] \quad \checkmark$$

$$= \frac{48}{5} \pi$$

$$= 30.159 \quad \checkmark$$

(12)

[24]

TOTAL: 200