



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

16 November 2022

This marking guideline consists of 20 pages.

QUESTION 1

1.1

$$Q = (x + y) \ln \left(\frac{x}{y} \right)$$

$$\begin{aligned} \frac{\partial Q}{\partial x} &= \ln \left(\frac{x}{y} \right) + (x + y) \times \frac{1}{x} \times \frac{1}{y} \\ &= \ln \left(\frac{x}{y} \right) + \frac{x + y}{x} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \frac{\partial Q}{\partial y} &= \ln \left(\frac{x}{y} \right) + \frac{x + y}{x} \times -xy^{-2} \\ &= \ln \left(\frac{x}{y} \right) - \frac{x + y}{y} \quad \checkmark \end{aligned}$$

$$x \left(\ln \left(\frac{x}{y} \right) + \frac{x + y}{x} \right) + y \left(\ln \left(\frac{x}{y} \right) - \frac{x + y}{y} \right) \quad \checkmark$$

$$x \ln \left(\frac{x}{y} \right) + x + y + y \ln \left(\frac{x}{y} \right) - x - y$$

$$= x \ln \left(\frac{x}{y} \right) + y \ln \left(\frac{x}{y} \right) \quad \checkmark$$

$$= (x + y) \ln \left(\frac{x}{y} \right) \quad \checkmark$$

$$= Q$$

(3)

1.2

$$z = 3x^2y^5$$

$$\begin{aligned} x &= 3 \\ \delta x &= -5 \end{aligned}$$

$$\begin{aligned} y &= 2 \\ \delta y &= 2.5 \end{aligned}$$

$$\therefore \frac{\partial z}{\partial x} = 6xy^5 \quad \checkmark \quad \text{and} \quad \frac{\partial z}{\partial y} = 15x^2y^4 \quad \checkmark$$

$$\begin{aligned} \delta z &= \frac{\partial z}{\partial x} \delta x + \frac{\partial z}{\partial y} \delta y \quad \checkmark \\ &= (6xy^5)(-5) + (15x^2y^4)(2.5) \quad \checkmark \\ &= 6(3)(2)^5(-5) + 15(3)^2(2)^4(2.5) \quad \checkmark \\ &= 2520 \quad \checkmark \end{aligned}$$

(3)

QUESTION 2

$$2.1 \quad \int -2x \operatorname{cosec}^2 2x dx = x \cot 2x - \int \cot 2x dx$$

$$\hookrightarrow x \cot 2x - \frac{1}{2} \ln |\sin 2x| + C$$

$$f(x) = -2x \Rightarrow f'(x) = -2$$

$$g'(x) = \operatorname{cosec}^2 2x \Rightarrow$$

$$g(x) = \frac{-1}{2} \cot 2x$$

(2)

$$2.2 \quad \int x^3 \cos x dx = x^3 \sin x - 3 \int x^2 \sin x dx$$

$$\hookrightarrow x^3 \sin x - 3 \left[-x^2 \cos x + 2 \int x \cos x dx \right]$$

$$\hookrightarrow x^3 \sin x + 3x^2 \cos x - 6 \left[x \sin x - \int \sin x dx \right]$$

$$\hookrightarrow x^3 \sin x + 3x^2 \cos x - 6x \sin x + 6(-\cos x) + C$$

$$\hookrightarrow x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + C$$

(5)

2.3

$$\int \frac{dx}{(x+1)(x+4)}$$

$$= - \int \frac{dx}{\frac{9}{4} - \left(x + \frac{5}{2}\right)^2} \quad \checkmark$$

$$= - \frac{1}{2 \left(\frac{3}{2}\right)} \ln \left[\frac{\frac{3}{2} + \left(x + \frac{5}{2}\right)}{\frac{3}{2} - \left(x + \frac{5}{2}\right)} \right] + C \quad \checkmark \checkmark$$

$$= - \frac{1}{3} \ln \left[\frac{\frac{3}{2} + \left(x + \frac{5}{2}\right)}{\frac{3}{2} - \left(x + \frac{5}{2}\right)} \right] + C \quad \checkmark$$

$$(x+1)(x+4) = x^2 + 5x + 4 \quad \checkmark$$

$$= x^2 + 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 4 \quad \checkmark$$

$$= x^2 + 5x + \frac{25}{4} - \frac{25}{4} + 4$$

$$= \left(x + \frac{5}{2}\right)^2 - \frac{9}{4} \quad \checkmark$$

$$= - \left(\frac{9}{4} - \left(x + \frac{5}{2}\right)^2 \right) \quad \checkmark$$

$$ax^2 + bx + c = \frac{4ac - b^2}{4a} + a \left(x + \frac{b}{2a}\right)^2 \quad \checkmark$$

$$\Rightarrow x^2 + 5x + 4 = \frac{4(4)(1) - (5)^2}{4(1)} + 1 \left(x + \frac{5}{2(1)}\right)^2 \quad \checkmark$$

$$\hookrightarrow -\frac{9}{4} + \left(x + \frac{5}{2}\right)^2 \quad \checkmark$$

(4)

2.4 $\int \sin^5 3x dx$

$$= \int \sin^4 3x \sin 3x dx$$

$$= \int (\sin^2 3x)^2 \sin 3x dx \checkmark$$

$$= \int (1 - \cos^2 3x)^2 \sin 3x dx \checkmark$$

$$= \int (1 - 2\cos^2 3x + \cos^4 3x) \sin 3x dx \checkmark$$

$$= \int \sin 3x dx - 2 \int \cos^2 3x \sin 3x dx + \int \cos^4 3x \sin 3x dx \checkmark$$

$$= \frac{-\cos 3x}{3} + \frac{2}{3} \int u^2 du - \frac{1}{3} \int u^4 du \checkmark \checkmark \checkmark$$

$$= \frac{-\cos 3x}{3} + \frac{2u^3}{9} + \frac{u^5}{15} + C \checkmark \checkmark$$

$$= \frac{-\cos 3x}{3} + \frac{2 \cos^3 3x}{9} - \frac{\cos^5 3x}{15} + C \checkmark$$

(5)

2.5 $\int 5x e^{-2x} dx = \frac{-5}{2} x e^{-2x} + \frac{5}{2} \int e^{-2x} dx$

$$\therefore -\frac{5}{2} x e^{-2x} + \frac{5}{2} \left[\frac{e^{-2x}}{-2} \right] + C$$

$$\therefore -\frac{5}{2} x e^{-2x} - \frac{5}{4} e^{-2x} + C$$

(2)

[18]

QUESTION 3

3.1

$$\int \frac{x^3 + 6x - 7}{(x^2 - 2x - 3)^2} dx$$

$$\text{Let } \frac{x^3 + 6x - 7}{(x+1)^2(x-3)^2} = \frac{A}{(x+1)^2} + \frac{B}{(x+1)} + \frac{C}{(x-3)^2} + \frac{D}{(x-3)} \checkmark$$

$$x^3 + 6x - 7 = A(x-3)^2 + B(x+1)(x-3)^2 + C(x+1)^2 + D(x-3)(x+1)^2 \checkmark$$

$$\text{Let } x = 3$$

$$\text{Let } x = -1$$

$$38 = 16C$$

$$-14 = 16A$$

$$C = \frac{19}{8} \checkmark$$

$$A = \frac{-7}{8} \checkmark$$

$$x^3: \quad 1 = B + D$$

$$x^2: \quad 0 = A + C - D - 5B$$

$$\frac{-3}{2} = -D - 5B$$

$$\frac{-1}{2} = -4B$$

$$B = \frac{1}{8} \checkmark$$

$$D = \frac{7}{8} \checkmark$$

$$= \frac{-7}{8} \int \frac{dx}{(x+1)^2} + \frac{1}{8} \int \frac{dx}{x+1} + \frac{19}{8} \int \frac{dx}{(x-3)^2} + \frac{7}{8} \int \frac{dx}{x-3} \checkmark$$

$$= -\frac{7}{8} \int (x+1)^{-2} dx + \frac{1}{8} \int \frac{dx}{x+1} + \frac{19}{8} \int (x-3)^{-2} dx + \frac{7}{8} \int \frac{dx}{x-3}$$

$$= \frac{7}{8(x+1)} + \frac{1}{8} \ln(x+1) - \frac{19}{8(x-3)} + \frac{7}{8} \ln(x-3) + C \checkmark \checkmark \checkmark \checkmark$$

(6)

3.2

$$\int \frac{x^2}{(1-x)(x^2+1)} dx$$

$$\text{Let } \frac{x^2}{(1-x)(x^2+1)} = \frac{A}{1-x} + \frac{Bx+C}{x^2+1} \quad \checkmark$$

$$x^2 = A(x^2+1) + (Bx+C)(1-x) \quad \checkmark$$

$$\text{Let } x=1:$$

$$1 = 2A$$

$$A = \frac{1}{2} \quad \checkmark$$

$$x^2: \quad 1 = A - B$$

$$B = \frac{-1}{2} \quad \checkmark$$

$$\text{constant: } 0 = A + C$$

$$C = \frac{-1}{2} \quad \checkmark$$

$$= \frac{1}{2} \int \frac{dx}{1-x} + \int \frac{\frac{-1}{2}x - \frac{1}{2}}{x^2+1} dx \quad \checkmark$$

$$= \frac{1}{2} \int \frac{dx}{1-x} - \frac{1}{2} \int \frac{xdx}{x^2+1} - \frac{1}{2} \int \frac{dx}{x^2+1} \quad \checkmark$$

$$= \frac{1}{2} \int \frac{-1}{1-x} dx - \frac{1}{4} \int \frac{2x}{x^2+1} dx - \frac{1}{2} \int \frac{dx}{x^2+1}$$

$$= \frac{-1}{2} \ln(1-x) - \frac{1}{4} \ln(x^2+1) - \frac{1}{2} \tan^{-1} x + C \quad \checkmark \checkmark \checkmark$$

(6)

[12]

QUESTION 4

$$4.1 \quad (t-1)^3 \frac{ds}{dt} + 4(t-1)^2 s = t+1$$

$$\frac{ds}{dt} + \left(\frac{4}{t-1} \right) s = \frac{t+1}{(t-1)^3}$$

$$IF = e^{4 \int \frac{dt}{t-1}}$$

$$e^{4 \ln(t-1)}$$

$$e^{\ln(t-1)^4}$$

$$(t-1)^4$$

$$\frac{d}{dt} \left[(t-1)^4 s \right] = \frac{t+1}{(t-1)^3} \times (t-1)^4 \quad \checkmark$$

$$(t-1)^4 s = \int (t^2 - 1) dt \quad \checkmark \checkmark$$

$$= \frac{t^3}{3} - t + C \quad \checkmark \checkmark$$

$$\therefore s = \frac{t^3}{3(t-1)^4} - \frac{t}{(t-1)^4} + \frac{C}{(t-1)^4} \quad \checkmark \quad (5)$$

4.2 $\frac{d^2s}{dt^2} + 4\frac{ds}{dt} + 13s = 39t^2 + 11t + 54$ when $t = 0; s = 1$ and $\frac{ds}{dt} = 8$ when $t = 0$

$$r^2 + 4r + 13 = 0 \checkmark$$

$$r = -2 \pm 3j \checkmark$$

$$s_c = e^{-2t} (A \cos 3t + B \sin 3t) \checkmark$$

$$s_p = Ct^2 + Dt + E \checkmark$$

$$s'_p = 2Ct + D \checkmark$$

$$s''_p = 2C \checkmark$$

$$2C + 8Ct + 4D + 13Ct^2 + 13Dt + 13E = 39t^2 + 11t + 54$$

$$t^2: \quad 13C = 39 \Rightarrow C = 3 \checkmark$$

$$t: \quad 8C + 13D = 11 \Rightarrow 13D = -13 \Rightarrow D = -1 \checkmark$$

$$\text{Constant: } 2C + 4D + 13E = 54 \Rightarrow 13E = 52 \Rightarrow E = 4 \checkmark$$

$$s_p = 3t^2 - t + 4$$

$$s = s_c + s_p$$

$$= e^{-2t} (A \cos 3t + B \sin 3t) + 3t^2 - t + 4 \checkmark$$

$$1 = A + 4 \Rightarrow A = -3 \checkmark$$

$$s' = -2e^{-2t} A \cos 3t - 3Ae^{-2t} \sin 3t - 2e^{-2t} B \sin 3t + 3Be^{-2t} \cos 3t \checkmark$$

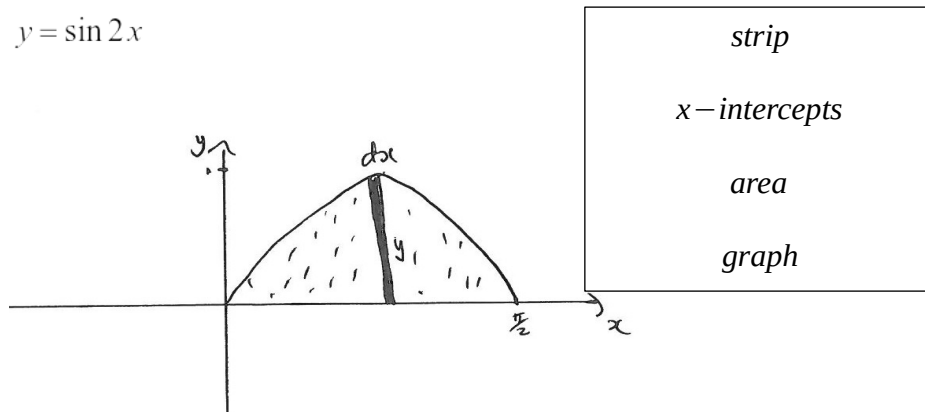
$$8 = -2A + 3B - 1 \Rightarrow 3 = 3B \Rightarrow B = 1 \checkmark$$

$$\therefore s = e^{-2t} (-3 \cos 3t + \sin 3t) + 3t^2 - t + 4 \checkmark$$

(7)
[12]

QUESTION 5

5.1 5.1.1 $y = \sin 2x$



(2)

5.1.2

$$dV_x = \pi y^2 dx \quad \checkmark$$

$$V_x = \pi \int_0^{\frac{\pi}{2}} \sin^2 2x dx \quad \checkmark$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} (1 - \cos 4x) dx \quad \checkmark$$

$$= \frac{\pi}{2} \left[x - \frac{\sin 4x}{4} \right]_0^{\frac{\pi}{2}} \quad \checkmark$$

$$\therefore \frac{\pi}{2} \left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} \right] - \frac{\pi}{2} [0 - 0]$$

$$= \frac{\pi^2}{4} \text{ or } 2.467 (\text{units})^3 \quad \checkmark$$

$$V_x = \pi \int_a^b y^2 dx$$

$$V_x = \pi \int_0^{\frac{\pi}{2}} \sin^2 2x dx$$

$$\therefore \pi \left[\frac{x}{2} - \frac{\sin 4x}{8} \right]_0^{\frac{\pi}{2}}$$

$$\therefore \pi \left[\frac{\pi}{4} - \frac{\sin 2\pi}{8} \right] - \pi [0 - 0]$$

$$\therefore \underline{\frac{\pi^2}{4}}$$

(3)

$$5.1.3 \quad dV_{moy} = x \pi y^2 dx$$

$$V_{moy} = \pi \int_0^{\frac{\pi}{2}} x \sin^2 2x dx \quad \checkmark \checkmark$$

$$= \pi \left[x \left(\frac{x}{2} - \frac{\sin 4x}{8} \right) - \int \left(\frac{x}{2} - \frac{\sin 4x}{8} \right) dx \right]_0^{\frac{\pi}{2}} \quad \checkmark \checkmark$$

$$= \pi \left[\frac{x^2}{2} - \frac{x \sin 4x}{8} - \left(\frac{x^2}{4} + \frac{\cos 4x}{32} \right) \right]$$

$$= \pi \left[\frac{x^2}{4} - \frac{x \sin 4x}{8} - \frac{\cos 4x}{32} \right]_0^{\frac{\pi}{2}}$$

$$= \pi \left[\frac{\pi^2}{8} - \frac{\frac{\pi}{2} \sin \pi}{8} - \frac{\cos \pi}{32} + \frac{1}{32} \right] \quad \checkmark$$

$$= 4.0721 (\text{units})^4 \quad \checkmark$$

$$\bar{x} = \frac{V_{moy}}{V}$$

$$= \frac{4.0721}{2.467} \quad \checkmark$$

$$= 0.912 \text{ units} \quad \checkmark$$

(5)

5.2 5.2.1 $y = 2x$ and $y = x^2$

$$x^2 - 2x = 0$$

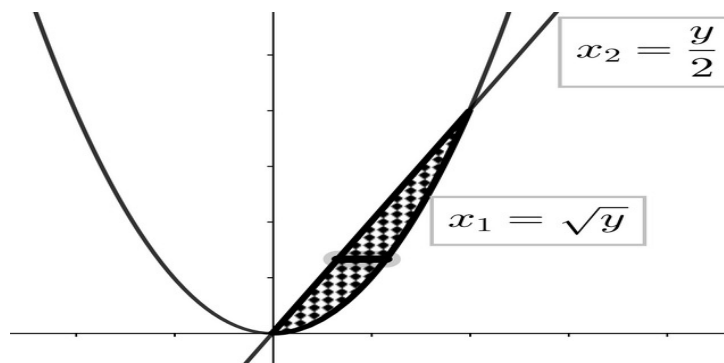
$$x(x - 2) = 0$$

$$x = 0 \text{ or } x = 2$$

$$y = 0 \quad y = 4$$

$$(0;0) \text{ and } (2;4)$$

strip area Each graph
--



(3)

5.2.2 $dA = (x_2 - x_1) dy$ ✓ $A_y = \int_c^d (x_1 - x_2) dy$

$$A = \int_0^4 \left(y^{\frac{1}{2}} - \frac{y}{2} \right) dy \quad \checkmark \checkmark$$

$$= \left[\frac{2y^{\frac{3}{2}}}{3} - \frac{y^2}{4} \right]_0^4 \quad \checkmark$$

$$= \left[\frac{2}{3}(4)^{\frac{3}{2}} - \frac{(4)^2}{4} - 0 \right] \quad \checkmark$$

$$= \frac{4}{3} \text{ or } 1.333(\text{units})^2 \quad \checkmark$$

(3)

5.2.3 $dI_x = \text{Area} \times (\text{distance})^2$ ✓ $I_x = \int_a^b r^2 dA$

$$I_x = \int_0^4 y^2 (x_2 - x_1) dy \quad \checkmark \checkmark$$

$$= \int_0^4 y^2 \left(y^{\frac{1}{2}} - \frac{y}{2} \right) dy \quad \checkmark$$

$$= \int_0^4 \left(y^{\frac{5}{2}} - \frac{y^3}{4} \right) dy \quad \checkmark$$

$$= \left[\frac{2}{7} y^{\frac{7}{2}} - \frac{y^4}{8} \right]_0^4 \quad \checkmark$$

$$= \left[\frac{2}{7} (4)^{\frac{7}{2}} - \frac{(4)^4}{8} - 0 \right] \quad \checkmark$$

$$= \frac{32}{7} \text{ or } 4.571 (\text{units})^4 \quad \checkmark$$

(4)

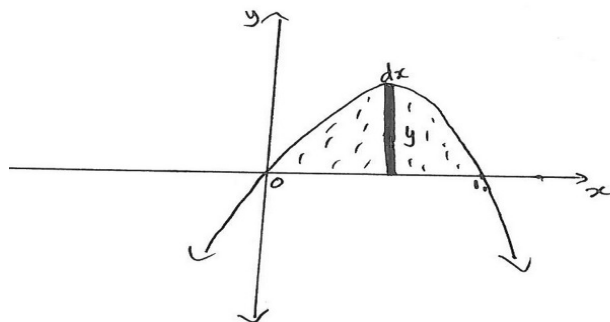
5.2.4 $I_x = 4.571 \frac{A}{A} \quad \checkmark$

$$= \frac{4.571}{1.333} A$$

$$= 3.429 (\text{units})^4 \quad \checkmark$$

(1)

5.3 5.3.1



strip
x-intercepts
area
graph

(2)

$$5.3.2 \quad dV_x = \pi y^2 dx$$

$$V_x = \pi \int_0^1 y^2 dx \quad \checkmark$$

$$= \pi \int_0^1 (-x^2 + x)^2 dx \quad \checkmark$$

$$= \pi \int_0^1 (x^4 - 2x^3 + x^2) dx \quad \checkmark$$

$$= \pi \left[\frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3} \right]_0^1 \quad \checkmark$$

$$= \pi \left[\frac{1}{5} - \frac{1}{2} + \frac{1}{3} - 0 \right] \quad \checkmark$$

$$= \frac{\pi}{30}$$

$$= 0.105 \text{ (units)}^3 \quad \checkmark$$

(3)

$$5.3.3 \quad dI_x = \frac{1}{2} r^2 dm$$

$$I_x = \frac{\rho\pi}{2} \int_0^1 y^4 dx \quad \checkmark \checkmark$$

$$= \frac{\rho\pi}{2} \int_0^1 (x^4 - 2x^3 + x^2)^2 dx \quad \checkmark$$

$$= \frac{\rho\pi}{2} \int_0^1 (x^8 - 4x^7 + 6x^6 - 4x^5 + x^4) dx \quad \checkmark \checkmark$$

$$= \frac{\rho\pi}{2} \left[\frac{x^9}{9} - \frac{x^8}{2} + \frac{6x^7}{7} - \frac{2x^6}{3} + \frac{x^5}{5} \right]_0^1 \quad \checkmark$$

$$= \frac{\rho\pi}{2} \left[\frac{1}{9} - \frac{1}{2} + \frac{6}{7} - \frac{2}{3} + \frac{1}{5} \right] \quad \checkmark$$

$$= \frac{\rho\pi}{1260}$$

(4)

$$= 0.002493 \rho \text{ (units)}^5 \checkmark$$

5.4 5.4.1 $y = ax^2 - 5$ ✓ through (4;-1)

$$-1 = a(4)^2 - 5$$

$$a = \frac{1}{4}$$

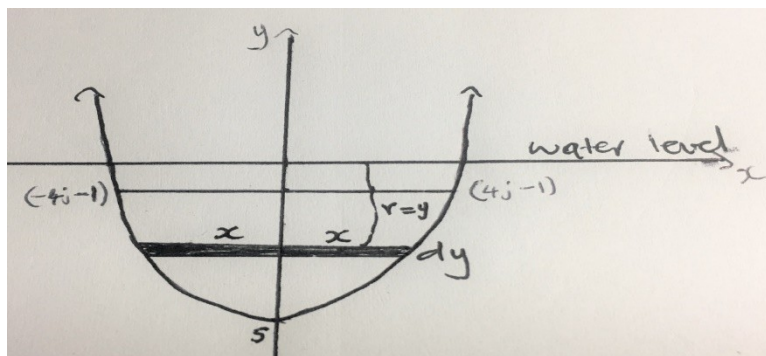
$$y = \frac{1}{4}x^2 - 5$$
 ✓

$$x = 2\sqrt{y+5}$$

Strip

Top of gate at $y = -1$

Bottom of gate at $y = -5$



(3)

5.4.2 First moment

$$= \int_a^b r dA$$

$$= 4 \int_{-5}^{-1} y \sqrt{y+5} dy \quad \checkmark$$

$$= 4 \int_{-5}^{-1} (u-5) u^{\frac{1}{2}} du \quad \checkmark$$

$$= 4 \int_{-5}^{-1} \left(u^{\frac{3}{2}} - 5u^{\frac{1}{2}} \right) du \quad \checkmark$$

$$= 4 \left[\frac{2}{5} u^{\frac{5}{2}} - \frac{10}{3} u^{\frac{3}{2}} \right]_{-5}^{-1} \quad \checkmark$$

$$= 4 \left[\frac{2}{5} (y+5)^{\frac{5}{2}} - \frac{10}{3} (y+5)^{\frac{3}{2}} \right]_{-5}^{-1} \quad \checkmark$$

$$= 4 \left[\frac{2}{5} (4)^{\frac{5}{2}} - \frac{10}{3} (4)^{\frac{3}{2}} - 0 \right] \quad \checkmark$$

$$= -55.467 \text{ m}^3 \quad \checkmark$$

$$dA = 2x dy = 4\sqrt{y+5}$$

$$\text{let } u = y+5$$

$$\Rightarrow u-5 = y$$

$$\Rightarrow du = dy$$

$$y = -1 \Rightarrow u = 4$$

$$y = -5 \Rightarrow u = 0$$

$$4 \left[\frac{2}{5} u^{\frac{5}{2}} - \frac{10}{3} u^{\frac{3}{2}} \right]_0^4 \quad \checkmark$$

$$4 \left[\frac{2}{5} (4)^{\frac{5}{2}} - \frac{10}{3} (4)^{\frac{3}{2}} - 0 \right] \quad \checkmark$$

(3)

5.4.3 Second moment

$$= \int_a^b r^2 dA$$

$$= 4 \int_{-5}^{-1} y^2 \sqrt{y+5} dy$$

$$dA = 2 \times dy = 4\sqrt{y+5}$$

$$= 4 \int_{-5}^{-1} (u-5)^2 u^{\frac{1}{2}} du \quad \checkmark$$

$$\text{let } u = y+5$$

$$\Rightarrow u-5 = y$$

$$= 4 \int_{-5}^{-1} (u^2 - 10u + 25) u^{\frac{1}{2}} du \quad \checkmark$$

$$\Rightarrow du = dy$$

$$= 4 \int_{-5}^{-1} \left(u^{\frac{5}{2}} - 10u^{\frac{3}{2}} + 25u^{\frac{1}{2}} \right) du \quad \checkmark$$

$$= 4 \left[\frac{2}{7} u^{\frac{7}{2}} - 4u^{\frac{5}{2}} + \frac{50}{3} u^{\frac{3}{2}} \right]_{-5}^{-1} \quad \checkmark$$

$$y = -1 \Rightarrow u = 4$$

$$y = -5 \Rightarrow u = 0$$

$$= 4 \left[\frac{2}{7} u^{\frac{7}{2}} - 4u^{\frac{5}{2}} + \frac{50}{3} u^{\frac{3}{2}} \right]_0^4 \quad \checkmark$$

$$= 4 \left[\frac{2}{7} (y+5)^{\frac{7}{2}} - 4(y+5)^{\frac{5}{2}} + \frac{50}{3} (y+5)^{\frac{3}{2}} \right]_{-5}^{-1} \quad \checkmark$$

$$= 4 \left[\frac{2}{7} (4)^{\frac{7}{2}} - 4(4)^{\frac{5}{2}} + \frac{50}{3} (4)^{\frac{3}{2}} \right] \quad \checkmark$$

$$= 4 \left[\frac{2}{7} (4)^{\frac{7}{2}} - 4(4)^{\frac{5}{2}} + \frac{50}{3} (4)^{\frac{3}{2}} \right] \quad \checkmark$$

$$= 167.619 \text{ m}^4 \quad \checkmark$$

$$y = \frac{167.619}{-55.467} \quad \checkmark$$

$$= -3.022 \text{ m} \quad \checkmark$$

(4)

(ALTERNATIVE ANSWERS TO QUESTION 5.4)

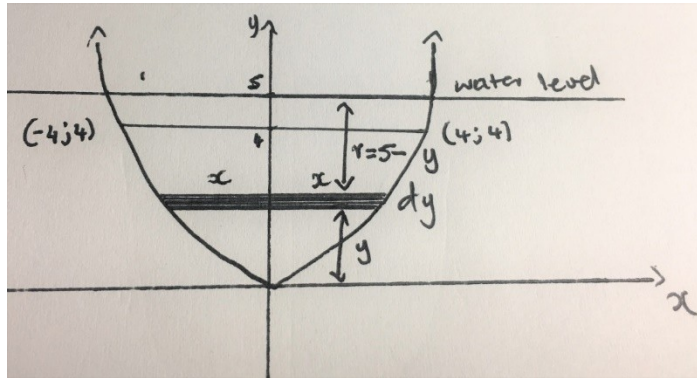
5.4

5.4.1

$$y = ax^2 \text{ through } (4; 4)$$

$$4 = 16a \Rightarrow a = \frac{1}{4}$$

$$\therefore y = \frac{1}{4}x^2 \Rightarrow x = 2\sqrt{y}$$

*Strip**Top of gate at $y = 4$* *Water level at $y = 5$* 

(3)

5.4.2

First moment

$$= \int_a^b r dA$$

$$= 4 \int_0^4 (5 - y) y^{\frac{1}{2}} dy \quad \checkmark \checkmark$$

$$= 4 \int_0^4 \left(5y^{\frac{1}{2}} - y^{\frac{3}{2}} \right) dy \quad \checkmark$$

$$= 4 \left[\frac{10}{3} y^{\frac{3}{2}} - \frac{2}{5} y^{\frac{5}{2}} \right]_0^4 \quad \checkmark$$

$$= 4 \left[\frac{10}{3} (4)^{\frac{3}{2}} - \frac{2}{5} (4)^{\frac{5}{2}} - 0 \right] \quad \checkmark$$

$$= 55.467 \text{ m}^3 \quad \checkmark$$

$$dA = 2x dy = 4\sqrt{y}$$

(3)

5.4.3 Second moment

$$\int_a^b r^2 dA$$

$$= 4 \int_0^4 (5-y)^2 y^{\frac{1}{2}} dy \quad \checkmark$$

$dA = 2x dy = 4\sqrt{y}$

$$= 4 \int_0^4 (25 - 10y + y^2) y^{\frac{1}{2}} dy \quad \checkmark$$

$$= 4 \int_0^4 \left(25y^{\frac{1}{2}} - 10y^{\frac{3}{2}} + y^{\frac{5}{2}} \right) dy \quad \checkmark$$

$$= 4 \left[\frac{50}{3} y^{\frac{3}{2}} - 4y^{\frac{5}{2}} + \frac{2}{7} y^{\frac{7}{2}} \right]_0^4 \quad \checkmark$$

$$= 4 \left[\frac{50}{3} (4)^{\frac{3}{2}} - 4(4)^{\frac{5}{2}} + \frac{2}{7} (4)^{\frac{7}{2}} - 0 \right] \quad \checkmark$$

$$= 167.619 \text{ m}^4 \quad \checkmark$$

$$y = \frac{167.619}{55.467} \quad \checkmark$$

$$= 3.022 \text{ m} \quad \checkmark$$

(4)
[40]

QUESTION 6

6.1 $x = 3 \cos t - \cos 3t$ and $y = 3 \sin t - \sin 3t$

$$\frac{dx}{dt} = -3 \sin t + 3 \sin 3t \quad \checkmark \quad \frac{dy}{dt} = 3 \cos t - 3 \cos 3t \quad \checkmark$$

$$\begin{aligned} \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= (-3 \sin t + 3 \sin 3t)^2 + (3 \cos t - 3 \cos 3t)^2 \\ &= 9 \sin^2 t - 18 \sin t \sin 3t + 9 \sin^2 3t + 9 \cos^2 t - 18 \cos t \cos 3t + 9 \cos^2 3t \quad \checkmark \\ &= 9(\sin^2 t + \cos^2 t) + 9(\sin^2 3t + \cos^2 3t) - 18(\cos t \cos 3t + \sin t \sin 3t) \quad \checkmark \\ &= 9(1) + 9(1) - 18 \cos(-2t) \\ &= 18 - 18 \cos 2t \quad \boxed{\cos(-2t) = \cos(t - 3t) = \cos t \cos 3t + \sin t \sin 3t} \\ &= 18(1 - \cos 2t) \quad \checkmark \\ &= 18(2 \sin^2 t) \quad \boxed{\cos 2t = 1 - 2 \sin^2 t} \\ &= 36 \sin^2 t \quad \checkmark \end{aligned}$$

Therefore,

$$\begin{aligned} S &= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \\ &= \int_0^\pi \sqrt{36 \sin^2 t} dt \quad \checkmark \checkmark \\ &= 6 \int_0^\pi \sin t dt \quad \checkmark \\ &= -6 \cos t \Big|_0^\pi \quad \checkmark \\ &= -6 \cos \pi - (-6 \cos 0) \quad \checkmark \\ &= -6(-1) - (-6(1)) \\ &= 12 \quad \checkmark \end{aligned}$$

(6)

6.2

$$y = \frac{1}{2}(e^x + e^{-x})$$

$$\frac{dy}{dx} = \frac{1}{2}(e^x - e^{-x}) \quad \checkmark$$

$$\left(\frac{dy}{dx}\right)^2 + 1 = \frac{1}{4}(e^x - e^{-x})^2 + 1$$

$$= \frac{e^{2x} - 2 + e^{-2x} + 4}{4} \quad \checkmark$$

$$= \frac{e^{2x} + 2 + e^{-2x}}{4}$$

$$\checkmark \frac{(e^x + e^{-x})^2}{4}$$

$$A_x = 2\pi \int y \sqrt{\left(\frac{dy}{dx}\right)^2 + 1} dx$$

$$= 2\pi \int_0^2 \frac{1}{2}(e^x + e^{-x}) \sqrt{\frac{(e^x + e^{-x})^2}{4}} dx \quad \checkmark \checkmark \checkmark$$

$$= \frac{\pi}{2} \int_0^2 (e^x + e^{-x})(e^x + e^{-x}) dx \quad \checkmark$$

$$= \frac{\pi}{2} \int_0^2 (e^{2x} + 2 + e^{-2x}) dx \quad \checkmark$$

$$= \frac{\pi}{2} \left[\frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2} \right]_0^2 \quad \checkmark \checkmark$$

$$= \frac{\pi}{2} \left[\frac{e^4}{2} + 4 - \frac{e^{-4}}{2} - \frac{1}{2} + \frac{1}{2} \right] \quad \checkmark$$

$$= 49.150 \quad \checkmark$$

(6)
[12]**TOTAL: 100**