



# higher education & training

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Department:  
Higher Education and Training  
**REPUBLIC OF SOUTH AFRICA**

## **MARKING GUIDELINE**

**NATIONAL CERTIFICATE**

**MATHEMATICS N5**

**21 November 2024**

**This marking guideline consists of 12 pages.**

## QUESTION 1

1.1 Determine the following limits:

$$\begin{aligned}
 1.1.1 \quad & \lim_{x \rightarrow \infty} \frac{\ln(1 + e^x)}{5x} \quad \frac{\infty}{\infty} \\
 &= \lim_{x \rightarrow \infty} \frac{e^x}{1 + e^x} \checkmark \\
 &= \lim_{x \rightarrow \infty} \frac{e^x}{5(1 + e^x)} \quad \frac{\infty}{\infty} \\
 &= \lim_{x \rightarrow \infty} \frac{e^x}{5e^x} \checkmark \\
 &= \frac{1}{5} \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 1.1.2 \quad & \lim_{x \rightarrow 0} \frac{3^x - 2^x}{x^2 - x} \\
 &= \lim_{x \rightarrow 0} \frac{3^x \ln 3 - 2^x \ln 2}{2x - 1} \checkmark \\
 &= \ln 2 - \ln 3 \text{ or } -0,405 \checkmark
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 1.2 \quad & f(x) = \frac{3x + \sin x}{\sqrt{2 - x}} \\
 & x < 2 \text{ or } (-\infty; 2) \checkmark
 \end{aligned} \tag{1}$$

**[6]**

## QUESTION 2

2.1      2.1.1       $y = \cot^4 \sqrt{\tan 3x^2}$

$$\frac{dy}{dx} = 4 \cot^3 \sqrt{\tan 3x^2} \times -\operatorname{cosec}^2 \sqrt{\tan 3x^2} \times \frac{1}{2\sqrt{\tan 3x^2}} \times \sec^2 3x^2 \times 6x \quad (5)$$

2.1.2

$$y = \tan \left( \sqrt{\frac{1 - e^{2x}}{1 + e^{2x}}} \right)$$

$$\frac{dy}{dx} = \sec^2 \left( \sqrt{\frac{1 - e^{2x}}{1 + e^{2x}}} \right) \times \frac{1}{2\sqrt{\frac{1 - e^{2x}}{1 + e^{2x}}}} \times \frac{2e^{2x}(1 + e^{2x}) - 2e^{2x}(1 - e^{2x})}{(1 + e^{2x})^2} \quad (5)$$

2.2

$$y = \left( \frac{1}{x} \right)^{3x^2+2}$$

$$y = x^{-(3x^2+2)}$$

$$\ln y = -(3x^2 + 2) \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = -6x \ln x - \frac{3x^2 + 2}{x}$$

$$\frac{dy}{dx} = y \left( -6x \ln x - \frac{3x^2 + 2}{x} \right)$$

$$\frac{dy}{dx} = \left( \frac{1}{x} \right)^{3x^2+2} \left( -6x \ln x - \frac{3x^2 + 2}{x} \right) \quad (5)$$

2.3      2.3.1       $x = 3 + \sqrt{x^2 + y^2}$

$$1 = 0 + \frac{1}{2\sqrt{x^2 + y^2}} \times \left( 2x + 2y \frac{dy}{dx} \right)$$

$$2x + 2y \frac{dy}{dx} = 2\sqrt{x^2 + y^2}$$

$$2y \frac{dy}{dx} = 2\sqrt{x^2 + y^2} - 2x$$

$$\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} - x}{y}$$

$$2.3.2 \quad \frac{dy}{dx} = \frac{\sqrt{3^2 + 4^2} - 3}{4} = \frac{1}{2} \checkmark$$

$$y = \frac{1}{2}x + c$$

$$4 = \frac{1}{2}(3) + c$$

$$c = \frac{5}{2} \checkmark$$

$$y = \frac{1}{2}x + \frac{5}{2} \checkmark$$

(3)  
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### QUESTION 3

3.1      3.1.1       $f(x) = 2x^3 - 3x - 2$   
 $f'(x) = 6x^2 - 3 = 0$   
 $x = \pm \frac{1}{\sqrt{2}}$

$$x = -0,707 \text{ or } x = 0,707$$

$$y = -0,586 \quad \checkmark \quad y = -3,414$$

Thus, the coordinates of the turning points are  $(-0,707; -0,586) \checkmark$  and  $(0,707; -3,414) \checkmark$

(3)

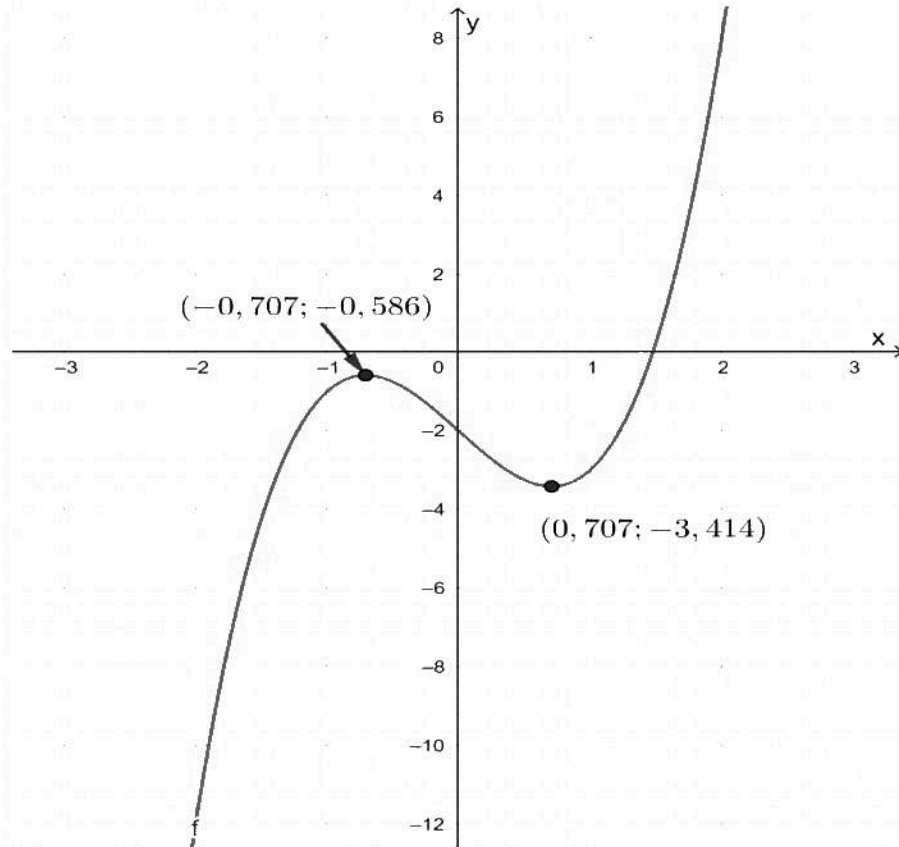
3.1.2

|     |     |    |    |    |   |
|-----|-----|----|----|----|---|
| $x$ | -2  | -1 | 0  | 1  | 2 |
| $y$ | -12 | -1 | -2 | -3 | 8 |

$\checkmark = 1$  mark for any 2 correct answers.

(2)

3.1.3



✓ = 1 mark for the shape.

✓ = 1 mark for the indication of the turning point on the graph.

(2)

3.1.4 Let  $x_0 = 1,4$

$$f(1,4) = -0,712$$

$$f'(1,4) = 8,760 \checkmark$$

$$x_1 = 1,4 - \frac{-0,712 \checkmark}{8,760}$$

$$= 1,481 \checkmark$$

$$\text{Let } x_1 = 1,481$$

$$f(1,481) = 0,057$$

$$f'(1,481) = 10,165$$

$$x_2 = 1,481 - \frac{0,057}{10,165}$$

$$= 1,476 \checkmark$$

(4)

$$\begin{aligned}
 3.2 \quad P &= 2l + 2w \\
 300 &= 2l + 2w \\
 150 &= l + w \checkmark \\
 w &= 150 - l \\
 A &= lw \\
 &= l(150 - l) \\
 &= 150l - l^2 \checkmark \\
 A' &= 150 - 2l = 0 \checkmark \\
 2l &= 150 \\
 l &= 75m \checkmark \\
 w &= 75m \checkmark
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 3.3 \quad 3.3.1 \quad s(t) &= e^{-t} \ln 3t \\
 s'(t) &= \frac{1}{t} e^{-t} - e^{-t} \ln 3t \checkmark
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 3.3.2 \quad s''(t) &= e^{-t} \ln 3t - \frac{2}{t} e^{-t} - \frac{1}{t^2} e^{-t} \checkmark \\
 s''(2) &= e^{-2} \ln 3(2) - \frac{2}{2} e^{-2} - \frac{1}{2^2} e^{-2} \checkmark \\
 &= 0,021m/s^2 \checkmark
 \end{aligned} \tag{3}$$

[20]

#### QUESTION 4

$$4.1 \quad 4.1.1 \quad \int \left[ 2 \sec^2 \left( \frac{\pi}{3} x \right) \tan \left( \frac{\pi}{3} x \right) \right] dx$$

$$\begin{aligned}
 \text{let } u &= \tan \left( \frac{\pi}{3} x \right) \\
 du &= \sec^2 \left( \frac{\pi}{3} x \right) \times \frac{\pi}{3} dx \checkmark \\
 \frac{3}{\pi} du &= \sec^2 \left( \frac{\pi}{3} x \right) dx
 \end{aligned}$$

$$\begin{aligned}
 &= 2 \times \frac{3}{\pi} \int u du \checkmark \\
 &= \frac{6}{\pi} \left[ \frac{1}{2} u^2 + C \right] \\
 &= \frac{3}{\pi} \tan^2 \left( \frac{\pi}{3} x \right) + C \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned} 4.1.2 \quad & \int [\sin(-6x) \sin(3x)] dx \\ &= \frac{1}{2} \int (\cos(-3x) - \cos(-6x)) dx \checkmark \\ &= \frac{1}{2} \left( \frac{1}{3} \sin(3x) \checkmark - \frac{1}{6} \sin(6x) \checkmark \right) + C \\ &= \frac{1}{6} \sin(3x) - \frac{1}{12} \sin(6x) + C \end{aligned} \quad (3)$$

$$\begin{aligned} 4.1.3 \quad & \int \sin^2 \left( \frac{3}{4} x \right) dx \\ &= \frac{1}{2} \int \left[ 1 - \cos \frac{3}{2} x \right] dx \checkmark \\ &= \frac{1}{2} x \checkmark - \frac{1}{3} \sin \frac{3}{2} x \checkmark + C \end{aligned} \quad (3)$$

$$\begin{aligned} 4.1.4 \quad & \int \frac{4x^3 - 6x^2 + 10x + 9}{2x - 1} dx \\ &= \int \left( 2x^2 - 2x + 4 + \frac{13}{2x - 1} \right) dx \checkmark \\ &= \frac{2}{3} x^3 \checkmark - x^2 \checkmark + 4x \checkmark + \frac{13}{2} \ln(2x - 1) \checkmark + C \end{aligned} \quad (5)$$

4.1.5

$$\int \cos(-x) e^{-4x} dx$$

|                                                                                                                                           |
|-------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{aligned} \text{let } u &= \cos(-x) & dv &= e^{-4x} dx \\ du &= -\sin(-x) dx & v &= -\frac{1}{4} e^{-4x} \checkmark \end{aligned}$ |
|-------------------------------------------------------------------------------------------------------------------------------------------|

$$\begin{aligned} \int \cos(-x) e^{-4x} dx &= -\frac{1}{4} e^{-4x} \cdot \cos(-x) - \int -\sin(-x) \cdot -\frac{1}{4} e^{-4x} dx \checkmark \\ &= -\frac{1}{4} e^{-4x} \cdot \cos(-x) - \frac{1}{4} \int \sin(-x) \cdot e^{-4x} dx \end{aligned}$$

|                                                                                                                                           |
|-------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{aligned} \text{let } u &= \sin(-x) & dv &= e^{-4x} dx \\ du &= -\cos(-x) dx & v &= -\frac{1}{4} e^{-4x} \checkmark \end{aligned}$ |
|-------------------------------------------------------------------------------------------------------------------------------------------|

$$\begin{aligned} &= -\frac{1}{4} e^{-4x} \cdot \cos(-x) \\ &\quad - \frac{1}{4} \left[ -\frac{1}{4} e^{-4x} \cdot \sin(-x) - \int -\cos(-x) \cdot -\frac{1}{4} e^{-4x} dx \right] \checkmark \\ &= -\frac{1}{4} e^{-4x} \cdot \cos(-x) + \frac{1}{16} e^{-4x} \cdot \sin(-x) + \frac{1}{16} \int \cos(-x) \cdot e^{-4x} dx \end{aligned}$$

$$\begin{aligned} \int \cos(-x) e^{-4x} dx + \frac{1}{16} \int \cos(-x) \cdot e^{-4x} dx &= -\frac{1}{4} e^{-4x} \cdot \cos(-x) \\ &\quad + \frac{1}{16} e^{-4x} \cdot \sin(-x) \end{aligned}$$

$$\frac{17}{16} \int \cos(-x) \cdot e^{-4x} dx = -\frac{1}{4} e^{-4x} \cdot \cos(-x) + \frac{1}{16} e^{-4x} \cdot \sin(-x)$$

$$\int \cos(-x) \cdot e^{-4x} dx = -\frac{4}{17} e^{-4x} \cdot \cos(-x) + \frac{1}{17} e^{-4x} \cdot \sin(-x) + C \checkmark \quad (5)$$

4.2

$$\int \frac{x-4}{2x^2-5x-3} dx$$

$$\frac{x-4}{(x-3)(2x+1)} = \frac{A}{x-3} + \frac{B}{2x+1}$$

$$x-4 = A(2x+1) + B(x-3) \checkmark$$

$$\text{let } x = 3; \quad B = -\frac{7}{2} \checkmark$$

or

$$x = -\frac{1}{2}; \quad A = \frac{9}{7} \checkmark$$

$$\int \left( \frac{-\frac{7}{2}}{x-3} + \frac{\frac{9}{7}}{2x+3} \right) dx$$

$$= -\frac{7}{2} \ln(x-3) \checkmark + \frac{9}{14} \ln(2x+3) \checkmark + C$$

(5)  
[15]

## QUESTION 5

5.1

$$\int_0^8 x f'(x) dx$$

$$\begin{array}{ll} \text{let } u = x & dv = f'(x) dx \\ du = dx & v = f(x) \checkmark \end{array}$$

$$\int_0^8 x f'(x) dx = [x f(x)]_0^8 - \int_0^8 f(x) dx \checkmark$$

$$= 8 \times f(8) - 0 \times f(0) - (-12) \checkmark$$

$$= 12 \checkmark$$

(4)

$$\begin{aligned}
 5.2 \quad & \int_0^{\infty} 4^{-x} dx \\
 &= \lim_{b \rightarrow \infty} \int_0^b 4^{-x} dx \checkmark \\
 &= \lim_{b \rightarrow \infty} \left[ -\frac{4^{-x}}{\ln 4} \right]_0^b \checkmark \\
 &= -\frac{1}{\ln 4} \lim_{b \rightarrow \infty} (4^{-b} - 4^0) \\
 &= \frac{1}{\ln 4} \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 5.3 \quad 5.3.1 \quad & A = \int_a^b (y_1 - y_2) dx \\
 & A = \int_2^4 (-x^2 + 7x - 8 - x) dx \\
 &= \int_2^4 (-x^2 + 6x - 8) dx \checkmark \\
 &= \left[ -\frac{1}{3}x^3 + 3x^2 - 8x \right]_2^4 \checkmark \\
 &= \frac{4}{3} \text{ units}^2 \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 5.3.2 \quad & V = \pi \int_a^b (y_1^2 - y_2^2) dx \\
 &= \pi \int_2^4 [(-x^2 + 7x - 8)^2 - x^2] dx \\
 &= \pi \int_2^4 (x^4 - 14x^3 + 15x^2 - 112x + 64) dx \checkmark \checkmark \\
 &= \pi \left[ \frac{1}{5}x^5 - \frac{7}{2}x^4 + \frac{57}{3}x^3 - 56x^2 + 64x \right]_2^4 \checkmark \\
 &= \frac{136\pi}{15} \text{ units}^3 \text{ or } 28,484 \text{ units}^3 \checkmark
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 5.4 \quad d)_z &= r^2 dA \\
 dA &= 2\pi x dx \checkmark \\
 x^2 \cdot 2\pi x dx &\checkmark \\
 2\pi \int_0^{15} x^3 dx &\checkmark \\
 &= 2\pi \left[ \frac{x^4}{4} \right]_0^{15} \checkmark \\
 &= 2\pi \left[ \frac{50625}{4} - 0 \right] \checkmark \\
 &= 79521.564 \text{ cm}^4 \checkmark
 \end{aligned}$$

(6)  
[20]

### QUESTION 6

$$\begin{aligned}
 6.1 \quad \frac{dy}{dx} &= \frac{2x + 2xy^2}{y} \quad y(0) = -2 \\
 y dy &= 2x(1 + y^2) dx \\
 \frac{y dy}{1 + y^2} &= 2x dx \checkmark \\
 \frac{1}{2} \ln(1 + y^2) \checkmark &= x^2 + C \checkmark \\
 \frac{1}{2} \ln(1 + (-2)^2) &= 0^2 + C \\
 C &= \ln \sqrt{5} \checkmark \\
 \frac{1}{2} \ln(1 + y^2) &= x^2 + \ln \sqrt{5} \checkmark
 \end{aligned}$$

(5)

$$6.2 \quad \frac{d^2y}{dx^2} = (x+2)^2 - 3$$

$$\frac{d^2y}{dx^2} = x^2 + 4x + 1 \checkmark$$

$$\frac{dy}{dx} = \frac{1}{3}x^3 + 2x^2 + x + A \checkmark$$

$$y = \frac{1}{12}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2 + Ax + B \checkmark$$

(3)

[8]

**TOTAL: 100**