



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

31 March 2023

This marking guideline consists of 13 pages.

QUESTION 1

1.1

1.1.1

$$y = \lim_{x \rightarrow 3} \left[\frac{1}{\ln(x-2)} - \frac{1}{(x-3)} \right]$$

$$\textcolor{red}{\hookrightarrow} \lim_{x \rightarrow 3} \left[\frac{(x-3) - \ln(x-2)}{(x-3)\ln(x-2)} \right] \frac{0}{0}$$

$$\textcolor{red}{\hookrightarrow} \lim_{x \rightarrow 3} \left[\frac{1 - \frac{1}{x-2}}{\ln(x-2) + (x-3) \times \frac{1}{x-2}} \right] \frac{0}{0}$$

$$\lim_{x \rightarrow 3} \left[\frac{\frac{1}{(x-2)^2}}{\frac{1}{x-2} + \frac{(x-2) - (x-3)}{(x-2)^2}} \right]$$

$$\textcolor{red}{\hookrightarrow} \frac{1}{2}$$

(4)

1.1.2

$$y = \lim_{x \rightarrow 0} \frac{\log(x+1) - \sin x}{3x} \frac{0}{0}$$

$$\textcolor{red}{\hookrightarrow} \lim_{x \rightarrow 0} \frac{\frac{1}{\ln 10} \times \frac{1}{x+1} - \cos x}{3}$$

$$\textcolor{red}{\hookrightarrow} \frac{\frac{1}{\ln 10} - 1}{3}$$

$$\textcolor{red}{\hookrightarrow} -0,189$$

(3)

1.2

$$y = \frac{x^3 + 5x - 10}{x^2 - 3x + 2}$$

$$x^2 - 3x + 2 = 0$$

$$x = 1 \vee x = 2$$

(2)
[6]

QUESTION 2

$$2.1 \quad f(x) = \frac{x-10}{x+2}$$

$$f'(x+h) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{\frac{(x+h)-10}{(x+h)+2} - \frac{x-10}{x+2}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{x^2+2x+2h-10-20-(x^2-10x+2h-10h-2x+20)}{(x+2)(x+h+2)}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{12h}{(x+2)(x+h+2)}}{h}$$

$$\lim_{h \rightarrow 0} \frac{12}{(x+2)(x+h+2)}$$

$$= \frac{12}{(x+2)^2}$$

(4)

$$2.2 \quad 2.2.1 \quad y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \times \left[\frac{-2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2} \right]$$

(4)

$$2.2.2 \quad y = \frac{e^{x \cos x}}{1+e^x}$$

$$\frac{dy}{dx} = \frac{(1+e^x)[e^{x \cos x}(\cos x + x \cdot -\sin x)] - e^{x \cos x} \times e^x}{(1+e^x)^2}$$

(4)

2.3 $y = (3x - 2^x)^{\cos 3x}$

$$\ln y = \cos 3x \times \ln(3x - 2^x)$$

$$\frac{1}{y} \frac{dy}{dx} = \cos 3x \times \frac{1}{3x - 2^x} \times (3 - 2^x \ln 2) - 3 \sin 3x \times \ln(3x - 2^x)$$

$$\frac{dy}{dx} = y \left[\frac{(3 - 2^x) \cos 3x}{3x - 2^x} - 3 \sin 3x \times \ln(3x - 2^x) \right]$$

$$\frac{d}{dx} (3x - 2^x)^{\cos 3x} = (3x - 2^x)^{\cos 3x} \left[\frac{(3 - 2^x) \cos 3x}{3x - 2^x} - 3 \sin 3x \times \ln(3x - 2^x) \right] \quad (4)$$

2.4 2.4.1 $\frac{9 - x^2}{(y - x)^3} = 1$

$$9 - x^2 = (y - x)^3$$

$$0 - 2x = 3(y - x)^2 \times \left(\frac{dy}{dx} - 1 \right)$$

$$\frac{dy}{dx} - 1 = \frac{-2x}{3(y - x)^2}$$

$$\frac{dy}{dx} = 1 - \frac{2x}{3(y - x)^2} \quad (3)$$

2.4.2 $\frac{dy}{dx} = 1 - \frac{2(1)}{3(1 - 3)^2} = \frac{5}{6}$

$$y = \frac{5}{6}x + c$$

$$3 = \frac{5}{6}(1) + c$$

$$c = \frac{13}{6}$$

$$y = \frac{5}{6}x + \frac{13}{6}$$

(3)
[22]

QUESTION 3

3.1 3.1.1 $f(x) = x^3 - 2x^2 - 3x + 5$

$$f'(x) = 3x^2 - 4x - 3 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-3)}}{2(3)}$$

$$x = -0,535 \vee x = 1,869$$

$$y = 7,024 \quad y = -1,065$$

Thus, the coordinates of the turning points are $(-0,535; 7,024)$ and $(1,869; -1,065)$

(3)

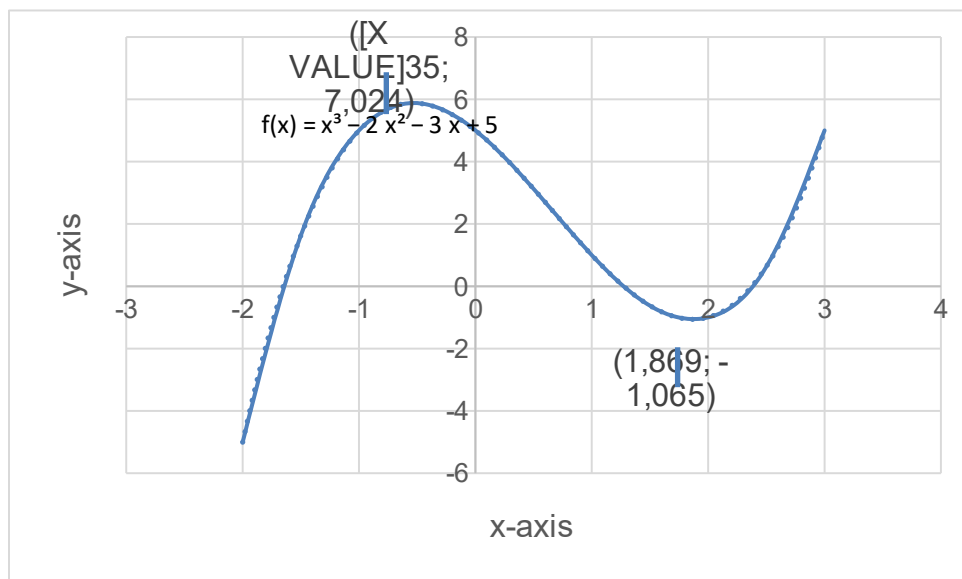
3.1.2

x	-2	-1	0	1	2	3
y	-5	5	5	1	-1	5

= 1 Mark for any 3 correct answers

(2)

3.1.3



= 1 Mark for the shape

= 1 Mark for the indication of the turning point on the graph

(2)

$$3.1.4 \quad \text{Let } x_0 = 1,3$$

$$f(1,3) = -0,083$$

$$f'(1,3) = -3,13$$

$$x_1 = 1,3 - \frac{-0,083}{-3,13}$$

$$\hookrightarrow 1,273$$

$$f(1,273) = 0,003$$

$$f'(1,273) = -3,230$$

$$x_2 = 1,273 - \frac{0,003}{-3,230}$$

$$\hookrightarrow 1,274$$

(4)

$$3.2 \quad \text{Perimeter} = 2l + w$$

$$160 = 2l + w$$

$$w = 160 - 2l$$

$$\text{Area} = w \times l$$

$$A = l(160 - 2l)$$

$$\hookrightarrow 160l - 2l^2$$

$$A' = 160 - 4l = 0$$

$$l = 40 \text{ m}$$

$$w = 160 - 2 \times 40 = 80 \text{ m}$$

(5)

$$3.3 \quad V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 4 \pi r^2 \frac{dr}{dt}$$

$$7 \text{ cm}^3/\text{s} = 4 \pi (4 \text{ cm})^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{7 \text{ cm}^3/\text{s}}{4 \pi (4 \text{ cm})^2}$$

$$\hookrightarrow 0,035 \text{ cm/s}$$

(4)

[20]

QUESTION 4

4.1

4.1.1

$$\int \frac{e^{\sqrt{\sin x}}}{\sec x \cdot \sqrt{\sin x}} dx$$

$$\text{let } u = \sqrt{\sin x}$$

$$du = \frac{1}{2\sqrt{\sin x}} \cdot \cos x dx$$

$$2 du = \frac{1}{\sqrt{\sin x}} \cdot \cos x dx$$

$$\int 2 e^u du$$

$$2 e^u + C$$

$$2 e^{\sqrt{\sin x}} + C$$

(3)

4.1.2

$$\int \sin 3x \sin(-2x) dx$$

$$\int \frac{1}{2} (\sin x + \sin 5x) dx$$

$$\int \frac{1}{2} \left(-\cos x - \frac{\cos 5x}{5} \right) + C$$

$$-\frac{\cos x}{2} - \frac{\cos 5x}{5} + C$$

(3)

4.1.3 $\int \cos^2 x \sin^3 x \, dx$

$\hookrightarrow \int \sin x \sin^2 x \cos^2 x \, dx$

let $u = \cos x$

$du = -\sin x$

$\hookrightarrow -\int (1-u^2)u^2 \, du$

$\hookrightarrow \int (u^4 - u^2) \, du$

$\hookrightarrow \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$

$\hookrightarrow \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$

(5)

4.1.4 $\int \frac{3x^3 - 2x^2 + x - 5}{x+2} \, dx$

$\hookrightarrow \int \left(3x^2 - 8x + 17 - \frac{39}{x+2} \right) \, dx$

$\hookrightarrow x^3 - 4x^2 + 17x - 39 \ln(x+2) + C$

(5)

4.1.5 $\int \frac{x^3 e^{x^2}}{(x^2+1)^2} \, dx$

let $u = x^2 e^{x^2} \, dv = \frac{x}{(x^2+1)^2}$

$du = (2x^3 e^{x^2} + 2x e^{x^2}) \, dx \quad v = \frac{-1}{2(x^2+1)}$

$\int \frac{x^3 e^{x^2}}{(x^2+1)^2} \, dx$

$\hookrightarrow -\frac{1}{2(x^2+1)} \times x^2 e^{x^2} - \int \frac{-1}{2(x^2+1)} \times (2x^3 e^{x^2} + 2x e^{x^2}) \, dx$

$\hookrightarrow -\frac{x^2 e^{x^2}}{2(x^2+1)} + \int \frac{2x e^{x^2}(x^2+1)}{2(x^2+1)} \, dx$

$\hookrightarrow -\frac{x^2 e^{x^2}}{2(x^2+1)} + \int x e^{x^2} \, dx$

$\hookrightarrow -\frac{x^2 e^{x^2}}{2(x^2+1)} + \frac{1}{2} e^{x^2} + C$

(4)

4.2

$$\int \frac{11-3x}{x^2+2x-3} dx$$

$$\frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$$11-3x = A(x+3) + B(x-1)$$

$$\text{let } x = -3; B = -5$$

$$\downarrow$$

$$x = 1; A = 2$$

$$\int \left(\frac{2}{x-1} + \frac{-5}{x+3} \right) dx$$

$$\downarrow 2 \ln(x-1) - 5 \ln(x+3) + C$$

(5)
[25]**QUESTION 5**

5.1

$$I = \int_0^{\frac{\pi}{4}} \frac{\sec x \tan x}{1 + \sec^2 x} dx$$

$$\text{let } u = \sec x$$

$$du = \sec x \tan x dx$$

$$x = 0 \leftrightarrow u = 1$$

$$x = \frac{\pi}{4} \leftrightarrow u = \sqrt{2}$$

$$I = \int_1^{\sqrt{2}} \frac{1}{1+u^2} du$$

$$\downarrow [\tan^{-1} u]_1^{\sqrt{2}}$$

$$\downarrow 0,170$$

(5)

5.2

$$L(3) = \frac{3}{s}$$

(1)

5.3

5.3.1

$$A = \int_a^b (y_1 - y_2) dx$$

$$\textcircled{!} \int_0^2 \left[\left(\sqrt{4-x^2} \right) - \left(\frac{1}{2}x^2 - 2x + 2 \right) \right] dx$$

$$\textcircled{!} \int_0^2 \left(\sqrt{4-x^2} - \frac{1}{2}x^2 + 2x - 2 \right) dx$$

$$\textcircled{!} \left[2 \sin^{-1} \left(\frac{x}{2} \right) - \frac{1}{6}x^3 + x^2 - 2x \right]_0^2$$

$$\textcircled{!} \pi - \frac{4}{3}$$

$$\textcircled{!} 1,808 \text{ units}^2$$

(3)

5.3.2

$$V = \pi \int_a^b (y_1^2 - y_2^2) dx$$

$$\textcircled{!} \pi \int_0^2 \left[\left(\sqrt{4-x^2} \right)^2 - \left(\frac{1}{2}x^2 - 2x + 2 \right)^2 \right] dx$$

$$\textcircled{!} \pi \int_0^2 \left(4 - x^2 - \frac{1}{4}x^4 + 2x^3 - 6x^2 + 8x - 4 \right) dx$$

$$\textcircled{!} \pi \int_0^2 \left(-\frac{1}{4}x^4 + 2x^3 - 7x^2 + 8x \right) dx$$

$$\textcircled{!} \pi \left[-\frac{1}{20}x^5 + \frac{1}{2}x^4 - \frac{7}{3}x^3 + 4x^2 \right]_0^2$$

$$\textcircled{!} \frac{56\pi}{15} = 11,729 \text{ units}^3$$

(5)

$$5.4 \quad m = \sigma A \quad dm = \sigma dA$$

$$dI_z = r^2 dm$$

$$\int x^2 \sigma dm$$

$$\int x^2 \sigma 2\pi x dx$$

$$\int_0^4 2\pi\sigma x^3 dx$$

$$\int_0^4 2\pi\sigma \left[\frac{1}{4} x^4 \right]_0^4$$

$$\int_0^4 2\pi\sigma \left[\frac{4^4}{4} - 0 \right] = 128\pi\sigma \vee 402,124 \text{ units}^4$$

(5)
[19]**QUESTION 6**

$$6.1 \quad \frac{dy}{dx} = 2x - 1 + 2xy - y, \text{ if } y(0) = 2$$

$$\frac{dy}{dx} = (2x - 1)(y + 1)$$

$$\frac{dy}{y+1} = (2x - 1) dx$$

$$\ln(y+1) = x^2 - x + C$$

(3)

$$6.2 \quad \frac{1}{(x+3)^2-2} \frac{d^2 y}{dx^2} = 1, \text{ if } \frac{dy}{dx} \Big|_{x=1} = 0 \wedge y(1) = 2$$

$$\frac{1}{(x+3)^2-2} \frac{d^2 y}{dx^2} = 1$$

$$\frac{d^2 y}{dx^2} = (x+3)^2 - 2$$

$$\frac{d^2 y}{dx^2} = x^2 + 6x + 7$$

$$\frac{dy}{dx} = \frac{1}{3}x^3 + 3x^2 + 7x + A$$

$$0 = \frac{1}{3}(1)^3 + 3(1)^2 + 7(1) + A$$

$$A = \frac{-31}{3}$$

$$\frac{dy}{dx} = \frac{1}{3}x^3 + 3x^2 + 7x - \frac{31}{3}$$

$$y = \frac{1}{12}x^4 + x^3 + \frac{7}{2}x^2 - \frac{31}{3}x + B$$

$$2 = \frac{1}{12}(1)^4 - (1)^3 + \frac{7}{2}(1)^2 - \frac{31}{3}(1) + B$$

$$B = \frac{31}{4}$$

$$y = \frac{1}{12}x^4 + x^3 + \frac{7}{2}x^2 - \frac{31}{3}x + \frac{31}{4}$$

(5)
[8]

TOTAL: 100