



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

30 June 2022

This marking guideline consists of 11 pages.

QUESTION 1

1.1 $y = \lim_{x \rightarrow 0} \frac{x}{\frac{\pi}{2} - \cos^{-1}(3x)} \frac{0}{0}$

$\dot{\lim}_{x \rightarrow 0} \frac{1}{0 + \frac{3}{\sqrt{1+9x^2}}}$

$\dot{\lim}_{x \rightarrow 0} \frac{1}{3}$ (2)

1.2 1.2.1 $\ln y = \lim_{x \rightarrow 4} \frac{\sqrt{8-x}-2}{\sqrt{5-x}-1} \frac{0}{0}$

$\dot{\lim}_{x \rightarrow 4} \frac{\frac{-1}{2\sqrt{8-x}}}{\frac{-1}{2\sqrt{5-x}}}$

$\ln y = \frac{1}{2}$ (2)

1.2.2 $y = e^{\frac{1}{2}} = 1,649$ (1)
[5]

QUESTION 2

2.1 2.1.1 $f(x) = \sqrt{x+2}$ (2)

$f(x+h) = [(x+2)+h]^{\frac{1}{2}}$

$\dot{\lim}_{h \rightarrow 0} \frac{(x+2)^{\frac{1}{2}} h^0 + \frac{1}{2}(x+2)^{-\frac{1}{2}} h^1 + \frac{-1}{4}(x+2)^{-\frac{3}{2}} h^2 + \frac{3}{8}(x+2)^{-\frac{5}{2}} h^3}{0! + 1! + 2! + 3! + \dots}$

$\dot{\lim}_{h \rightarrow 0} (x+2)^{\frac{1}{2}} + \frac{1}{2}(x+2)^{-\frac{1}{2}} h - \frac{1}{8}(x+2)^{-\frac{3}{2}} h^2 + \frac{1}{16}(x+2)^{-\frac{5}{2}} h^3 + \dots$

2.1.2 $f(x+h) - f(x) = \frac{1}{2}(x+2)^{-\frac{1}{2}} h - \frac{1}{8}(x+2)^{-\frac{3}{2}} h^2 + \frac{1}{16}(x+2)^{-\frac{5}{2}} h^3$ (1)

2.1.3 $\frac{f(x+h) - f(x)}{h} = \frac{1}{2}(x+2)^{-\frac{1}{2}} - \frac{1}{8}(x+2)^{-\frac{3}{2}} h + \frac{1}{16}(x+2)^{-\frac{5}{2}} h^2 + \dot{\lim}_{h \rightarrow 0} \dots$ (1)

2.1.4 $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \frac{1}{2}(x+2)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x+2}}$ (1)

2.2 2.2.1 $y = \log_3 \left(e^{\frac{x}{5}} \tan 7x \right)$

$$\frac{dy}{dx} = \frac{1}{\ln 3} \times \frac{1}{e^{\frac{x}{5}} \tan 7x} \times \left(\frac{1}{5} e^{\frac{x}{5}} \tan 7x + e^{\frac{x}{5}} \times \sec^2 7x \times 7 \right) \quad (3)$$

2.2.2 $y = \cot(\log_\pi(x-5) + x^3)$

$$\frac{dy}{dx} = -\operatorname{cosec}^2(\log_\pi(x-5) + x^3) \times \left(\frac{1}{\ln \pi} \times \frac{1}{x-5} + 3x^2 \right) \quad (3)$$

2.3

$$y = \frac{\sqrt{1+\sin 2x}}{2x - \tan x}$$

$\ln y = \ln \sqrt{1+\sin 2x} - \ln(2x - \tan x)$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{\sqrt{1+\sin 2x}} \times \frac{1}{2\sqrt{1+\sin 2x}} \times 2\cos 2x - \frac{1}{2x - \tan x} \times (2 - \sec^2 x)$$

$$\frac{dy}{dx} = y \left[\frac{\cos 2x}{1+\sin 2x} - \frac{2 - \sec^2 x}{2x - \tan x} \right]$$

$$\frac{\sqrt{1+\sin 2x}}{2x - \tan x} \left[\frac{\cos 2x}{1+\sin 2x} - \frac{2 - \sec^2 x}{2x - \tan x} \right] \quad (4)$$

2.4 2.4.1 $y^2 + x^2 = -(3xy + 1)$

$$2y \frac{dy}{dx} + 2x = - \left(3y + 3x \frac{dy}{dx} + 0 \right)$$

$$2y \frac{dy}{dx} + 2x = -3y - 3x \frac{dy}{dx}$$

$$(2y + 3x) \times \frac{dy}{dx} = -3y - 2x$$

$$\frac{dy}{dx} = \frac{-3y - 2x}{2y + 3x} \quad (4)$$

$$2.4.2 \quad m = \frac{-3(1) - 2(-1)}{2(1) + 3(-1)} = 1$$

$$y = mx + c$$

$$y = x + c$$

$$1 = -1 + c$$

$$c = 2$$

$$y = x + 2$$

∴

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 1(x + 1)$$

$$y = x + 1 + 1$$

$$y = x + 2$$

(3)
[22]

QUESTION 3

$$3.1 \quad 3.1.1 \quad f(x) = 2x^3 - x^2 - 4x + 1$$

$$f'(x) = 6x^2 - 2x - 4 = 0$$

$$f''(x) = 12x - 2 = 0$$

$$12x = 2$$

$$x = \frac{1}{6} \quad y = \frac{17}{54}$$

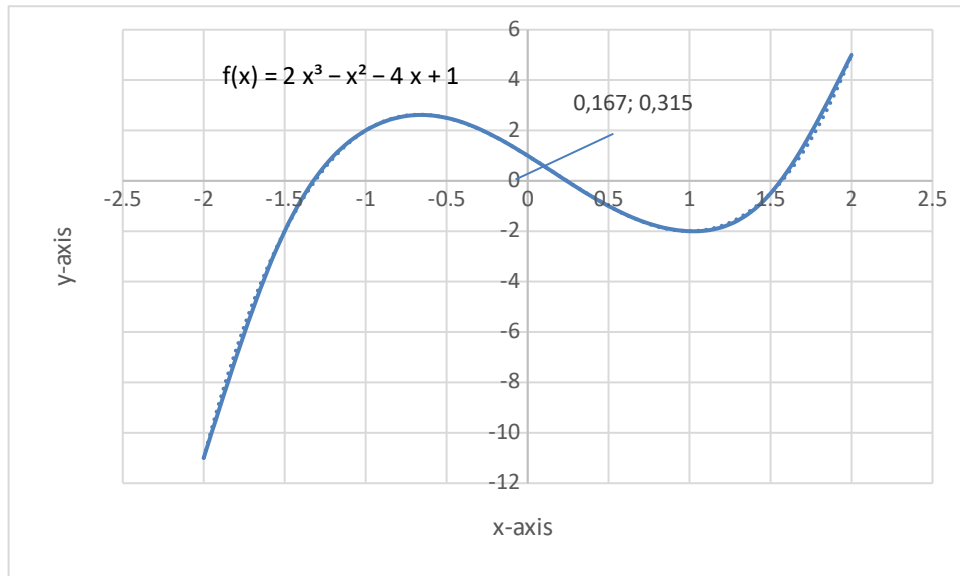
Thus, the coordinate of the point of inflection is $\left(\frac{1}{6}, \frac{17}{54}\right)$. (2)

3.1.2

x	-2	-1	0	1	2
y	-11	2	1	-2	5

ONE mark each for any THREE correct answers ✓✓✓ (3)

3.1.3



✓ ONE mark for the shape

✓ ONE mark for the indication of the point of inflection on the graph (2)

3.1.4 Let $x_0 = 1,5$

$$f(1,5) = -0,5$$

$$f'(1,5) = 6,5$$

$$x_1 = 1,5 - \frac{-0,5}{6,5}$$

$$\hookrightarrow 1,577$$

$$f(1,577) = 0,049$$

$$f'(1,577) = 7,768$$

$$x_2 = 1,577 - \frac{0,049}{7,768}$$

$$\hookrightarrow 1,571$$

(4)

3.2 $V = \pi r^2 h$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

$$\hookrightarrow \pi (5m)^2 (-0,3m/s)$$

$$\hookrightarrow -\frac{15\pi}{2} m^3/s$$

$$\hookrightarrow -23,562 m^3/s$$

(4)

- 3.3 3.3.1 $s(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 - 3t$
- $velocity = s'(t) = t^2 - t - 3 = 3$
- $t^2 - t - 6 = 0$
- $(t+2)(t-3) = 0$
- $t = -2 \vee t = 3$
- but $t \geq 0$
- thus, $t = 3 \text{ s}$ (3)
- 3.3.2 $acceleration = s''(t) = 2t - 1 = 0$
- $t = \frac{1}{2}$
- thus, $t = 0,5 \text{ s}$ (2)
- [20]

QUESTION 4

- 4.1 4.1.1 $\int \frac{x^2+x}{\sqrt{x-1}} dx$
-
- let $u = x - 1$
 $du = dx$
 $\rightarrow x = u + 1$
-
- $\therefore \int \frac{(u+1)^2 + u + 1}{\sqrt{u}} du$
- $\therefore \int \frac{u^2 + 3u + 2}{\sqrt{u}} du$
- $\therefore \int \left(u^{\frac{3}{2}} + 3u^{\frac{1}{2}} + 2u^{-\frac{1}{2}} \right) du$
- $\therefore \frac{2}{5} u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + 4u^{\frac{1}{2}} + C$
- $\therefore \frac{2}{5} (x-1)^{\frac{5}{2}} + 2(x-1)^{\frac{3}{2}} + 4(x-1)^{\frac{1}{2}} + C$ (5)

$$4.1.2 \quad \int \frac{20x^3 + 2x^2 + 4x + 15}{5x + 3} dx$$

$$\hookrightarrow \int \left(4x^2 - 2x + 2 + \frac{9}{5x + 3} \right) dx$$

$$\hookrightarrow \frac{4}{3}x^3 - x^2 + 2x + \frac{9}{5} \ln(5x + 3) + C \quad (5)$$

$$4.1.3 \quad y = \cos 4x \cos \frac{3}{2}x$$

$$\hookrightarrow \int \frac{1}{2} \left[\cos \left(4x + \frac{3}{2}x \right) + \cos \left(4x - \frac{3}{2}x \right) \right] dx$$

$$\hookrightarrow \frac{1}{2} \int \left(\frac{2}{11} \cos \frac{11}{2}x + \frac{2}{5} \cos \frac{5}{2}x \right) dx$$

$$\hookrightarrow \frac{1}{11} \sin \frac{11}{2}x + \frac{1}{5} \sin \frac{5}{2}x + C \quad (3)$$

$$4.1.4 \quad \int \ln 7x \, dx$$

$$\text{let } f(x) = \ln 7x \quad g'(x) = 1 \, dx$$

$$f'(x) = \frac{1}{x} \quad g(x) = x$$

$$\int x^2 e^{5x} dx = x \ln 7x - \int x \times \frac{1}{x} dx$$

$$\hookrightarrow x \ln 7x - x + C \quad (4)$$

$$4.1.5 \quad \int \cos^2 \left(\frac{x}{\pi} \right) dx$$

$$\hookrightarrow \int \frac{1}{2} \left[1 + \cos \left(\frac{2}{\pi}x \right) \right] dx$$

$$\hookrightarrow \frac{1}{2}x + \frac{\pi}{4} \sin \left(\frac{2}{\pi}x \right) + C \quad (3)$$

4.2

$$\int \frac{3x+5}{x^2-6x+8} dx$$

$$\frac{3x+5}{(x-2)(x-4)} = \frac{A}{x-2} + \frac{B}{x-4}$$

$$3x+5 = A(x-4) + B(x-2)$$

let $x=4$; $B = \frac{17}{2}$

$x=2$; $A = \frac{-11}{2}$

$$\int \left(\frac{-11}{2} \frac{1}{x-2} + \frac{17}{2} \frac{1}{x-4} \right) dx$$

$$- \frac{11}{2} \ln(x-2) + \frac{17}{2} \ln(x-4) + C$$

(5)
[25]

QUESTION 5

5.1

$$\int_0^{\infty} e^{-st} \cdot f(t) dt$$

$$\frac{1}{4} \int_0^{\infty} t e^{-st} dt$$

let $u=t$ $dv=e^{-st} dt$

$$du=dt \quad v=\frac{-1}{s} e^{-st}$$

$$\frac{1}{4} \left[\frac{-t}{s} e^{-st} \right]_0^{\infty} + \frac{1}{4s} \int_0^{\infty} e^{-st} dt$$

$$- \frac{1}{4s} \left[t e^{-st} \right]_0^{\infty} - \frac{1}{4s^2} \left[e^{-st} \right]_0^{\infty}$$

$$- \frac{1}{4s} (0-0) - \frac{1}{4s^2} (0-1)$$

$$\frac{1}{4s^2}$$

(5)

$$5.2 \quad 5.2.1 \quad \frac{x+3}{2} = \frac{3-2x-x^2}{2}$$

$$x+3=3-2x-x^2$$

$$x^2+3x=0$$

$$x(x+3)=0$$

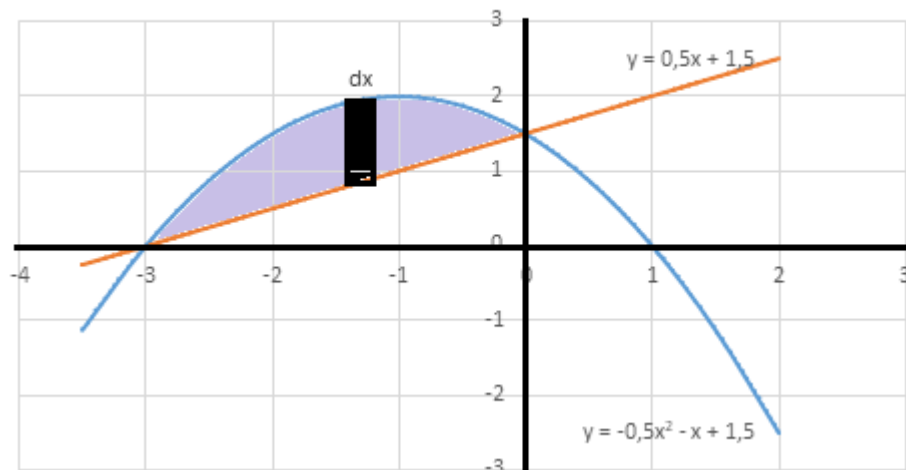
$$x=0 \vee x=-3$$

$$y=\frac{3}{2} \quad y=0$$

Thus, the coordinates of the points of intersection are $(0; 1,5)$ and $(-3; 0)$. (2)

5.2.2

Bounded Area



✓ ONE mark for indication of enclosed area

✓ ONE mark for indication of the vertical or horizontal strip

(2)

5.2.3

$$A = \int_a^b (y_1 - y_2) dx$$

$$\int_{-3}^0 \left[\left(\frac{3-2x-x^2}{2} \right) - \left(\frac{x+3}{2} \right) \right] dx$$

$$\int_{-3}^0 \frac{1}{2} (-3x - x^2) dx$$

$$\int_{-3}^0 \left[\frac{-1}{3} x^3 - \frac{3}{2} x^2 \right]_{-3}^0$$

$$\int_{-3}^0 \frac{9}{4} = 2,25 \text{ units}^2$$

(3)

$$\begin{aligned}
 5.2.4 \quad V &= \pi \int_a^b (y_1^2 - y_2^2) dx \\
 &= \pi \int_{-3}^0 \left[\left(\frac{3-2x-x^2}{2} \right)^2 - \left(\frac{x+3}{2} \right)^2 \right] dx \\
 &= \frac{\pi}{4} \int_{-3}^0 (x^4 + 4x^3 - 3x^2 - 18x) dx \\
 &= \left[\frac{1}{5}x^5 + x^4 - x^3 - 9x^2 \right]_{-3}^0 \\
 &= \frac{27}{5} \pi = 16,965 \text{ units}^3
 \end{aligned}$$

(4)

$$\begin{aligned}
 5.3 \quad d)_z &= r^2 dA \\
 &= x^2 \cdot 2\pi x dx \\
 &= 2\pi \int_0^8 x^3 dx \\
 &= 2\pi \left[\frac{x^4}{4} \right]_0^8 \\
 &= 2\pi \left[\frac{4096}{4} - 0 \right] \\
 &= 2048\pi \text{ cm}^4 \\
 &= 6433,982 \text{ cm}^4
 \end{aligned}$$

(4)
[20]

QUESTION 6

$$\begin{aligned}
 6.1 \quad \frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} &= \cos x \\
 \frac{dy}{dx} &= \cos x - \frac{1}{\sqrt{1-x^2}} \\
 y &= \sin x + \sin^{-1} x + C
 \end{aligned}$$

(2)

$$6.2 \quad \frac{d^2 y}{dx^2} = 24x^2 + 2x + 10$$

$$\frac{dy}{dx} = 8x^3 + x^2 + 10x + A$$

$$-3 = 8(0)^3 + (0)^2 + 10(0) + A$$

$$A = -3$$

$$\frac{dy}{dx} = 8x^3 + x^2 + 10x - 3$$

$$y = 2x^4 + \frac{1}{3}x^3 + 5x^2 - 3x + B$$

$$5 = 2(0)^4 + \frac{1}{3}(0)^3 + 5(0)^2 - 3(0) + B$$

$$B = 5$$

$$y = 2x^4 + \frac{1}{3}x^3 + 5x^2 - 3x + 5$$

(6)
[8]

TOTAL: 100