



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N5

30 Julie 2024

Hierdie nasienriglyn bestaan uit 10 bladsye.

VRAAG 1

$$\begin{aligned}
 1.1.1 \quad \ln y &= \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{3x^2} : \frac{0}{0} \\
 &= \lim_{x \rightarrow 0} \frac{\sec^2 x + \cos x}{6x} \checkmark : \frac{0}{0} \\
 &= \lim_{x \rightarrow 0} \frac{2 \sec x \cdot \sec x \cdot \tan x - \sin x}{6} \checkmark \\
 &= 0 \checkmark
 \end{aligned}$$

(3)

$$\begin{aligned}
 1.1.2 \quad \ln y &= 0 \\
 y &= 1 \checkmark
 \end{aligned}$$

(1)

$$1.2 \quad f(x) = \frac{e^x - 1}{\sin x}, \quad [0^\circ; 270^\circ]$$

$$x = 0^\circ \checkmark \text{ or } x = 180^\circ \checkmark$$

(2)

[6]

VRAAG 2

$$2.1 \quad f(x) = \cos x$$

$$f(x + h) = \cos x \cos h - \sin x \sin h \checkmark$$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \checkmark \\
 &= \lim_{h \rightarrow 0} \left[\cos x \cdot \frac{\cos h - 1}{h} - \sin x \cdot \frac{\sin h}{h} \right] \\
 &= \cos x \cdot \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} - \sin x \cdot \lim_{h \rightarrow 0} \frac{\sin h}{h} \checkmark \\
 &= \cos x \cdot 0 - \sin x \cdot 1 \checkmark \\
 &= -\sin x
 \end{aligned}$$

(4)

2.2 2.2.1 $y = \sin^6(\ln(4x^5))$

$$\frac{dy}{dx} = 6 \sin^5(\ln(4x^5)) \times \cos(\ln(4x^5)) \times \frac{1}{4x^5} \times 20x^4 \quad (3)$$

2.2.2 $y = \left(\frac{x^2 + 4}{x^3 - 3x + 6} \right)^4 + \sqrt{2x - 5}$

$$\begin{aligned} \frac{dy}{dx} = 4 \left(\frac{x^2 + 4}{x^3 - 3x + 6} \right)^3 \times \frac{2x(x^3 - 3x + 6) - (x^2 + 4)(3x^2 - 3)}{(x^3 - 3x + 6)^2} \\ + \frac{1}{2\sqrt{2x - 5}} \times 2 \end{aligned} \quad (5)$$

2.3 $y = (\cos x)^{-2x}$

$$\ln y = -2x \ln \cos x$$

$$\frac{1}{y} \frac{dy}{dx} = -2 \ln \cos x - 2x \times \frac{1}{\cos x} \times (-\sin x)$$

$$\frac{1}{y} \frac{dy}{dx} = -2 \ln \cos x + 2x \tan x$$

$$\frac{dy}{dx} = y(2x \tan x - 2 \ln \cos x)$$

$$\frac{dy}{dx} = (\cos x)^{-2x} (2x \tan x - 2 \ln \cos x) \quad (5)$$

2.4 $\tan 3y - y^3 \cos x + 3xy^2 = -4$

$$\sec^2 3y \frac{dy}{dx} - \left[y^3 (-\sin x) + 3y^2 \cos x \frac{dy}{dx} \right] + 3y^2 + 6xy \frac{dy}{dx} = 0$$

$$\sec^2 3y \frac{dy}{dx} + y^3 \sin x - 3y^2 \cos x \frac{dy}{dx} + 3y^2 + 6xy \frac{dy}{dx} = 0$$

$$[\sec^2 3y - 3y^2 \cos x + 6xy] \frac{dy}{dx} = -y^3 \sin x - 3y^2$$

$$\frac{dy}{dx} = \frac{-y^3 \sin x - 3y^2}{\sec^2 3y - 3y^2 \cos x + 6xy} \quad (5)$$

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VRAAG 3

3.1

3.1.1

$$f(x) = x^3 - 3x^2 - 3x + 6$$

$$f'(x) = 3x^2 - 6x - 3 = 0$$

$$x^2 - 2x - 1 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = -0,414 \quad \text{or} \quad x = 2,414$$

$$y = 6,657 \quad \checkmark \quad y = -4,657$$

Dus, die koördinate van die draaipunte is $(-0,414 ; 6,657)$ en $(2,414; -4,657)$. ✓

(3)

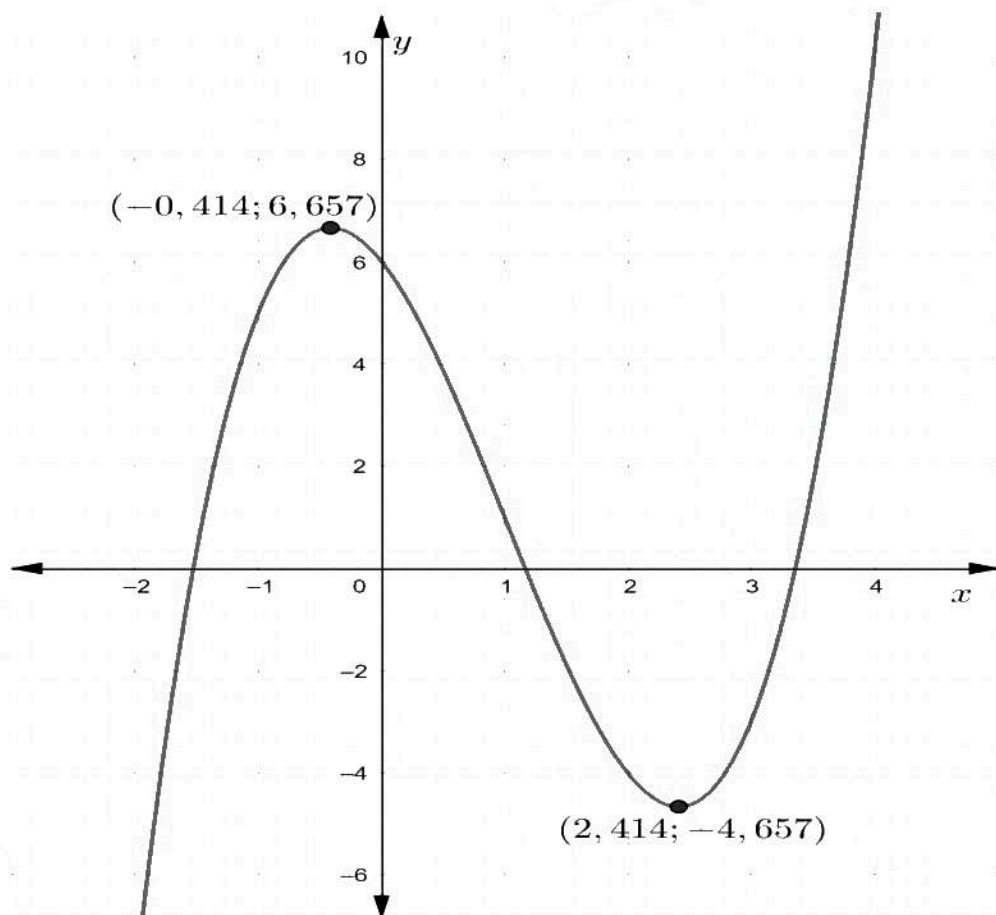
3.1.2

x	-2	-1	0	1	2	3	4
y	-8	5	6	1	-4	-3	10

✓✓ = 2 punte vir enige 2 korrekte antwoorde

(2)

3.1.3



✓ = 1 punt vir die vorm

✓ = 1 punt vir die aanduiding van die draaipunte op die grafiek

(2)

3.1.4 Laat $x_0 = -1,5$

$$f(-1,5) = 0,375$$

$$f'(-1,5) = 12,750 \checkmark$$

$$x_1 = -1,5 - \frac{0,375}{12,750} \checkmark$$

$$= -1,529 \checkmark$$

(3)

3.2 3.2.1 $\frac{dV}{dt} = \frac{1}{12\pi} \left(2Ch \frac{dC}{dt} \checkmark + C^2 \frac{dh}{dt} \checkmark \right)$

Dit verteenwoordig die verandering van die volume van die boom, of hoe vinnig die boom groei. $\checkmark$

(3)

3.2.2 $\frac{dV}{dt} = \frac{1}{12\pi} (2 \times 5 \times 22 \times 0,2 + 5^2 \times 4) \checkmark$

$$= \frac{1}{12\pi} (44 + 100) \checkmark$$

$$= \frac{12}{\pi} \text{ or } 3,820 \text{ m/jaar} \checkmark$$

(3)

3.3 $S = x + \frac{1}{x} \checkmark$

$$S' = 1 - \frac{1}{x^2} = 0 \checkmark$$

$$x^2 - 1 = 0 \checkmark$$

$$x = \pm 1$$

$$\therefore x = 1 \checkmark$$

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VRAAG 4

4.1 4.1.1 $\int \cos 3x \sin(-7x) dx$

$$= \frac{1}{2} \int (\sin(-4x) - \sin 10x) dx \checkmark$$

$$= \frac{1}{8} \cos 4x \checkmark + \frac{1}{20} \cos 10x \checkmark + C$$

$$\begin{aligned}
 4.1.2 \quad & \int (\sin^4 x \operatorname{cosec} x) dx \\
 &= \int \sin^3 x dx \checkmark \\
 &= \int (1 - \cos^2 x) \sin x dx \checkmark \\
 &= \int \sin x dx - \int \cos^2 x \sin x dx \checkmark \\
 &= -\cos x \checkmark + \frac{\cos^2 x}{2} \checkmark + C
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 4.1.3 \quad & \int \frac{x^3 - 3x^2 - 10x + 24}{x + 4} dx \\
 &= \int \left(x^2 - 7x + 18 - \frac{48}{x + 4} \right) dx \checkmark \\
 &= \frac{1}{3}x^3 \checkmark - \frac{7}{2}x^2 \checkmark + 18x \checkmark - 48 \ln(x + 4) \checkmark + C
 \end{aligned} \tag{5}$$

$$4.1.4 \quad \int x^3 \ln 4x dx$$

$ \begin{aligned} \text{let } u &= \ln 4x & dv &= x^3 dx \\ du &= \frac{1}{4x} \times 4 dx & v &= \frac{1}{4} x^4 \checkmark \\ &= \frac{1}{4} dx \end{aligned} $

$$\begin{aligned}
 \int x^3 \ln 4x dx &= \frac{1}{4} x^4 \cdot \ln 4x - \int \frac{1}{4} x^4 \cdot \frac{1}{x} dx \checkmark \\
 &= \frac{1}{4} x^4 \cdot \ln 4x - \frac{1}{4} \int x^3 dx \checkmark \\
 &= \frac{1}{4} x^4 \cdot \ln 4x - \frac{1}{16} x^4 \checkmark + C
 \end{aligned} \tag{4}$$

4.2

$$\int \frac{2x-1}{(x+1)^3} dx$$

$$\frac{2x-1}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$$2x-1 = A(x+1)^3 + B(x+1)^2 + C \checkmark$$

$$A = -1 \checkmark$$

$$B = 3 \checkmark$$

$$C = -3 \checkmark$$

$$= \int \left[\frac{-1}{x+1} + \frac{3}{(x+1)^2} - \frac{3}{(x+1)^3} \right] dx$$

$$= -\ln(x+1) \checkmark - \frac{3}{(x+1)} \checkmark + \frac{3}{2(x+1)^2} \checkmark + C$$

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[24]

VRAAG 5

5.1

$$\int_0^2 f'(x) [f(x)]^2 dx$$

$$\text{let } u = f(x)$$

$$du = f'(x) \checkmark$$

$$u = f(0) = 1 \quad \text{and} \quad u = f(2) = 5$$

$$= \int_1^5 u^2 du \checkmark$$

$$= \left[\frac{1}{3} u^3 \right]_1^5 \checkmark$$

$$= \frac{1}{3} \times 5^3 - \frac{1}{3} \times 1^3$$

$$= \frac{124}{3} \checkmark$$

(4)

$$\begin{aligned}
 5.2 \quad \int_0^{\infty} e^{-2t} \cdot p dt \checkmark &= p \int_0^{\infty} e^{-2t} dt \\
 &= p \left[-\frac{e^{-2t}}{2} \right]_0^{\infty} \checkmark \\
 &= p \left(0 + \frac{1}{2} \right) \checkmark \\
 &= \frac{p}{2}
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 5.3 \quad 5.3.1 \quad A &= \int_a^b (y_1 - y_2) dx \\
 &= \int_{-1,577}^{-0,422} [-x - (3x^2 + 5x + 2)] dx \checkmark \\
 &= \int_{-1,577}^{-0,422} (-3x^2 - 6x - 2) dx \\
 &= [-x^3 - 3x^2 - 2x]_{-1,577}^{-0,422} \checkmark \\
 &= 0,770 \text{ units}^2 \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 5.3.2 \quad V &= \pi \int_a^b (y_1^2 - y_2^2) dx \\
 &= \pi \int_{-1,577}^{-0,422} [(-x)^2 - (3x^2 + 5x + 2)^2] dx \checkmark \\
 &= \pi \int_{-1,577}^{-0,422} (-9x^4 - 30x^3 - 26x^2 - 20x - 4) dx \checkmark \checkmark \\
 &= \pi \left[-\frac{9}{5}x^5 - \frac{15}{2}x^4 - \frac{26}{3}x^3 - 10x^2 - 4x \right]_{-1,577}^{-0,422} \checkmark \\
 &= 2,902 \text{ units}^3 \checkmark
 \end{aligned} \tag{5}$$

$$5.4 \quad m = \sigma A$$

$$dm = \sigma dA$$

$$dI_z = r^2$$

$$= x^2 \sigma dm \checkmark$$

$$= x^2 \sigma 2\pi x dx \checkmark$$

$$= 2\pi\sigma \int_0^{\frac{r}{2}} x^3 dx \checkmark$$

$$= 2\pi\sigma \left[\frac{1}{4} x^4 \right]_0^{\frac{r}{2}} \checkmark$$

$$= 2\pi\sigma \left[\frac{r^4}{64} - 0 \right]$$

$$= \frac{1}{32} \pi \sigma r^4 \text{units}^4 \checkmark$$

(5)
[20]

VRAAG 6

$$6.1 \quad \frac{dy}{dx} = \frac{2xy}{y^2 - 1} \quad y(2) = 1$$

$$(y^2 - 1) dy = 2xy dx$$

$$\left(y - \frac{1}{y} \right) dy = 2x dx \checkmark$$

$$\frac{y^2}{2} - \ln y \checkmark = x^2 + C \checkmark$$

$$\frac{1^2}{2} - \ln 1 = 2^2 + C$$

$$C = -\frac{7}{2} \checkmark$$

$$\frac{y^2}{2} - \ln y = x^2 - \frac{7}{2} \checkmark$$

(5)

6.2

$$14 + 2x \frac{d^2y}{dx^2} = 2x^3 - 6x^2$$

$$\frac{d^2y}{dx^2} = x^2 - 3x - 7 \checkmark$$

$$\frac{d y}{d x} = \frac{1}{3} x^3 - \frac{3}{2} x^2 - 7x + A \checkmark$$

$$y = \frac{1}{12} x^4 - \frac{1}{2} x^3 - \frac{7}{2} x^2 + Ax + B \checkmark$$

(3)

[8]

TOTAAL: 100