



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N5

28 Maart 2025

Hierdie nasienriglyn bestaan uit 11 bladsye.

VRAAG 1

$$\begin{aligned}
 1.1 \quad y &= \lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x} \quad \frac{0}{0} \\
 &= \lim_{x \rightarrow 0} \frac{\sec^2 x - 1}{1 - \cos x} \checkmark \quad \frac{0}{0} \\
 &= \lim_{x \rightarrow 0} \frac{\sec x (\sec x \tan x)}{\sin x} \checkmark \\
 &= \lim_{x \rightarrow 0} \frac{2}{\cos^2 x} \times \frac{1}{\sin x} \times \frac{\sin x}{\cos x} \\
 &= \lim_{x \rightarrow 0} \frac{2}{\cos^3 x} \checkmark \\
 &= 2 \checkmark
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 1.2 \quad f(x) &= \frac{3}{e^{x^2} - e^x} \\
 e^{x^2} - e^x &= 0 \\
 x^2 &= x \\
 x(x - 1) &= 0 \\
 x = 0 \checkmark \text{ of } x = 1 \checkmark
 \end{aligned} \tag{2}$$

[6]

VRAAG 2

$$\begin{aligned}
 2.1 \quad f(x) &= \frac{4x - 3}{x + 2} \tag{4} \\
 f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{4(x+h)-3}{(x+h)+2} - \frac{4x-3}{x+2}}{h} \checkmark \\
 &= \lim_{h \rightarrow 0} \frac{(4x+4h-3)(x+2) - (4x-3)(x+h+2)}{(x+h+2)(x+2)h} \\
 &= \lim_{h \rightarrow 0} \frac{4x^2+5x+4xh+8h-6 - (4x^2+4xh+5x-3h-6)}{(x+h+2)(x+2)h} \checkmark \\
 &= \lim_{h \rightarrow 0} \frac{11h}{(x+h+2)(x+2)h} \checkmark = \frac{11}{(x+2)^2} \checkmark
 \end{aligned}$$

$$2.2 \quad 2.2.1 \quad y = \ln\left(\frac{2x+1}{2x-1}\right) + 2 \tan^{-1}(2x)$$

$$\frac{dy}{dx} = \frac{2x-1}{2x+1} \checkmark \times \frac{2(2x-1) - 2(2x+1) \checkmark}{(2x-1)^2 \checkmark} + 2 \times \frac{1}{1+(2x)^2} \checkmark \times 2 \checkmark \quad (5)$$

$$2.2.2 \quad y = \{\sin[\tan(\cos x^3)]\}^5$$

$$\frac{dy}{dx} = 5\{\sin[\tan(\cos x^3)]\}^4 \checkmark$$

$$\times \cos[\tan(\cos x^3)] \checkmark$$

$$\times \sec^2(\cos x^3) \checkmark \times (-\sin x^3) \checkmark \times 3x^2 \checkmark \quad (5)$$

$$2.3 \quad y = (2x+1)^{\sin 4\pi x}$$

$$\ln y = \sin 4\pi x \ln(2x+1)$$

$$\frac{1}{y} \frac{dy}{dx} \checkmark = 4\pi \cos 4\pi x \times \ln(2x+1) \checkmark + \sin 4\pi x \times \frac{1}{2x+1} \times 2 \checkmark$$

$$\frac{dy}{dx} = y \left[4\pi \cos 4\pi x \ln(2x+1) + \frac{2 \sin 4\pi x}{2x+1} \right] \checkmark$$

$$= (2x+1)^{\sin 4\pi x} \left[4\pi \cos 4\pi x \ln(2x+1) + \frac{2 \sin 4\pi x}{2x+1} \right] \quad (4)$$

$$2.4 \quad \operatorname{cosec}^{-1} x + \cot^{-1}(x+y) = \pi$$

$$-\frac{1}{x\sqrt{x^2-1}} - \frac{1}{1+(x+y)^2} \left(1 + \frac{dy}{dx}\right) = 0 \checkmark$$

$$-\frac{1}{1+(x+y)^2} \left(1 + \frac{dy}{dx}\right) = \frac{1}{x\sqrt{x^2-1}} \checkmark$$

$$1 + \frac{dy}{dx} = -\frac{1+(x+y)^2}{x\sqrt{x^2-1}} \checkmark$$

$$\frac{dy}{dx} = -\frac{1+(x+y)^2}{x\sqrt{x^2-1}} - 1 \checkmark \quad (4)$$

[22]

VRAAG 3

3.1 3.1.1 $f(x) = x^3 - 2x + 3$

$$f'(x) = 3x^2 - 2 = 0 \checkmark$$

$$x = \pm \sqrt{\frac{2}{3}}$$

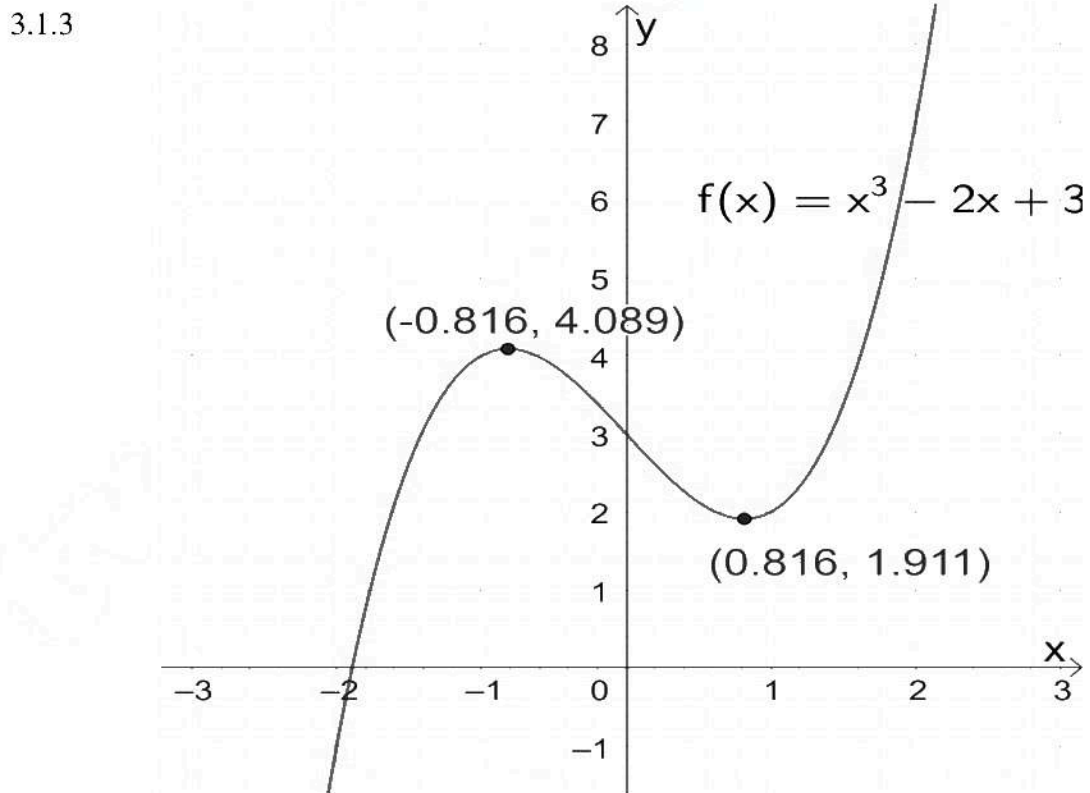
$$y = 1,911 \quad y = 4,089$$

Die koördinate van die draaipunte is $\left(-\sqrt{\frac{2}{3}}; 4,089\right) \checkmark$ en $\left(\sqrt{\frac{2}{3}}; 1,911\right) \checkmark$ (3)

3.1.2

x	-2	-1	0	1	2
y	-1	4	3	2	7

$\checkmark = 1$ punt vir enige 2 korrekte antwoorde (2)



$\checkmark = 1$ punt vir die vorm

$\checkmark = 1$ punt vir die aanduiding van die draaipunt op die grafiek. (2)

3.1.4 Laat $x_0 = -1,75$

$$f(-1,75) = 1,141$$

$$f'(-1,75) = 7,188✓$$

$$x_1 = -1,75 + \frac{1,141}{7,188}✓$$

$$= -1,909✓$$

$$f(-1,909) = -0,136$$

$$f'(-1,909) = 8,929$$

$$x_2 = -1,909 - \frac{0,136}{8,929}$$

$$= -1,893✓$$

(4)

3.2 Laat x = die breedte van die kamp.

y = die sykant van elk van die kleiner reghoekige kampe

$$P = 4x + 6y$$

$$120 = 4x + 6y$$

$$y = \frac{120 - 4x}{6}✓$$

$$A = x \times 3y$$

$$= x \times 3 \left(\frac{120 - 4x}{6} \right)$$

$$= 60x - 2x^2✓$$

$$\frac{dA}{dx} = 60 - 4x = 0✓$$

$$x = 15 \quad y = 10$$

Vir maksimum oppervlakte is breedte = 15✓ en lengte = 30✓

$$\text{Oppervlakte} = 450 \text{ m}^2✓$$

(6)

3.3 $C = 2\pi r$

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt} \checkmark$$

$$\frac{dA}{dt} = 2\pi(-4\text{cm}) \checkmark$$

$$= -8\pi \text{ cm/s} = -25,133 \text{ cm/s} \checkmark$$

(3)
[20]

VRAAG 4

4.1 4.1.1 $\int \frac{\cos x}{2 \sin x - 1} dx$

laat $u = 2 \sin x - 1$

$$du = 2 \cos x dx \checkmark$$

$$= \frac{1}{2} \int \frac{1}{u} du \checkmark$$

$$= \frac{1}{2} \ln u + C$$

$$= \frac{1}{2} \ln(2 \sin x - 1) + C \checkmark$$

Alternatief

laat $f(x) = 2 \sin x - 1$

$$f'(x) = 2 \cos x \checkmark$$

$$\begin{aligned} \int \frac{\cos x}{2 \sin x - 1} dx &= \frac{1}{2} \int \frac{2 \cos x}{2 \sin x - 1} dx \checkmark \\ &= \frac{1}{2} \ln(2 \sin x - 1) + C \checkmark \end{aligned}$$

(3)

4.1.2 $\int \sin\left(\frac{7}{3}x\right) \sin\left(\frac{1}{4}x\right) dx$

$$= \frac{1}{2} \int \left[\cos\left(\frac{25}{12}x\right) - \cos\left(\frac{31}{12}x\right) \right] dx \checkmark$$

$$= \frac{6}{25} \sin\left(\frac{25}{12}x\right) \checkmark - \frac{6}{31} \sin\left(\frac{31}{12}x\right) \checkmark + C$$

(3)

4.1.3

$$y = \frac{e^x}{\sqrt{3 - e^{2x}}}$$

$$= \int \frac{e^x}{\sqrt{3 - e^{2x}}} dx$$

$$\text{laat } u = e^x \therefore du = e^x dx \therefore dx = \frac{du}{e^x} \text{ en } e^{2x} = u^2 \checkmark$$

$$= \int \frac{1}{\sqrt{(\sqrt{3})^2 - u^2}} du \checkmark$$

$$= \sin^{-1} \frac{u}{\sqrt{3}} + c \checkmark$$

$$= \sin^{-1} \frac{e^x}{\sqrt{3}} + c \checkmark$$

(4)

4.1.4

$$\int \frac{2x^3 + 8x^2 - 6x + 10}{x - 2} dx$$

$$= \int \left(2x^2 + 12x + 18 + \frac{46}{x - 2} \right) dx \checkmark$$

$$= \frac{2}{3}x^3 \checkmark + 6x^2 \checkmark + 18x \checkmark + 46 \ln(x - 2) \checkmark + C$$

(5)

4.1.5

$$\int \log_2 x \, dx$$

laat $u = \log_2 x$	$dv = dx$	OF $f(x) = \log_2 x$	$g'(x) = 1$
$du = \frac{1}{x \ln 2} dx$	$v = x \checkmark$	$f'(x) = \frac{1}{x \ln 2}$	$g(x) = x$

$$\int \log_2 x \, dx = x \log_2 x - \int \frac{1}{x \ln 2} \times x dx \checkmark$$

$$= x \log_2 x \checkmark - \frac{1}{\ln 2} x \checkmark + C$$

(4)

4.2

$$v = \int \frac{dv}{1 - (kv)^2}$$

$$\frac{1}{(1 + kv)(1 - kv)} = \frac{A}{1 + kv} + \frac{B}{1 - kv}$$

$$1 = A(1 - kv) + B(1 + kv) \checkmark$$

$$\text{laat } v = \frac{1}{k}; \quad A = \frac{1}{2} \checkmark$$

or

$$v = -\frac{1}{k}; \quad B = \frac{1}{2} \checkmark$$

$$= \frac{1}{2} \int \left(\frac{1}{1 + kv} + \frac{1}{1 - kv} \right) dv$$

$$= \frac{1}{2k} [\ln(1 + kv) \checkmark - \ln(1 - kv) \checkmark] + C$$

$$= \frac{1}{2k} \ln \left(\frac{1 + kv}{1 - kv} \right) + C$$

(5)
[24]

VRAAG 5

5.1

$$\int_1^{\infty} \frac{1}{x^3} dx$$

$$= \lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^3} dx \checkmark$$

$$= \lim_{b \rightarrow \infty} \left[-\frac{1}{2x^2} \right]_1^b \checkmark$$

$$= \lim_{b \rightarrow \infty} \left[-\frac{1}{2b^2} + \frac{1}{2} \right] \checkmark$$

$$= \frac{1}{2} \checkmark$$

**May accept direct
working without $\lim_{b \rightarrow \infty}$**

$$\int_1^{\infty} \frac{1}{x^3} dx$$

$$= \left[-\frac{1}{2x^2} \right]_1^{\infty} \checkmark$$

$$= \left[-\frac{1}{\infty} + \frac{1}{2} \right] \checkmark \checkmark$$

$$= \frac{1}{2} \checkmark$$

(4)

5.2 5.2.1 $(x - 2)^2 = -x^2 + 2x$

$$2x^2 - 6x + 4 = 0$$

$$x^2 - 3x + 2 = 0$$

$$(x - 1)(x - 2) = 0$$

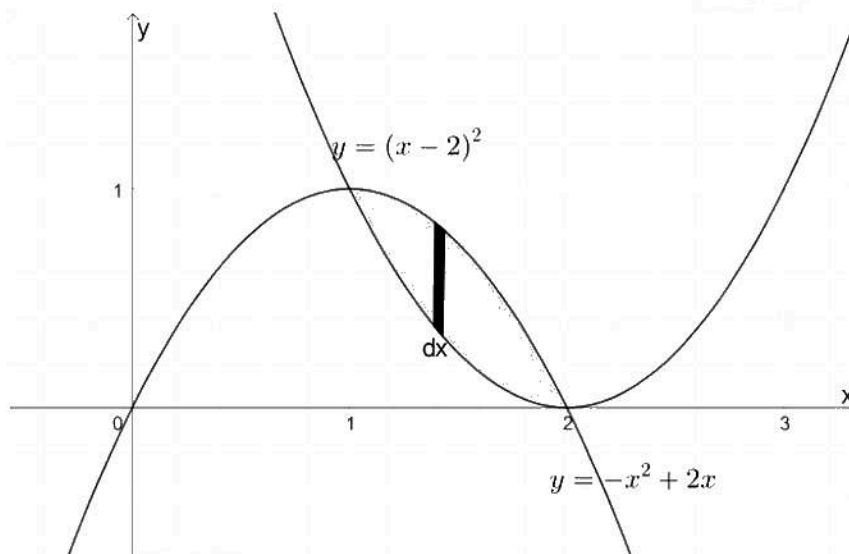
$$x = 1 \quad \text{of} \quad x = 2$$

$$y = 1 \quad \quad y = 0 \quad \checkmark$$

Die koördinate van die kruispunte vir die begrensde oppervlakte is (1; 1)✓
en (2; 0)✓.

(3)

5.2.2



✓ = 1 punt vir aanduiding van die korrekte begrensde oppervlakte

✓ = 1 punt vir aanduiding van die vertikale of horisontale strook

(2)

5.2.3

$$A = \int_a^b (y_1 - y_2) dx$$

$$= \int_1^2 [(-x^2 + 2x) - (x - 2)^2] dx \quad \checkmark$$

$$= \int_1^2 (-2x^2 + 6x - 4) dx \quad \checkmark$$

$$= \left[-\frac{2}{3}x^3 + 3x^2 - 4x \right]_1^2 \quad \checkmark$$

$$= \frac{1}{3} \text{ eenhede}^2 \quad \text{Of} \quad = 0,333 \text{ eenhede}^2 \quad \checkmark$$

$$\begin{aligned}
 5.2.4 \quad V &= \pi \int_a^b (y_1^2 - y_2^2) dx \\
 &= \pi \int_1^2 [(-x^2 + 2x)^2 - (x^2 - 4x + 4)^2] dx \checkmark \\
 &= \pi \int_1^2 (4x^3 - 20x^2 + 32x - 16) dx \checkmark \checkmark \\
 &= \pi \left[x^4 - \frac{20}{3}x^3 + 16x^2 - 16x \right]_1^2 \checkmark \\
 &= \frac{\pi}{3} \text{ eenhede}^3 \quad OF = 1,047 \text{ eenhede}^3 \checkmark
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 5.3 \quad d)_z &= r^2 dA \\
 &= x^2 \cdot 2\pi x dx \checkmark \\
 I_z &= 2\pi \int_0^1 x^3 dx \checkmark \\
 &= 2\pi \left[\frac{x^4}{4} \right]_0^1 \checkmark \\
 &= \frac{\pi}{2} \text{ cm}^4 \quad OF = 1,571 \text{ cm}^4 \checkmark
 \end{aligned} \tag{4}$$

[22]

VRAAG 6

$$\begin{aligned}
 6.1 \quad \frac{dy}{dx} &= \frac{3x(y-5)}{x^2-1} \\
 \frac{dy}{y-5} &= \frac{3x}{x^2-1} dx \checkmark \\
 \ln(y-5) \checkmark &= \frac{3}{2} \ln(x^2-1) \checkmark + C
 \end{aligned} \tag{3}$$

6.2 $\frac{d^2y}{dx^2} = \sin^2 3x$

$$\frac{d^2y}{dx^2} = \frac{1}{2} - \frac{1}{2} \cos 6x \quad \checkmark$$

$$\frac{dy}{dx} = \frac{1}{2}x - \frac{1}{12} \sin 6x + A \quad \checkmark$$

$$y = \frac{1}{4}x^2 + \frac{1}{72} \cos 6x + Ax + B \quad \checkmark$$

(3)

[6]

TOTAAL: 100