



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

2 April 2024

This marking guideline consists of 11 pages.

QUESTION 1

1.1

$$\begin{aligned} & \lim_{x \rightarrow 0} \left(\frac{e^x}{e^x - 1} - \frac{1}{x} \right) \\ &= \lim_{x \rightarrow 0} \frac{xe^x - (e^x - 1)}{x(e^x - 1)} \quad \checkmark \quad \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \frac{xe^x - e^x + e^x}{(e^x - 1) + xe^x} \quad \checkmark \\ &= \lim_{x \rightarrow 0} \frac{xe^x}{e^x - 1 + xe^x} \quad \frac{0}{0} \\ &= \lim_{x \rightarrow 0} \frac{xe^x + e^x}{e^x + xe^x + e^x} \quad \checkmark \\ &= \frac{1}{2} \quad \checkmark \end{aligned} \quad (4)$$

1.2

$$\begin{aligned} f(x) &= \frac{x^2 - 5x + 4}{e^{x^2-9} - 1} \\ e^{x^2-9} - 1 &= 0 \\ e^{x^2-9} &= e^0 \\ x^2 - 9 &= 0 \\ x &= \pm 3 \quad \checkmark \checkmark \end{aligned} \quad \begin{array}{l} (2) \\ [6] \end{array}$$

QUESTION 2

2.1

$$f(x) = \frac{5}{7x+2}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{5}{7(x+h)+2} - \frac{5}{7x+2}}{h} \checkmark$$

$$= \lim_{h \rightarrow 0} \frac{\frac{5(7x+2) - 5(7x+7h+2)}{(7x+7h+2)(7x+2)}}{h} \checkmark$$

$$= \lim_{h \rightarrow 0} \frac{\frac{35x+10-35x-35h-10}{(7x+7h+2)(7x+2)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-35h}{(7x+7h+2)(7x+2)} \times \frac{1}{h} \checkmark$$

$$= \lim_{h \rightarrow 0} \frac{-35}{(7x+7h+2)(7x+2)}$$

$$= -\frac{35}{(7x+2)^2} \checkmark$$

(4)

2.2

2.2.1 $y = \frac{1}{5} \sec^{-4}(4+x^3)$

$$\frac{dy}{dx} = -\frac{4}{5} \sec^{-5}(4+x^3) \checkmark \times \sec(4+x^3) \tan(4+x^3) \checkmark \times 3x^2 \checkmark \quad (3)$$

2.2.1

$$y = \frac{\sqrt{2x^3+1}}{\sqrt{2x^3-1}}$$

$$\frac{dy}{dx} = \frac{\frac{1}{2\sqrt{2x^3+1}} \checkmark \times 6x^2 \checkmark \times \sqrt{2x^3-1} - \frac{1}{2\sqrt{2x^3-1}} \checkmark \times 6x^2 \checkmark \times \sqrt{2x^3+1}}{(\sqrt{2x^3-1})^2} \quad (4)$$

2.3

$$y = \frac{(3x+5)^3}{\sqrt{x^4-1}}$$

$$\ln y = 3 \ln(3x+5) - \frac{1}{2} \ln(x^4-1)$$

$$\frac{1}{y} \frac{dy}{dx} \checkmark = 3 \times \frac{1}{3x+5} \times 3 \checkmark - \frac{1}{2} \times \frac{1}{x^4-1} \times 4x^3 \checkmark$$

$$\frac{dy}{dx} = y \left(\frac{9}{3x+5} - \frac{2x^3}{x^4-1} \right) \checkmark$$

$$\frac{dy}{dx} = \frac{(3x+5)^3}{\sqrt{x^4-1}} \left(\frac{9}{3x+5} - \frac{2x^3}{x^4-1} \right) \quad (4)$$

2.4

2.4.1 $(y^2 + x^2)^3 = 8x^2y^2$

$$3(y^2 + x^2)^2 \times \left(2x + 2y \frac{dy}{dx} \right) \checkmark = 16xy^2 + 16x^2 \frac{dy}{dx} \checkmark$$

$$6x(y^2 + x^2)^2 + 6y(y^2 + x^2)^2 \frac{dy}{dx} = 16xy^2 + 16x^2y \frac{dy}{dx}$$

$$\left[6y(y^2 + x^2)^2 - 16x^2y \right] \frac{dy}{dx} = 16xy^2 - 6x(y^2 + x^2)^2 \checkmark$$

$$\frac{dy}{dx} = \frac{16xy^2 - 6x(y^2 + x^2)^2}{6y(y^2 + x^2)^2 - 16x^2y} \checkmark \quad (4)$$

2.4.2

$$m = \frac{dy}{dx} = \frac{16(-1)(1)^2 - 6(-1)((1)^2 + (-1)^2)^2}{6(1)((1)^2 + (-1)^2)^2 - 16(-1)^2(1)} = 1 \checkmark$$

$$y = x + c$$

$$-1 = 1 + c$$

$$c = -2 \checkmark$$

$$y = x - 2 \checkmark$$

(3)
[22]

QUESTION 3

3.1 3.1.1 $f(x) = x^3 - 2x + 3$
 $f'(x) = 3x^2 - 2 = 0$
 $x = \pm \sqrt{\frac{2}{3}}$
 $x = 0,817 \text{ or } x = -0,817$
 $y = 1,911 \quad \checkmark \quad y = 4,089$

Thus, the coordinates of the turning points are $(0,817 ; 1,911)\checkmark$ and $(-0,817;4,089).\checkmark$

(3)

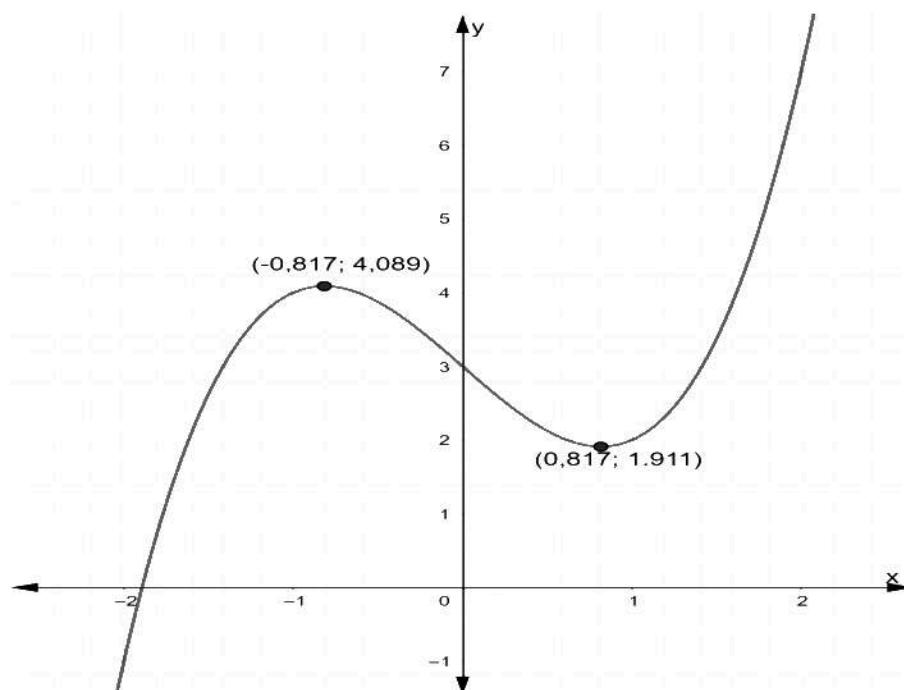
3.1.2

x	-2	-1	0	1	2
y	-1	4	3	2	7

$\checkmark\checkmark = 2$ Marks for the any 2 correct answers.

(2)

3.1.3



$\checkmark = 1$ Mark for the shape.

$\checkmark = 1$ Mark for the indication of the turning point on the graph.

(2)

3.1.4 Let $x_0 = -1,9$

$$f(-1,9) = -0,059$$

$$f'(-1,9) = 8,830\checkmark$$

$$x_1 = -1,9 - \frac{-0,059}{8,830}\checkmark$$

$$= -1,893\checkmark$$

$$3.2 \quad xy = 256 \checkmark$$

$$y = \frac{256}{x}$$

$$S = 4x + y \checkmark$$

$$= 4x + \frac{256}{x}$$

$$= 4x + 256x^{-1}$$

$$S' = 4 - 256x^{-2} = 0 \checkmark$$

$$4x^2 - 256 = 0$$

$$x^2 = 64 \quad ;$$

$$x = \pm 8$$

$$\therefore x = 8 \checkmark \quad y = 32 \checkmark$$

(5)

$$3.3 \quad V = s^3$$

$$s = \sqrt[3]{V}$$

$$A = 6s$$

$$A = 6\sqrt[3]{V} \checkmark$$

$$\frac{dA}{dt} = 6 \times \frac{2}{3} \times \frac{1}{\sqrt[3]{V}} \frac{dV}{dt} \checkmark$$

$$= \frac{4}{\sqrt[3]{V}} \frac{dV}{dt} \checkmark$$

$$\frac{dA}{dt} = \frac{4}{\sqrt[3]{15^3}} (-9) \checkmark$$

$$= -2,4 \text{ cm}^2/\text{s} \checkmark$$

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QUESTION 4

4.1

4.1.1

$$\int \frac{\tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x}}{\sqrt{x}} dx$$

$$\begin{aligned} \text{let } u &= \tan \sqrt{x} \\ 2du &= \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx \checkmark \end{aligned}$$

$$= 2 \int u^4 du \checkmark$$

$$= \frac{2}{5} u^5 + C$$

$$= \frac{2}{5} (\tan \sqrt{x})^5 \checkmark + C$$

(3)

4.1.2

$$\int \cos 2x \cos(-6x) dx$$

$$= \int \frac{1}{2} [\cos(2x - 6x) + \cos(2x + 6x)] dx$$

$$= \frac{1}{2} \int (\cos -4x + \cos 8x) dx \checkmark$$

$$= \frac{\sin 3x}{8} \checkmark + \frac{\sin 8x}{16} \checkmark + C$$

(3)

4.1.3

$$\int \sin^2\left(\frac{15}{4}x\right) dx$$

$$= \frac{1}{2} \int \left[1 - \cos\left(\frac{15}{2}x\right)\right] dx \checkmark$$

$$= \frac{1}{2} x \checkmark - \frac{1}{15} \sin\left(\frac{15}{2}x\right) \checkmark + C$$

(3)

4.1.4

$$\int \frac{x^3 + 9x^2 + 24x + 16}{x - 1} dx$$

$$= \int \left(x^2 + 10x + 34 + \frac{50}{x - 1}\right) dx \checkmark$$

$$= \frac{1}{3} x^3 \checkmark + 5x^2 \checkmark + 34x \checkmark + 50 \ln(x - 1) \checkmark + C$$

4.1.5 $\int \cos x \ln(\tan x) dx$

$$\begin{aligned} \text{let } u &= \ln(\tan x) & dv &= \cos x dx \\ du &= \frac{1}{\tan x} \times \sec^2 x dx & v &= \sin x \checkmark \\ &= \frac{1}{\sin x} \cdot \frac{1}{\cos x} dx \end{aligned}$$

$$\int \cos x \ln(\tan x) dx = \sin x \ln(\tan x) - \int \sin x \times \frac{1}{\sin x} \cdot \frac{1}{\cos x} dx \checkmark$$

$$= \sin x \ln(\tan x) - \int \sec x dx \checkmark$$

$$= \sin x \ln(\tan x) - \ln(\tan x + \sec x) \checkmark + C \quad (4)$$

4.2 $\int \frac{x^2 + 2x + 1}{(2x - 3)^3} dx$

$$\frac{x^2 + 2x + 1}{(2x - 3)^3} = \frac{A}{(2x - 3)^3} + \frac{B}{(2x - 3)^2} + \frac{C}{2x - 3}$$

$$x^2 + 2x + 1 = A + B(2x - 3) + C(2x - 3)^2 \checkmark$$

$$x^2 + 2x + 1 = 4Cx^2 + (2B - 12C)x + (A - 3B + 9C)$$

$$\begin{aligned} 4C &= 1; & C &= \frac{1}{4} \checkmark \\ 2B - 12C &= 2; & B &= \frac{5}{2} \checkmark \\ A - 3B + 9C &= 1, & A &= \frac{25}{4} \checkmark \end{aligned}$$

$$\int \left(\frac{\frac{25}{4}}{(2x - 3)^3} + \frac{\frac{5}{2}}{(2x - 3)^2} + \frac{\frac{1}{4}}{2x - 3} \right) dx$$

$$= -\frac{25}{16} \frac{1}{(2x - 3)^2} \checkmark - \frac{5}{4} \frac{1}{(2x - 3)} \checkmark + \frac{1}{8} \ln(2x - 3) \checkmark + C$$

(7)
[25]

QUESTION 5

5.1

$$\int_1^5 x^4 \ln x \, dx$$

$\text{let } u = \ln x \quad dv = x^4 dx$ $du = \frac{1}{x} dx \quad v = \frac{1}{5} x^5 \quad \checkmark$

$$\int_1^5 x^4 \ln x \, dx = \frac{1}{5} x^5 \ln x - \int_1^5 \frac{1}{5} x^5 \cdot \frac{1}{x} \, dx \quad \checkmark$$

$$= \frac{1}{5} x^5 \ln x - \frac{1}{5} \int_1^5 x^4 \, dx \quad \checkmark$$

$$= \left[\frac{1}{5} x^5 \ln x - \frac{1}{25} x^5 \right]_1^5 \quad \checkmark$$

$$= \left[\frac{1}{5} (5)^5 \ln 5 - \frac{1}{25} (5)^5 \right] - \left[\frac{1}{5} (1)^5 \ln 1 - \frac{1}{25} (1)^5 \right] = 880,939 \quad \checkmark \quad (5)$$

5.2

$$\mathcal{L}\left(\frac{t}{7}\right) = \frac{1}{7s^2} \quad (1)$$

5.3

5.3.1

$$A = \int_a^b (y_1 - y_2) \, dx$$

$$= \int_0^6 \left[\left(9 - \frac{x^2}{4} \right) - \left(9 - \frac{3}{2}x \right) \right] \, dx$$

$$= \int_0^6 \left(-\frac{x^2}{4} + \frac{3}{2}x \right) \, dx \quad \checkmark$$

$$= \left[-\frac{1}{12}x^3 + \frac{3}{4}x^2 \right]_0^6 \quad \checkmark$$

$$= 9 \text{ units}^2 \quad \checkmark \quad (3)$$

5.3.2

$$\begin{aligned}
 V &= \pi \int_a^b (y_1^2 - y_2^2) dx \\
 &= \pi \int_0^6 \left[\left(9 - \frac{x^2}{4}\right)^2 - \left(9 - \frac{3}{2}x\right)^2 \right] dx \\
 &= \pi \int_0^6 \left(81 - \frac{9}{2}x^2 + \frac{1}{16}x^4 - 81 + \frac{9}{4}x^2 + 27x\right) dx \checkmark \checkmark \\
 &= \pi \int_0^6 \left(\frac{1}{16}x^4 - \frac{27}{4}x^2 + 27x\right) dx \checkmark \\
 &= \pi \left[\frac{1}{80}x^5 - \frac{9}{4}x^3 + \frac{27}{2}x^2 \right]_0^6 \checkmark \\
 &= \frac{485\pi}{2} \text{ units}^3 \text{ or } 305,363 \text{ units}^3 \checkmark
 \end{aligned}
 \tag{5}$$

5.4

$$I_y = r^2 dA$$

$$dA = 8dx \checkmark$$

$$I_y = x^2 \cdot 8dx \checkmark$$

$$= 8 \int_0^{12} x^2 dx \checkmark$$

$$= 8 \left[\frac{x^3}{3} \right]_0^{12} \checkmark$$

$$= 4608 \text{ units}^4 \checkmark$$

(5)
[19]

QUESTION 6

6.1

$$x \frac{dy}{dx} = x + 1 + xy + y$$

$$x \frac{dy}{dx} = (x + 1)(y + 1)$$

$$\frac{dy}{y + 1} = \frac{x + 1}{x} dx \checkmark$$

$$\frac{dy}{y + 1} = \left(1 + \frac{1}{x}\right) dx$$

$$\ln(y + 1) \checkmark = x + \ln x + C \checkmark$$

$$6.2 \quad \frac{1}{(2-5x)^2} \frac{d^2y}{dx^2} = -3, \quad \text{if } \frac{dy}{dx} \Big|_{x=1} = 0 \text{ and } y(1) = -\frac{1}{4}$$

$$\frac{d^2y}{dx^2} = -75x^2 + 60x - 12$$

$$\frac{dy}{dx} = -\frac{75}{3}x^3 + 30x^2 - 12x + A \checkmark$$

$$0 = -\frac{75}{3}(1)^3 + 30(1)^2 - 12(1) + A$$
$$A = 7 \checkmark$$

$$\frac{dy}{dx} = -\frac{75}{3}x^3 + 30x^2 - 12x + 7$$

$$y = -\frac{75}{12}x^4 + 10x^3 - 6x^2 + 7x + B \checkmark$$

$$-\frac{1}{4} = -\frac{75}{12}(1)^4 + 10(1)^3 - 6(1)^2 + 7(1) + B$$
$$B = -5 \checkmark$$

$$y = -\frac{75}{12}x^4 + 10x^3 - 6x^2 + 7x - 5 \checkmark$$

(5)

[8]

TOTAL: 100