



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

2 April 2020

This marking guideline consists of 11 pages.

QUESTION 1

1.1 1.1.1 $\lim_{x \rightarrow \frac{1}{9}} \frac{9x-1}{3\sqrt{x}-1} \left(\frac{0}{0} \right)$

$\hookrightarrow \lim_{x \rightarrow \frac{1}{9}} \frac{9}{\frac{3}{2\sqrt{x}}}$

$\hookrightarrow \lim_{x \rightarrow \frac{1}{9}} 6\sqrt{x}$

$\hookrightarrow 2$ (3)

1.1.2 $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x) (\infty - \infty)$

$\hookrightarrow \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$

$\hookrightarrow \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{\sin x} \right) \left(\frac{0}{0} \right)$

$\hookrightarrow \lim_{x \rightarrow 0} \left(\frac{\sin x}{\cos x} \right)$

$\hookrightarrow 0$ (4)

1.2 $f(x) = \frac{x^2 + 4x - 12}{x^2 - 2x}$

$x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0 \vee x = 2$ (2)

[9]

QUESTION 2

2.1 $y = \operatorname{arc} \cot x$ (2)

$\cot y = x$

$-\operatorname{cosec}^2 y \frac{dy}{dx} = 1$

$\hookrightarrow -\frac{1}{1 + \cot^2 y}$

$$6 - \frac{1}{1+x^2}$$

$$2.2 \quad 2.2.1 \quad y = \ln(\cos^5(3x^4))$$

$$\frac{dy}{dx} = \frac{1}{\cos^5(3x^4)} \times 5 \cos^4(3x^4) \times (-\sin 3x^4) \times 12x^3 \quad (4)$$

$$2.2.2 \quad y = \left(\frac{8x - x^6}{x^3} \right)^{-\frac{4}{5}}$$

$$\frac{dy}{dx} = \frac{-4}{5} \left(\frac{8x - x^6}{x^3} \right)^{-\frac{9}{5}} \times \frac{x^3(8 - 6x^5) - 3x^2(8x - x^6)}{(x^3)^2} \quad (4)$$

$$2.2.3 \quad y = \sqrt{1 + \sqrt{1 + x^2}}$$

$$\frac{dy}{dx} = \frac{1}{2} (1 + \sqrt{1 + x^2})^{-\frac{1}{2}} \times \frac{1}{2} (1 + x^2)^{-\frac{1}{2}} \times 2x \quad (3)$$

$$2.3 \quad y = \left(1 + \frac{1}{x} \right)^x$$

$$\ln y = x \ln \left(1 + \frac{1}{x} \right)$$

$$\frac{1}{y} \frac{dy}{dx} = \ln \left(1 + \frac{1}{x} \right) + x \times \frac{1}{\left(1 + \frac{1}{x} \right)^2} \times -x^{-2}$$

$$\frac{dy}{dx} = y \left[\ln \left(1 + \frac{1}{x} \right) - \frac{x}{x^2 \left(1 + \frac{1}{x} \right)} \right]$$

$$\frac{dy}{dx} = \left(1 + \frac{1}{x} \right) \left[\ln \left(1 + \frac{1}{x} \right) - \frac{1}{x \left(1 + \frac{1}{x} \right)} \right] \quad (4)$$

$$2.4 \quad 2.4.1 \quad 25x^2 + 9y^2 - 70x - 30y + 49 = 0 \quad (3)$$

$$50x + 18y \frac{dy}{dx} - 70 - 30 \frac{dy}{dx} + 0 = 0$$

$$18y \frac{dy}{dx} - 30 \frac{dy}{dx} = 70 - 50x$$

$$\frac{dy}{dx} (18y - 30) = 70 - 50x$$

$$\frac{dy}{dx} = \frac{30-25x}{9y-15}$$

$$2.4.2 \quad \frac{dy}{dx} = \frac{-5}{4}$$

$$y = mx + c$$

$$y = \frac{-5}{4}x + c$$

$$3 = \frac{-5}{4}(2) + c$$

$$c = \frac{11}{2}$$

$$y = \frac{-5}{4}x + \frac{11}{2}$$

(3)
[23]

QUESTION 3

3.1 3.1.1

$$f(x) = x^3 - 3x + 1$$

$$f'(x) = 3x^2 - 3 = 0$$

$$3(x^2 - 1) = 0$$

$$x = -1 \vee x = 1$$

$$y = 3 \vee y = -1$$

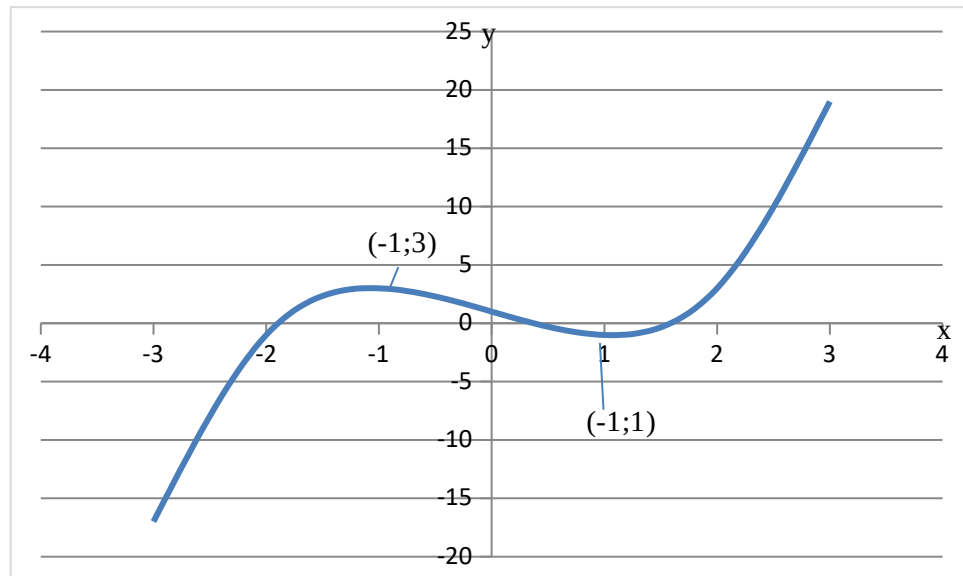
Thus, the coordinates of the turning points are $(-1; 3)$ and $(1; -1)$ (2)

3.1.2

x	-2	-1	0	1	2
y	-1	3	1	-1	3

= 1 mark each for any 3 correct answers (3)

3.1.3



= 1 mark for shape

= 1 mark for indication of turning point on graph

(2)

3.1.4 Let $x_0 = 1.5$

$$f(1.5) = -0.125$$

$$f'(1.5) = 3.75$$

$$x_1 = 1.5 - \frac{(-0.125)}{3.75}$$

$$\hat{=} 1.533$$

$$\text{Let } x_1 = 1.533$$

$$f(1.53) = 0.005$$

$$f'(1.533) = 4.053$$

$$x_1 = 1.533 - \frac{0.005}{4.053}$$

$$\hat{=} 1.532$$

(4)

$$3.2 \quad V = \frac{1}{3} \pi r^2 h$$

$$\textcolor{red}{\cdot} \frac{1}{3} \pi \left(\frac{h}{2} \right)^2 h$$

$$\textcolor{red}{\cdot} \frac{\pi h^3}{12}$$

$$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt}$$

$$\pi = \frac{\pi}{4} (6)^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{9} \text{ m/s}$$

(4)
[15]**QUESTION 4**

4.1 4.1.1

$$\int \frac{4}{\sec x e^{\sin x}} dx$$

$$\text{let } u = \sin x$$

$$du = \cos x \, dx$$

$$\textcolor{red}{\cdot} \int \frac{1}{e^u} du$$

$$\textcolor{red}{\cdot} \int e^{-u} du$$

$$\textcolor{red}{\cdot} -e^{-u} + C$$

$$\textcolor{red}{\cdot} e^{-\sin x} + C$$

(3)

4.1.2

$$\int \frac{(\cos^{-1} 2x)^4}{\sqrt{1-4x^2}} dx$$

(3)

$$\text{let } u = \cos^{-1} 2x$$

$$\frac{-du}{2} = \frac{1}{\sqrt{1-4x^2}} dx$$

$$\textcolor{red}{\cdot} -\frac{1}{2} \int u^4 du$$

$$\int -\frac{1}{10}u^5 + C$$

$$\int -\frac{1}{10}(\cos^{-1} 2x)^5 + C$$

4.1.3 $\int \frac{x^3 + 2x^2 + 9x - 17}{x+4} dx$

$$\int \left(x^2 - 2x + 17 + \frac{51}{x+4} \right) dx$$

$$\int \frac{1}{3}x^3 - x^2 + 17x + 51 \ln(x+4) + C \quad (5)$$

4.1.4 $\int \cos 3\pi x \cdot \cos \pi x dx$

$$\int \frac{1}{2} \int \cos 4\pi x + \cos 2\pi x dx$$

$$\int \frac{1}{8\pi} \sin 4\pi x + \frac{1}{4\pi} \sin 2\pi x + C \quad (3)$$

4.1.5 $\int x \sqrt{1+2x} dx$

$let u = 1 + 2x$

$$\frac{du}{2} = dx$$

$$x = \frac{u-1}{2}$$

$$\int \frac{u-1}{2} \sqrt{u} du$$

$$\frac{1}{4} \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}} \right) du$$

$$\frac{1}{10} u^{\frac{5}{2}} - \frac{1}{6} u^{\frac{3}{2}} + C$$

$$\frac{1}{10} (1+2x)^{\frac{5}{2}} - \frac{1}{6} (1+2x)^{\frac{3}{2}} + C \quad (4)$$

4.2

$$\int \frac{3x}{(2x+1)(x+4)} dx$$

$$\frac{3x}{(2x+1)(x+4)} = \frac{A}{2x+1} + \frac{B}{x+4}$$

$$3x = A(x+4) + B(2x+1)$$

$$\text{let } x = -4, B = \frac{12}{7}$$

$$\text{let } x = \frac{-1}{2}, A = \frac{-3}{7}$$

$$-\frac{3}{7} \int \frac{1}{2x+1} dx + \frac{12}{7} \int \frac{1}{x+4} dx$$

$$-\frac{3}{14} \ln(2x+1) + \frac{12}{7} \ln(x+4) + C$$

(5)
[23]**QUESTION 5**

5.1

$$\int_0^{\infty} e^{-st} \cdot f(t) dt$$

(5)

$$\frac{1}{\pi} \int_0^{\infty} t e^{-st} dt$$

$$\text{let } u = t \quad dv = e^{-st} dt$$

$$du = dt \quad v = \frac{-1}{s} e^{-st}$$

$$\frac{1}{\pi} \left[\frac{-t}{s} e^{-st} \right]_0^{\infty} + \frac{1}{\pi s} \int_0^{\infty} t e^{-st} dt$$

$$-\frac{1}{s\pi} \left[t e^{-st} \right]_0^{\infty} - \frac{1}{s^2 \pi} \left[e^{-st} \right]_0^{\infty}$$

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$$i - \frac{1}{s\pi}(0-0) - \frac{1}{s^2\pi}(0-1)$$

$$i \frac{1}{s^2\pi}$$

5.2 5.2.1

$$2x^2 = x + 1$$

$$2x^2 - x - 1 = 0$$

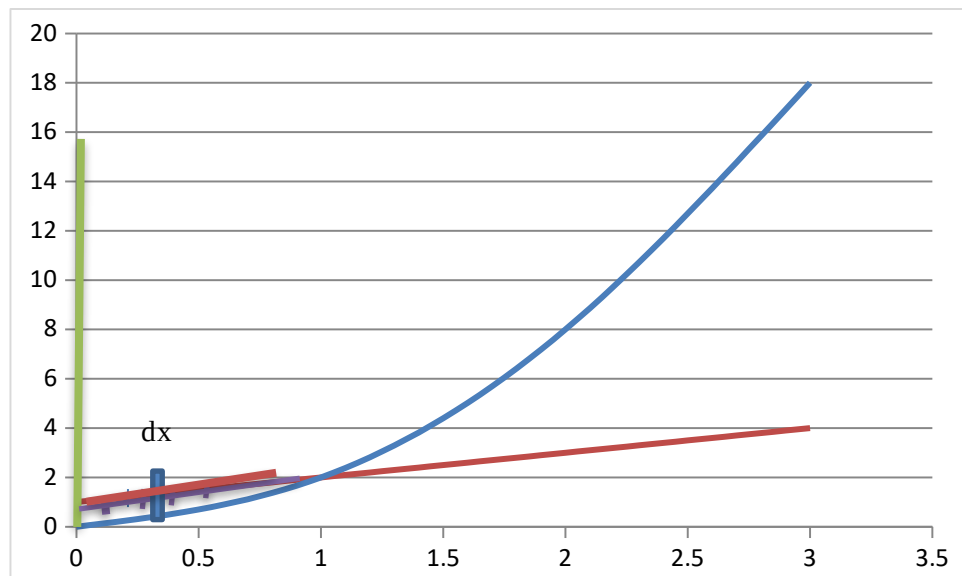
$$(2x+1)(x-1) = 0$$

$$x = -\frac{1}{2} \vee x = 1$$

$$y = \frac{1}{2} \vee y = 2$$

Thus, the coordinates of the points of intersection are $\left(-\frac{1}{2}; \frac{1}{2}\right)$ and $(1; 2)$ (2)

5.2.2



= 1 mark for indication of enclosed area

= 1 mark for indication of vertical or horizontal strip (2)

5.2.3

$$A = \int_a^b (y_1 - y_2) dx \quad (3)$$

$$i \int_0^1 (x+1-2x^2) dx$$

$$\checkmark \left[\frac{1}{2}x^2 + x - \frac{2}{3}x^3 \right]_0^1$$

$$\checkmark \frac{5}{6} \vee 0.833 \text{ units}^2$$

5.2.4

$$V = \pi \int_a^b (y_1^2 - y_2^2) dx$$

$$V = \pi \int_0^1 [(x+1)^2 - (2x^2)^2] dx$$

$$\checkmark \pi \int_0^1 (x^2 + 2x + 1 - 4x^4) dx$$

$$\checkmark \pi \left[\frac{1}{3}x^3 + x^2 + x - \frac{4}{5}x^5 \right]_0^1$$

$$\checkmark \frac{23\pi}{15} \vee 4.817 \text{ units}^3$$

(4)

5.3

$$\checkmark y = r^2 dA$$

$$\checkmark x^2 \cdot 3 dx$$

$$\checkmark 3 \int_0^6 x^2 dx$$

$$\checkmark 3 \left[\frac{x^3}{3} \right]_0^6$$

$$\checkmark 216 \text{ units}^4$$

(4)
[20]

QUESTION 6

$$6.1 \quad \frac{dy}{dx} = \frac{x(e^{x^2} + 2)}{6y^2}$$

$$6y^2 dy = x(e^{x^2} + 2) dx$$

$$6y^2 dy = (xe^{x^2} + 2x) dx$$

$$2y^3 = \frac{1}{2}e^{x^2} + x^2 + C$$

$$2(1)^3 = \frac{1}{2}e^{(0)^2} + (0)^2 + c$$

$$c = \frac{3}{2}$$

$$2y^3 = \frac{1}{2}e^{x^2} + x^2 + \frac{3}{2}$$

(4)

$$6.2 \quad x^2 \cdot \frac{d^2y}{dx^2} = x^6 + x^3 - 2x^2$$

$$\frac{d^2y}{dx^2} = x^4 + x - 2$$

$$\frac{dy}{dx} = \frac{1}{5}x^5 + \frac{1}{2}x^2 - 2x + A$$

$$\frac{3}{5} = \frac{1}{5}(1)^5 + \frac{1}{2}(1)^2 - 2(1) + A$$

$$A = 2$$

$$\frac{dy}{dx} = \frac{1}{5}x^5 + \frac{1}{2}x^2 - 2x + 2$$

$$y = \frac{1}{30}x^6 + \frac{1}{6}x^3 - x^2 + 2x + B$$

$$\frac{1}{5} = \frac{1}{30}(1)^6 + \frac{1}{6}(1)^3 - (1)^2 + 2(1) + B$$

$$B = -1$$

$$y = \frac{1}{30}x^6 + \frac{1}{6}x^3 - x^2 + 2x - 1$$

(6)
[10]

