



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

2 August 2023

This marking guideline consists of 11 pages.

QUESTION 1

1.1

1.1.1

$$y = \lim_{x \rightarrow 0} \frac{e^{-x^2} - \ln(1+x) - 1}{\cos x - \sin x - 1} \cdot \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{2xe^{-x^2} - \frac{1}{1+x}}{-\sin x - \cos x}$$

1

(2)

1.1.2

$$y = \lim_{x \rightarrow 0} \frac{2^{-x} - 1}{3^{-x} - 1}$$

$$\lim_{x \rightarrow 0} \frac{-2^{-x} \ln 2}{-3^{-x} \ln 2}$$

0,631

(2)

1.2

$$y = \frac{x^2 - x + 4}{\ln(x)}$$

$$\ln x \neq 0$$

$$x \neq 1$$

$$x \in \mathbb{R}; x \neq 1 \vee (-\infty; 1) \cup (1; \infty) \vee \{x/x < 1 \cup x > 1\}$$

(2)

[6]

QUESTION 2

2.1 $f(x) = \frac{5}{3x+2}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{5}{3(x+h)+2} - \frac{5}{3x+2}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{5(3x+2) - 5[3(x+h)+2]}{[3(x+h)+2](3x+2)}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{15x+10 - 15x - 15h - 10}{[3(x+h)+2](3x+2)}}{h}$$

$$\lim_{h \rightarrow 0} \frac{-15h}{[3(x+h)+2](3x+2)} \times \frac{1}{h}$$

$$\lim_{h \rightarrow 0} \frac{-15}{[3(x+h)+2](3x+2)}$$

$$-\frac{15}{(3x+2)^2} \quad (4)$$

2.2 2.2.1

$$y = \sin^5[\tan(\cos x^3)]$$

$$\frac{dy}{dx} = 5 \sin^4[\tan(\cos x^3)] \times \cos[\tan(\cos x^3)] \times \sec^2(\cos x^3) \times (-\sin x^3) \times 3x^2 \quad (5)$$

2.2.2

$$y = \sqrt{x^3 + \sqrt{x^2 + \sqrt{3-x}}}$$

$$\frac{dy}{dx} = \frac{1}{2} [x^3 + \sqrt{x^2 + \sqrt{3-x}}]^{-\frac{1}{2}} \times \left[3x^2 + \frac{1}{2} (x^2 + \sqrt{3-x})^{-\frac{1}{2}} \right] \times \left[2x + \frac{1}{2} (3-x)^{-\frac{1}{2}} \right] \times -1 \quad (4)$$

2.3

$$y = \frac{(9-3x)^4}{x^2 \cos 2x}$$

$$\ln y = 4 \ln(9-3x) - 2 \ln x - \ln(\cos 2x)$$

$$\frac{1}{y} \frac{dy}{dx} = 4 \times \frac{1}{9-3x} \times (-3) - 2 \times \frac{1}{x} - \frac{1}{\cos 2x} \times (-\sin 2x)$$

$$\frac{dy}{dx} = y \left[\frac{-12}{9-3x} - \frac{2}{x} + \tan 2x \right]$$

$$\frac{(9-3x)^4}{x^2 \cos 2x} \left[\frac{-12}{9-3x} - \frac{2}{x} + \tan 2x \right] \quad (4)$$

2.4

$$e^{xy} = \log 4x - e^{5y}$$

$$e^{xy} \times \left[y + x \frac{dy}{dx} \right] = \frac{1}{x \ln 10} - 5e^{5y} \frac{dy}{dx}$$

$$x e^{xy} \frac{dy}{dx} + 5e^{5y} \frac{dy}{dx} = \frac{1}{x \ln 10} - y e^{xy}$$

$$\frac{dy}{dx} [x e^{xy} + 5e^{5y}] = \frac{1}{x \ln 10} - y e^{xy}$$

$$\frac{dy}{dx} = \frac{\frac{1}{x \ln 10} - y e^{xy}}{x e^{xy} + 5e^{5y}} \quad (5)$$

[22]

QUESTION 3

3.1

3.1.1

$$f(x) = x^3 + x^2 - x + 7$$

$$f'(x) = 3x^2 + 2x - 1 = 0$$

$$(x+1)(3x-1) = 0$$

$$x = -1 \vee x = \frac{1}{3}$$

$$y = 8 \vee y = \frac{184}{27}$$

Thus, the coordinates of the turning points are $(-1; 8)$ and $\left(\frac{1}{3}; \frac{184}{27}\right)$ (3)

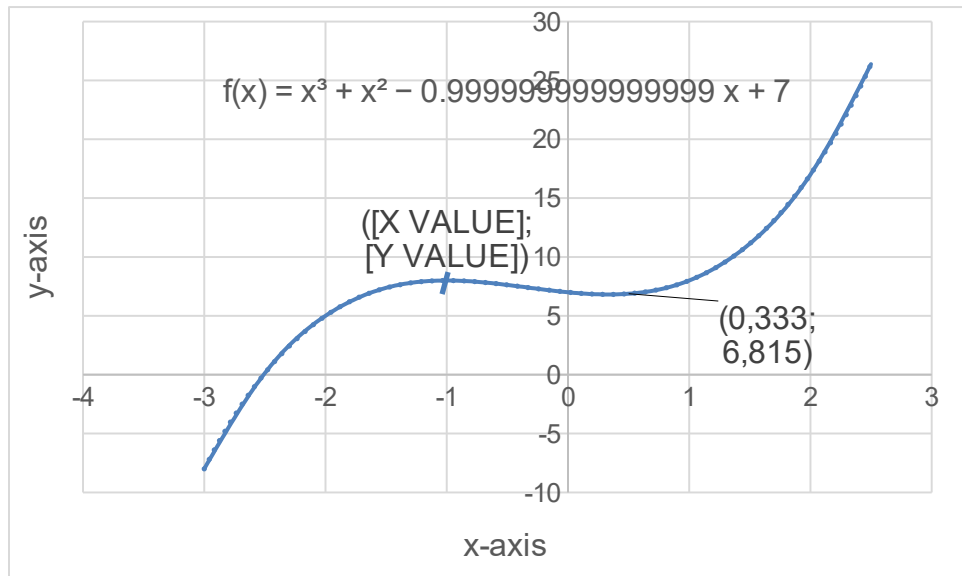
3.1.2

x	-3	-2	-1	0	1	2	3
y	-8	5	8	7	8	17	40

= 1 Mark for the every 3 correct answers

(2)

3.1.3



1 Mark for the shape

1 Mark for the indication of the turnings point on the graph

(2)

3.1.4 Let $x_0 = -2,7$

$$f(-2,7) = -2,693$$

$$f'(-2,7) = 15,47$$

$$x_1 = -2,7 - \frac{-2,693}{15,47}$$

$$x_1 = -2,526$$

(3)

$$3.2 \quad x + y = 18 \dots\dots\dots (1)$$

$$P = x^2 y \dots\dots\dots (2)$$

$$y = 18 - x \dots\dots\dots (3)$$

Substitute (3) into (2)

$$P = x^2(18 - x)$$

$$\therefore 18x^2 - x^3$$

$$P' = 36x - 3x^2 = 0$$

$$3x(12 - x) = 0$$

$$x = 0 \vee x = 12$$

$$y = 18 \vee y = 6$$

For, $x = 0 \wedge y = 18$, the product will minimum

Thus, numbers $x = 12 \wedge y = 6$, will maximise the product.

(6)

3.3

$$V = \frac{1}{3} \pi r^2 h$$

$$r = \frac{h}{2}$$

$$V = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{12} \pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4} \pi h^2 \frac{dh}{dt}$$

$$15 \text{ cm}^3/\text{s} = \frac{1}{4} \pi (3 \text{ cm})^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{15}{\frac{9\pi}{4}}$$

$$\therefore \frac{20}{3\pi} \text{ cm/s} \vee 2,122 \text{ cm/s}$$

(4)

[20]

QUESTION 4

4.1 4.1.1

$$\int \frac{\tan x}{\ln(\cos x)} dx$$

$$\text{let } u = \ln(\cos x)$$

$$du = \frac{1}{\cos x} \cdot -\sin x dx$$

$$\int -\frac{1}{u} du$$

$$-\ln u + C$$

$$-\ln(\ln(\cos x)) + C$$

(3)

4.1.2
$$\int \cos(ax) \cos(bx) dx$$

$$\int \frac{1}{2} [\cos(a+b)x - \cos(a-b)x] dx$$

$$-\frac{\sin(a+b)x}{2(a+b)} - \frac{\sin(a-b)x}{2(a-b)} + C$$

(3)

4.1.3
$$\int \cos^3 x \sin^2 x dx$$

$$\int \cos x \cos^2 x \sin^2 x dx$$

(5)

$$\int \cos x (1 - \sin^2 x) \sin^2 x dx$$

$$u = \sin x$$

$$du = \cos x dx$$

$$\int (1 - u^2) u^2 du$$

$$\int (u^2 - u^4) du$$

$$\frac{1}{3} u^3 - \frac{1}{5} u^5 + C$$

$$\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$$

$$4.1.4 \quad \int \frac{x^3 + 4x^2 + 6}{x+1} dx$$

$$\hookrightarrow \int \left(x^2 + 3x - 3 + \frac{7}{x+1} \right) dx$$

$$\hookrightarrow \frac{1}{3}x^3 - \frac{3}{2}x^2 - 3x + 7\ln(x+1) + C \quad (5)$$

$$4.1.5 \quad \int e^{6x} \sin e^{3x} dx$$

$$\text{let } u = e^{3x} \quad dv = e^{3x} \sin e^{3x} dx$$

$$du = 3e^{3x} dx \quad v = e^{3x} \cos e^{3x}$$

$$\int x^5 \ln 3x dx = e^{3x} \times -\frac{1}{3} \cos e^{3x} - \int -\frac{1}{3} e^{3x} \cos e^{3x} \times 3e^{3x} dx$$

$$\hookrightarrow -\frac{1}{3} e^{3x} \cos e^{3x} + \frac{1}{3} \sin e^{3x} + C \quad (4)$$

$$4.2 \quad \int \frac{2x+3}{(x-2)^2} dx$$

$$\frac{2x+3}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$$

$$2x+3 = A(x-2) + B$$

$$\text{let } x=2; B=7$$

$$\hookrightarrow$$

$$x=0; A=2$$

$$\int \left(\frac{2}{x-2} + \frac{7}{(x-2)^2} \right) dx$$

$$\hookrightarrow 2 \ln(x-2) - \frac{7}{x-2} + C \quad (5)$$

[25]

QUESTION 5

$$5.1 \quad \int_0^{\infty} x e^{-2x} dx$$

$$\textcolor{red}{\hookrightarrow} \lim_{b \rightarrow \infty} \int_0^b x e^{-2x} dx$$

$$\text{let } u = x \quad dv = e^{-2x} dx$$

$$du = dx \quad v = \frac{-e^{-2x}}{2}$$

$$\textcolor{red}{\hookrightarrow} \lim_{b \rightarrow \infty} \left\{ \left[-x \frac{e^{-2x}}{2} \right]_0^b - \int_0^b \frac{-e^{-2x}}{2} dx \right\}$$

$$\textcolor{red}{\hookrightarrow} \lim_{b \rightarrow \infty} \left\{ \left(-b \frac{e^{-2b}}{2} - 0 \right) - \left[\frac{e^{-2x}}{4} \right]_0^b \right\}$$

$$\textcolor{red}{\hookrightarrow} 0 - \lim_{b \rightarrow \infty} \left(\frac{-e^{-2b}}{4} - \frac{1}{4} \right)$$

$$\textcolor{red}{\hookrightarrow} \frac{1}{4}$$

(6)

$$5.2 \quad 5.2.1 \quad A = \int_a^b (y_1 - y_2) dx$$

$$\textcolor{red}{\hookrightarrow} \int_{0,879}^{5,121} \left[(6x - x^2) - (x^2 - 6x + 9) \right] dx$$

$$\textcolor{red}{\hookrightarrow} \int_{0,879}^{5,121} (6x - x^2 - x^2 + 6x - 9) dx$$

$$\textcolor{red}{\hookrightarrow} \int_{0,879}^{5,121} (-2x^2 + 12x - 9) dx$$

$$\textcolor{red}{\hookrightarrow} \left[\frac{-2}{3} x^3 + 6x^2 - 9x \right]_{0,879}^{5,121}$$

$$\textcolor{red}{\hookrightarrow} 25,456 \text{ units}^2$$

(3)

$$5.2.4 \quad V = \pi \int_a^b (y_1^2 - y_2^2) dx$$

$$\checkmark \pi \int_{0,879}^{5,121} [(6x - x^2)^2 - (x^2 - 6x + 9)^2] dx$$

$$\checkmark \pi \int_{0,879}^{5,121} (36x^2 - 12x^3 + x^4 - x^4 + 12x^3 - 54x^2 - 108x - 81) dx$$

$$\checkmark \pi \int_{0,879}^{5,121} (-18x^2 + 108x - 81) dx$$

$$\checkmark \pi [-6x^3 + 54x^2 - 81x]_{0,879}^{5,121}$$

$$\checkmark 719,747 \text{ units}^3 \quad (5)$$

$$5.3 \quad I_y = r^2 dA; dA = 9 dx$$

$$\checkmark x^2 \cdot 9 dx$$

$$\checkmark 9 \int_0^6 x^2 dx$$

$$\checkmark 9 \left[\frac{x^3}{3} \right]_0^6$$

$$\checkmark 648 \text{ units}^4 \quad (5)$$

[19]

QUESTION 6

6.1 $y \frac{dy}{dx} = 3\sqrt{x y^2 + 9x}, \text{ if } y(0) = 4$

$$y \frac{dy}{dx} = 3\sqrt{x} \sqrt{y^2 + 9}$$

$$\frac{y}{\sqrt{y^2 + 9}} dy = 3\sqrt{x} dx$$

$$\sqrt{y^2 + 9} = 2x^{\frac{3}{2}} + C$$

$$\sqrt{0^2 + 9} = 2(4)^{\frac{3}{2}} + C$$

$$C = -13$$

$$\therefore \sqrt{y^2 + 9} = 2x^{\frac{3}{2}} - 13$$

(3)

6.2 $\frac{d^2 y}{dx^2} = 2\left(x - \frac{3}{2}\right)^2 - \frac{1}{2}, \text{ if } \frac{dy}{dx} \Big|_{x=0} = 3 \wedge y(1) = 2$

$$\frac{d^2 y}{dx^2} = 2\left(x - \frac{3}{2}\right)^2 - \frac{1}{2}$$

$$\frac{d^2 y}{dx^2} = 2x^2 - 6x + 4$$

$$\frac{dy}{dx} = \frac{2}{3}x^3 - 3x^2 + 4x + A$$

$$3 = \frac{2}{3}(1)^3 - 3(1)^2 + 4(1) + A \therefore A = 3$$

$$\frac{dy}{dx} = \frac{2}{3}x^3 - 3x^2 + 4x + 3$$

$$y = \frac{1}{6}x^4 - x^3 + 2x^2 + 3x + B$$

$$2 = \frac{1}{6}(1)^4 - (1)^3 + 2(1)^2 - 3(1) + B \therefore B = \frac{-13}{6}$$

$$y = \frac{1}{6}x^4 - x^3 + 2x^2 + 3x - \frac{13}{6}$$

(5)
[8]**TOTAL 100**

