



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

20 November 2020

This marking guideline consists of 11 pages.

QUESTION 1

- 1.1 1.1.1 $\lim_{x \rightarrow 0} x \cot x (0, \infty)$
- $\textcolor{red}{\lim}_{x \rightarrow 0} \frac{x}{\tan x} \left(\frac{0}{0} \right)$
- $\textcolor{red}{\lim}_{x \rightarrow 0} \frac{1}{\sec^2 x}$
- $\textcolor{red}{\lim}_{x \rightarrow 0} \cos^2 x$
- $\textcolor{red}{1}$ (4)
- 1.2 1.2.1 $\log_4 y \int_0^1 e^t dt$
- $\textcolor{red}{\lim}_{x \rightarrow 0} \frac{-x}{2x}$
- $\textcolor{red}{\lim}_{x \rightarrow 0} \frac{1 - e^{-x}}{2x}$
- $\textcolor{red}{\lim}_{x \rightarrow 0} \frac{e^{-x}}{2}$
- $\textcolor{red}{\frac{1}{2}}$ (3)
- 1.2.2 $y = \sqrt{4}$
- $\textcolor{red}{2}$ (1)
- 1.3 $f(x) = \frac{x^3 - 6x + 4}{x^2 - 1}$
- $x^2 - 1 = 0$
- $x = \pm 1$ (2)
- [10]

QUESTION 2

2.1 $f(x) = \frac{1-x}{2-x}$

$$f'(x+h) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{1-(x+h)}{2-(x+h)} - \frac{1-x}{2-x}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{1-(x+h)}{2-(x+h)} \times \frac{2-x}{2-x} - \frac{1-x}{2-x} \times \frac{2-x-h}{2-x-h}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{2-2x-2h-x+x^2+xh-2+x+h+2x-x^2-xh}{(2-x)(2-x-h)}}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{-h}{(2-x)(2-x-h)}}{h}$$

$$\lim_{h \rightarrow 0} \frac{-1}{(2-x)(2-x-h)}$$

$$-\frac{1}{(2-x)^2}$$

(5)

2.2 $y = \arccos x$

$$\cos y = x$$

$$-\sin y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{-1}{\sin y}$$

$$-\frac{1}{\sqrt{1-\cos^2 y}}$$

$$-\frac{1}{\sqrt{1-x^2}}$$

(3)

2.3 2.3.1 $y = \sqrt{\sin(7x + \ln 5x)}$

$$\frac{dy}{dx} = \frac{1}{2} (\sin(7x + \ln 5x))^{\frac{-1}{2}} \times \cos(7x + \ln 5x) \times \left(7 + \frac{1}{5x}\right) \times 5 \quad (4)$$

2.3.2 $y = 4 \ln(\ln(\sec^2 x))$

$$\frac{dy}{dx} = 4 \times \frac{1}{\ln(\sec^2 x)} \times \frac{1}{\sec^2 x} \times 2 \sec x \times \sec x \tan x \quad (4)$$

2.4 $y = (3x^2 + 2)^{\frac{1}{x}}$

$$\ln y = \frac{1}{x} \ln(3x^2 + 2)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x} \times \frac{1}{3x^2 + 2} \times 6x - \frac{1}{x^2} \times \ln(3x^2 + 2)$$

$$\frac{dy}{dx} = y \left[\frac{6}{3x^2 + 2} - \frac{\ln(3x^2 + 2)}{x^2} \right]$$

$$\frac{dy}{dx} = (3x^2 + 2)^{\frac{1}{x}} \left[\frac{6}{3x^2 + 2} - \frac{\ln(3x^2 + 2)}{x^2} \right] \quad (4)$$

2.5 2.5.1 $\frac{x+y}{x-y} = x$

$$x+y = x(x-y)$$

$$1 + \frac{dy}{dx} = (x-y) + x \left(1 - \frac{dy}{dx}\right)$$

$$1 + \frac{dy}{dx} = x - y + x - x \frac{dy}{dx}$$

$$x \frac{dy}{dx} + \frac{dy}{dx} = x - y + x - 1$$

$$\frac{dy}{dx} (x+1) = x - y + x - 1$$

$$\frac{dy}{dx} = \frac{x - y + x - 1}{x+1} \quad (4)$$

2.5.2 $\frac{dy}{dx} = -2$

$$y = -2x + C$$

$$y = -2x + 1$$

(2)
[26]**QUESTION 3**

3.1 3.1.1 $f(x) = 2x^3 + x^2 - 3x + 3$

$$f'(x) = 6x^2 + 2x - 3$$

$$f''(x) = 12x + 2 = 0$$

$$12x = -2$$

$$x = \frac{-1}{6} \quad y = \frac{68}{27}$$

Thus, coordinate of point of inflection is $\left(\frac{-1}{6}; \frac{68}{27}\right)$

(2)

3.1.2

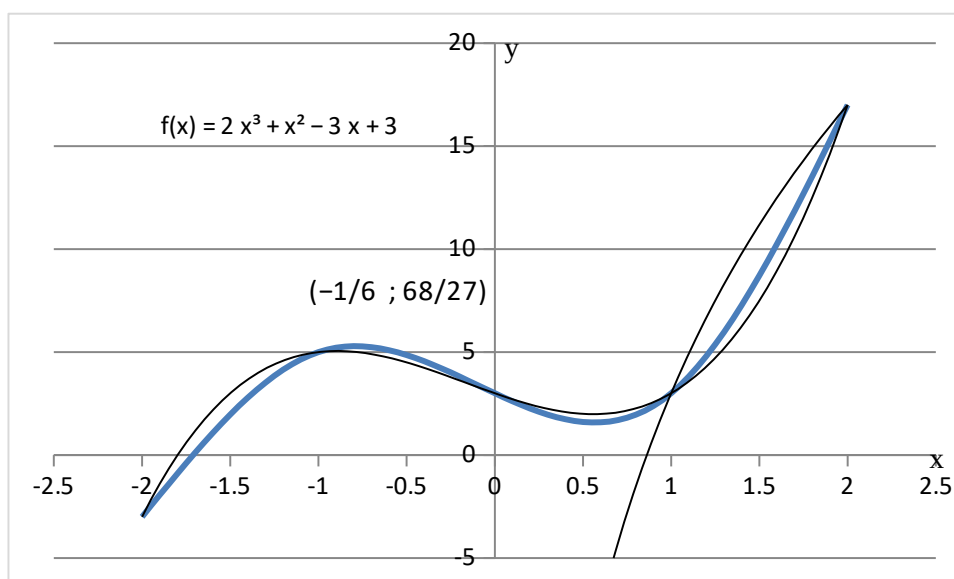
x	-2	-1	0	1	2
y	-3	5	3	3	17

(1 mark each for any 3 correct

answers)

(3)

3.1.3



1 mark for shape

1 mark for indication of turning point on graph

(2)

$$3.1.4 \quad \text{Let } x_0 = -1,6$$

$$f(-1,6) = 2,168$$

$$f'(-1,6) = 9,16$$

$$x_1 = -1,6 - \frac{2,168}{9,16}$$

$$\approx -1,837$$

$$\text{Let } x_1 = -1,837$$

$$f(-1,837) = -0,508$$

$$f'(-1,837) = 13,567$$

$$x_1 = -1,837 - \frac{(-0,508)}{13,567}$$

$$\approx -1,799$$

(4)

$$3.2 \quad f(t) = 4t^2 \ln t$$

$$f'(t) = 8t \ln t + 4t$$

$$f'(\text{left } t \text{ right}) = 8 \ln \{t\} + 12$$

$$f'(\text{left } 3,5 \text{ right}) = 8 \ln \{\text{left } 3,5 \text{ right}\} + 12$$

$$\approx 22,022 \text{ m/s}^2$$

(4)
[15]

QUESTION 4

4.1 4.1.1

$$\int (e^x + e^{-x})^{\frac{1}{4}} (e^x - e^{-x}) dx$$

$$\text{let } u = e^x + e^{-x}$$

$$du = (e^x - e^{-x}) dx$$

$$\int u^{\frac{1}{4}} du$$

$$\frac{4}{5} u^{\frac{5}{4}} + C$$

$$\frac{4}{5} (e^x + e^{-x})^{\frac{5}{4}} + C \quad (3)$$

4.1.2

$$\int \frac{1}{x(1+2\ln x)} dx$$

$$\text{let } u = 1 + 2\ln x$$

$$\frac{du}{2} = \frac{1}{x} dx$$

$$\frac{1}{2} \int \frac{1}{u} du$$

$$\frac{1}{2} \ln u + C$$

$$\frac{1}{2} \ln(1 + 2\ln x) + C \quad (3)$$

4.1.3

$$\int \frac{2x^3 + 2x + 7x^2 + 9}{2x + 3} dx \quad (5)$$

$$\int \left(x^2 + 2x - 2 + \frac{15}{2x + 3} \right) dx$$

$$\int \frac{x^3}{3} + x^2 - 2x + \frac{15}{2} \ln(2x+3) + C$$

4.1.4

$$\int e^x \sqrt{1-e^{2x}} dx$$

$$\text{let } u = e^x$$

$$du = e^x dx$$

$$\int \sqrt{1-u^2} du$$

$$\frac{1}{2} \sin^{-1} u + \frac{u}{2} \sqrt{1-u^2} + C$$

$$\frac{1}{2} \sin^{-1} e^x + \frac{e^x}{2} \sqrt{1-e^{2x}} + C$$

(4)

4.2

$$\int \frac{9x+25}{(x+3)^2} dx$$

$$\frac{9x+25}{(x+3)^2} = \frac{A}{x+3} + \frac{B}{(x+3)^2}$$

$$9x+25 = A(x+3) + B$$

$$\text{let } x = -3, B = -2$$

$$\text{let } x = 0, A = 9$$

$$\int \left[\frac{9}{x+3} - \frac{2}{(x+3)^2} \right] dx$$

$$9 \ln(x+3) + \frac{2}{x+3} + C$$

(5)
[20]

QUESTION 5

5.1

$$\int_1^9 \frac{x-1}{\sqrt{x}} dx$$

$$\int_1^9 \left(\frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}} \right) dx$$

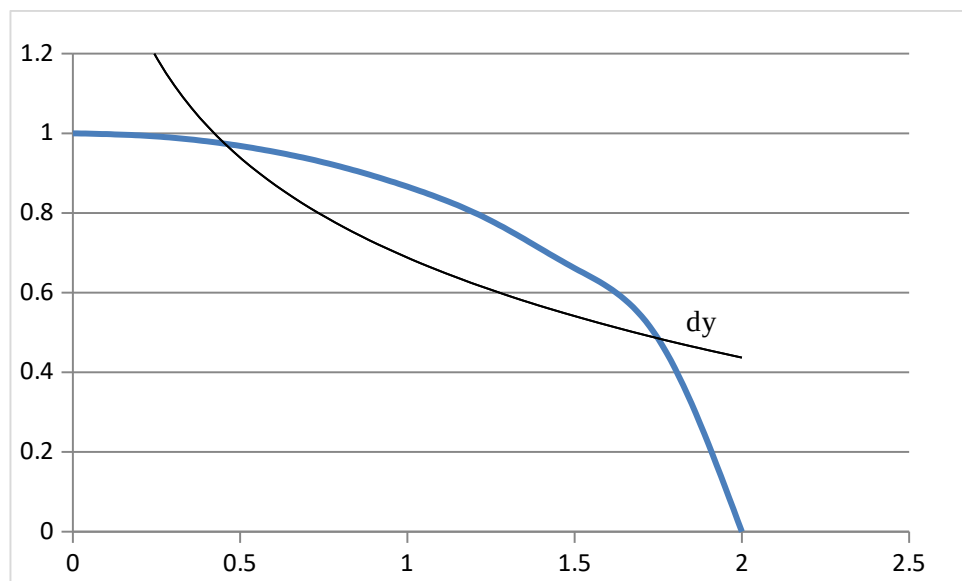
$$\int_1^9 \left(x^{\frac{1}{2}} - x^{-\frac{1}{2}} \right) dx$$

$$\left[\frac{2}{3} x^{\frac{3}{2}} - 2 x^{\frac{1}{2}} \right]_1^9$$

$$\frac{40}{3}$$

(4)

5.2 5.2.1



= 1 mark for indication of enclosed area

= 1 mark for indication of vertical or horizontal strip

(2)

5.2.2

$$A = \int_0^1 x \, dy$$

$$\hookrightarrow \int_0^1 \sqrt{4-4y^2} \, dy$$

$$\hookrightarrow 2 \int_0^1 \sqrt{1-y^2} \, dy$$

$$\hookrightarrow 2 \left[\frac{1}{2} \sin^{-1} y + \frac{y}{2} \sqrt{1-y^2} \right]_0^1$$

$$\hookrightarrow \frac{\pi}{2} \approx 1,571 \text{ units}^2$$

(4)

5.2.3

$$V = \pi \int_0^1 x^2 \, dy$$

$$\hookrightarrow \pi \int_0^1 (\sqrt{4-4y^2})^2 \, dy$$

$$\hookrightarrow \pi \int_0^1 (4-4y^2) \, dy$$

$$\hookrightarrow \pi \left[4y - \frac{4}{3} y^3 \right]_0^1$$

$$\hookrightarrow \frac{8\pi}{3} \approx 8,378 \text{ units}^2$$

(4)

5.3

$$dV = r^2 \, dA$$

$$x^2 \cdot 2\pi x \, dx$$

$$2\pi \int_0^7 x^3 \, dx$$

$$\hookrightarrow 2\pi \left[\frac{x^4}{4} \right]_0^7$$

$$\hookrightarrow 2\pi \left[\frac{2401}{4} - 0 \right]$$

$$\hookrightarrow 3771,482 \text{ cm}^4$$

(5)
[19]

QUESTION 6

$$6.1 \quad \frac{dy}{dx} = (1 + e^{-x})(y^2 - 1)$$

$$\frac{dy}{y^2 - 1} = (1 + e^{-x}) dx$$

$$\frac{1}{2} \ln \left(\frac{y-1}{y+1} \right) = x - e^{-x} + C$$

(3)

$$6.2 \quad \frac{d^2 y}{dx^2} = (x^2 + \pi)^2$$

$$\frac{d^2 y}{dx^2} = x^4 + 2\pi x^2 + \pi^2$$

$$\frac{dy}{dx} = \frac{1}{5}x^5 + \frac{2\pi}{3}x^3 + x\pi^2 + A$$

$$3 = \frac{1}{5}(0)^5 + \frac{2\pi}{3}(0)^3 + (0)\pi^2 + A$$

$$A = 3$$

$$\frac{dy}{dx} = \frac{1}{5}x^5 + \frac{2\pi}{3}x^3 + x\pi^2 + 3$$

$$y = \frac{1}{30}x^6 + \frac{\pi}{6}x^4 + \frac{\pi^2}{2}x^2 + 3x + B$$

$$-4 = \frac{1}{30}(0)^6 + \frac{\pi}{6}(0)^4 + \frac{\pi^2}{2}(0)^2 + 3(0) + B$$

$$B = -4$$

$$y = \frac{1}{30}x^6 + \frac{\pi}{6}x^4 + \frac{\pi^2}{2}x^2 + 3x - 4$$

(7)
[10]**TOTAL: 100**