



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

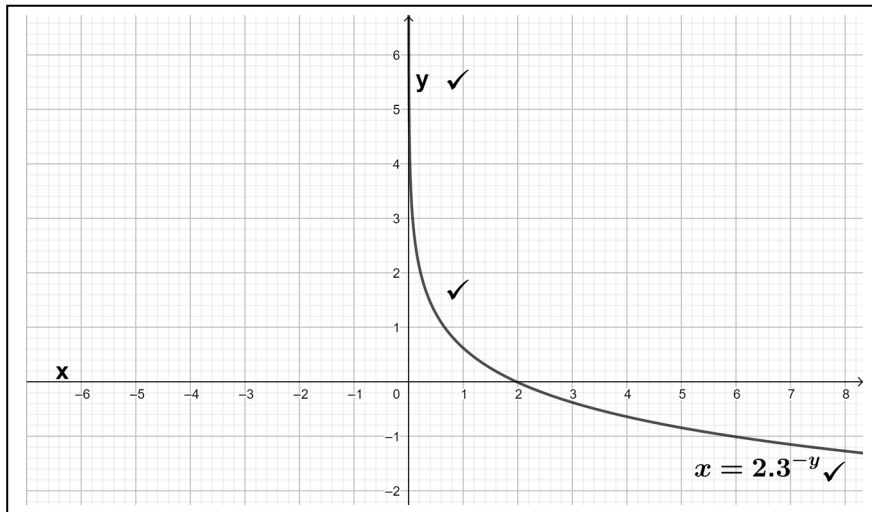
MATHEMATICS N4

1 August 2023

This marking guideline consists of 7 pages.

QUESTION 1

1.1



(3)

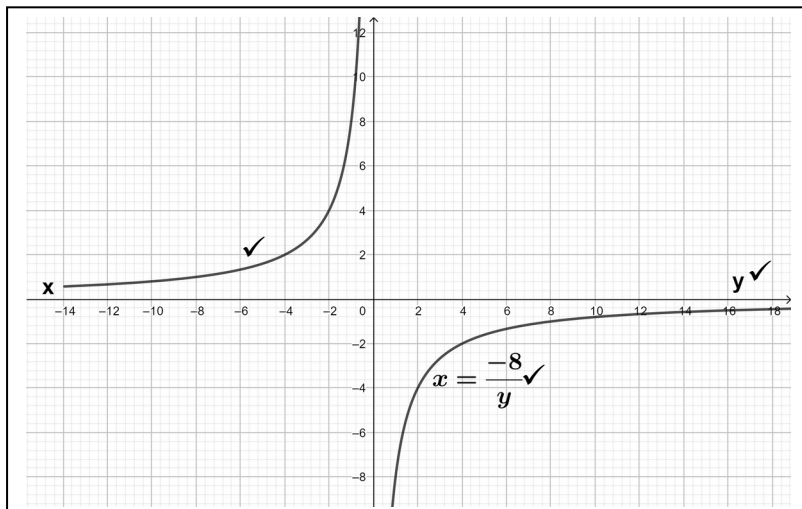
1.1.1 Domain of inverse $0 < x$.

1.1.2 Range of inverse $y \in \mathbb{R}$

(2 × 1)

(2)

1.2

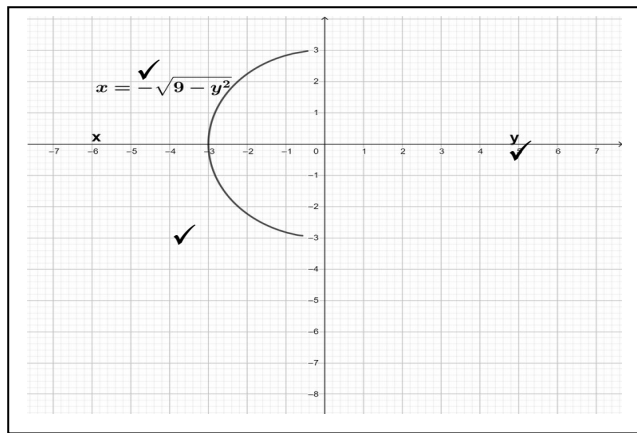


(3)

1.2.1 Not symmetrical about the x-axis.

(1)

1.3



(3)

1.3.1 Relation

(1)

1.4

$$9^{3x-1} = 4^{2x+2}$$

$$(3x-1)\ln 9 = (2x+2)\ln 4 \checkmark$$

$$6,592x - 2,197 = 2,773x + 2,773 \checkmark$$

$$3,819x = 4,97$$

$$\therefore x = 1,301 \checkmark$$

(3)

1.5

$$\frac{1}{x^7} + \frac{14}{x^8} + \frac{112}{x^9} + \frac{672}{x^{10}}$$

(4)

[20]**QUESTION 2**

2.1

$$\begin{vmatrix} 3 & -2 & 4 \\ 1 & 3 & -6 \end{vmatrix}$$

(10)

$$2.1. \quad \begin{vmatrix} 2 & -1 & -2 \end{vmatrix}$$

$$\Delta_0 = 3 \begin{vmatrix} 3 & -6 \\ -1 & -2 \end{vmatrix} + 2 \begin{vmatrix} 1 & -6 \\ 2 & -2 \end{vmatrix} + 4 \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} \checkmark \checkmark$$

$$= 3(-12) + 2(10) + 4(-7) \checkmark$$

$$= -44 \checkmark$$

$$\Delta_{I_3} = \begin{vmatrix} 3 & -2 & 2 \\ 1 & 3 & 8 \\ 2 & -1 & 0 \end{vmatrix}$$

$$= 3 \begin{vmatrix} 3 & 8 \\ -1 & 0 \end{vmatrix} + 2 \begin{vmatrix} 1 & 8 \\ 2 & 0 \end{vmatrix} + 2 \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} \checkmark \checkmark$$

$$= 3(8) + 2(-16) + 2(-7) \checkmark$$

$$= -22 \checkmark$$

$$\therefore I_3 = \frac{\Delta_{I_3}}{\Delta_0}$$

$$= \frac{-22}{-44} \checkmark$$

$$= \frac{1}{2} \checkmark$$

2.2

2.2.1

$$|r| = \sqrt{x^2 + y^2} \checkmark$$

$$= \sqrt{(-1)^2 + (-4)^2} \checkmark$$

$$= \sqrt{9}$$

$$= 3 \checkmark$$

$$\tan \theta = \frac{y}{x}$$

$$= \frac{-4}{-1} \checkmark$$

$$\therefore \theta = \tan^{-1} 4$$

$$= 75,963^\circ \checkmark$$

(5)

$$2.2.2 \quad = 3(\cos 75,963^\circ + i \sin 75,963^\circ) \checkmark \checkmark$$

(2)

2.3

$$\sqrt{j^4} - j^3$$

$$= (j^4)^{1/2} - j^2 \cdot j \checkmark$$

$$= j^2 - j^2 \cdot j \checkmark$$

$$= -1 - 1j$$

$$= -1 - j \checkmark$$

(3)

[20]

QUESTION 3

3.1

3.1.1

$$\sin(x + 45^\circ) = \frac{\sin x + \cos x}{\sqrt{2}}$$

$$LHS = \sin(x + 45^\circ)$$

$$= \sin x \cdot \cos 45^\circ + \sin 45^\circ \cdot \cos x \checkmark$$

$$= \sin x \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \cos x \checkmark$$

$$= \frac{\sin x + \cos x}{\sqrt{2}} \checkmark$$

$$= RHS$$

(3)

3.1.2

$$(\sin x + \cos x)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 \checkmark$$

$$\sin^2 x + \cos^2 x + 2 \sin x \cdot \cos x = \frac{1}{2}$$

$$1 + 2 \sin x \cos x = \frac{1}{2} \checkmark$$

$$\sin 2x = -\frac{1}{2} \checkmark$$

$$\text{ref } \angle = 30^\circ$$

$$\therefore 2x = 180^\circ + 30^\circ; 360^\circ - 30^\circ$$

$$2x = 210^\circ; 330^\circ$$

$$\therefore x = 105^\circ; 165^\circ \checkmark$$

PROVE:

$$\sin 105^\circ + \cos 105^\circ = 0,707 = \frac{1}{\sqrt{2}} \checkmark$$

$$\therefore x = 105^\circ \text{ is a solution } \in [0^\circ; 360^\circ] \checkmark$$

AND:

$$\sin 165^\circ + \cos 165^\circ = -0,707 \neq \frac{1}{\sqrt{2}}$$

$$\therefore x = 165^\circ \text{ is not a solution}$$

(6)

3.2

$$\begin{aligned} & \frac{\tan x \cdot \sin x \cdot (-\cos x)}{\sec x \cdot \cos x} \checkmark \checkmark \\ &= \frac{\frac{\sin x}{\cos x} \cdot \sin x \cdot -\frac{1}{\sin x}}{\frac{1}{\cos x} \cdot \cos x} \checkmark \\ &= -\tan x \checkmark \end{aligned}$$

(4)

3.3

$$\begin{aligned} \cos 2\theta &= \cos(\theta + \theta) \checkmark \\ &= \cos \theta \cdot \cos \theta - \sin \theta \cdot \sin \theta \\ &= \cos^2 \theta - \sin^2 \theta \checkmark \\ &= 1 - \sin^2 \theta - \sin^2 \theta \\ &= 1 - 2 \sin^2 \theta \checkmark \end{aligned}$$

(3)

$$\begin{aligned}
 3.4 \quad x^2 &= r^2 - y^2 \\
 &= (-13)^2 - (12)^2 \quad \checkmark \\
 &= 25 \\
 \therefore x &= 5 \quad \checkmark \\
 &= \sin \theta + \cos \theta \\
 &= \frac{-12}{13} + \frac{5}{12} \quad \checkmark \\
 &= \frac{-7}{13} \quad \checkmark
 \end{aligned}$$

(4)
[20]**QUESTION 4**

$$\begin{aligned}
 4.1 \quad f(x) &= 3x^3 \\
 \lim_{h \rightarrow 0} &= \frac{3(x+h)^3 - 3x^3}{h} \quad \checkmark \\
 &= \frac{3 \left[x^3 + \frac{3x^{3-1} \cdot h}{1!} + \frac{3(3-1)x^{3-2} \cdot h^2}{2!} + \frac{3(3-1)(3-2)x^{3-3} \cdot h^3}{3!} \right] - 3x^3}{h} \quad \checkmark \\
 \lim_{h \rightarrow 0} &= \frac{3[x^3 + 3x^2h + 3xh^2 + 2h^3] - 3x^3}{h} \\
 \lim_{h \rightarrow 0} &= \frac{3x^3 + 9x^2h + 9xh^2 + 6h^3 - 3x^3}{h} \quad \checkmark \\
 \lim_{h \rightarrow 0} &= \frac{h(9x^2 + 9x^2h + 6h^2)}{h} \quad \checkmark \\
 &= 9x^2
 \end{aligned}$$

(4)

$$\begin{aligned}
 4.2 \quad y &= \sqrt{u} = u^{1/2} \\
 \frac{dy}{du} &= \frac{1}{2} u^{-1/2} \quad \checkmark \\
 \therefore \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\
 u &= \cot x \\
 \frac{du}{dx} &= -\operatorname{cosec}^2 x \quad \checkmark
 \end{aligned}$$

(5)

$$= \frac{1}{2\sqrt{u}} \times -\operatorname{cosec}^2 x$$

✓

$$= \frac{-\operatorname{cosec}^2 x}{2\sqrt{\cot x}}$$

$$4.3 \quad \frac{dy}{dx} = 2^x \ln 2 + \frac{1}{2\pi x} + \frac{3}{2} e^{3x} + 0 \quad (4)$$

$$4.4 \quad \text{For TP let } y' = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)}$$

$$x = 2,43 \text{ or } x = -1,097$$

Y-values

$$y = (2,43)^3 - 2(2,43)^2 - 8(2,43) \quad \text{and} \quad y = (-1,097)^3 - 2(-1,097)^2 - 8(-1,097)$$

$$y = -16,9 \quad y = 5,04$$

$$\therefore \text{TP}_1: [2,43; -16,9] \text{ Minimum}$$

$$\text{TP}_2: [-1,097; 5,04] \text{ Maximum}$$

Points of inflection:

$$\frac{d^2 y}{dx^2} = 0$$

$$\therefore 6x - 4 = 0$$

$$6x = 4$$

$$x = \frac{2}{3} = 0,667$$

$$\therefore y = (0,667)^3 - 2(0,667)^2 - 8(0,667)$$

$$y = -5,93$$

$$\text{Points of inflection: } (0,667; -5,93)$$

(7)
[20]

QUESTION 5

5.1
$$\int_1^4 \left(x^{1/2} - 5 \right)^2 dx$$

$$= \int_1^4 \left(x^{1/2} - 5 \right) \left(x^{1/2} - 5 \right) dx \quad \checkmark$$

$$= \int_1^4 \left(x - 5x^{1/2} - 5x^{1/2} + 25 \right) dx \quad \checkmark$$

$$= \left[\frac{x^2}{2} - \frac{10x^{3/2}}{3/2} + 25x \right]_1^4 \quad \checkmark$$

$$= \frac{(4)^2}{2} - \frac{10(4)^{3/2}}{3/2} + 25(4) - \left(\frac{1^2}{2} - \frac{10(1)^{3/2}}{3/2} + 25(1) \right) \quad \checkmark$$

$$= 62,5 \quad \checkmark \quad (5)$$

5.2
$$\int (3 \sec x + \tan x)(2 \sec x + 5 \tan x) dx$$

$$= \int (6 \sec^2 x + 15 \sec x \cdot \tan x + 2 \sec x \cdot \tan x + 5 \tan^2 x) dx$$

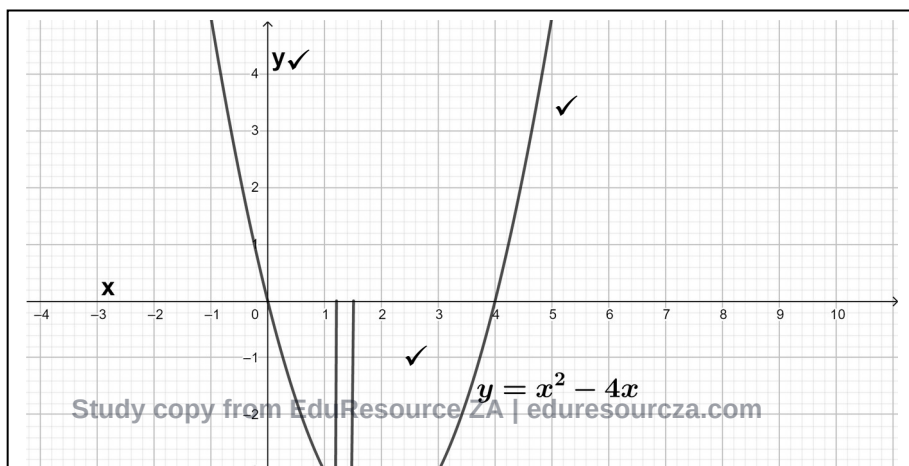
$$= \int (6 \sec^2 x + 17 \sec x \cdot \tan x + 5(\sec^2 x - 1)) dx \quad \checkmark$$

$$= \int (6 \sec^2 x + 17 \sec x \cdot \tan x + 5 \sec^2 x - 5) dx$$

$$= \int (11 \sec^2 x + 17 \sec x \cdot \tan x - 5) dx \quad \checkmark$$

$$= 11 \tan x + 17 \sec x - 5x + c \quad \checkmark \quad (5)$$

5.3 5.3.1





5.3.2

$$\begin{aligned} & \int_0^4 (x^2 - 4x) dx \\ &= \left[\frac{x^3}{3} - 2x^2 \right]_0^4 \quad \checkmark \\ &= \frac{(4)^3}{3} - 2(4)^2 - \left(\frac{(0)^3}{3} - 2(0)^2 \right) \quad \checkmark \\ &= \frac{-32}{3} \quad \checkmark \\ &= |-10,667| \quad \checkmark \\ &= 10,667 \quad \checkmark \end{aligned}$$

(5)
[20]

TOTAL: 100