



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

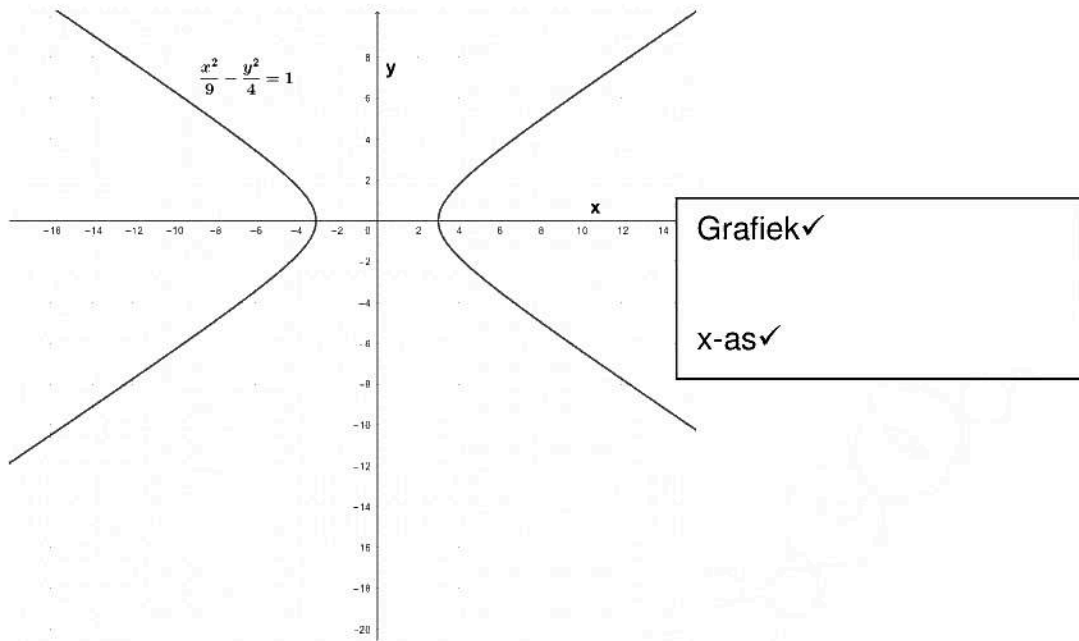
WISKUNDE N4

27 Maart 2025

Hierdie nasienriglyn bestaan uit 8 bladsye.

VRAAG 1

1.1



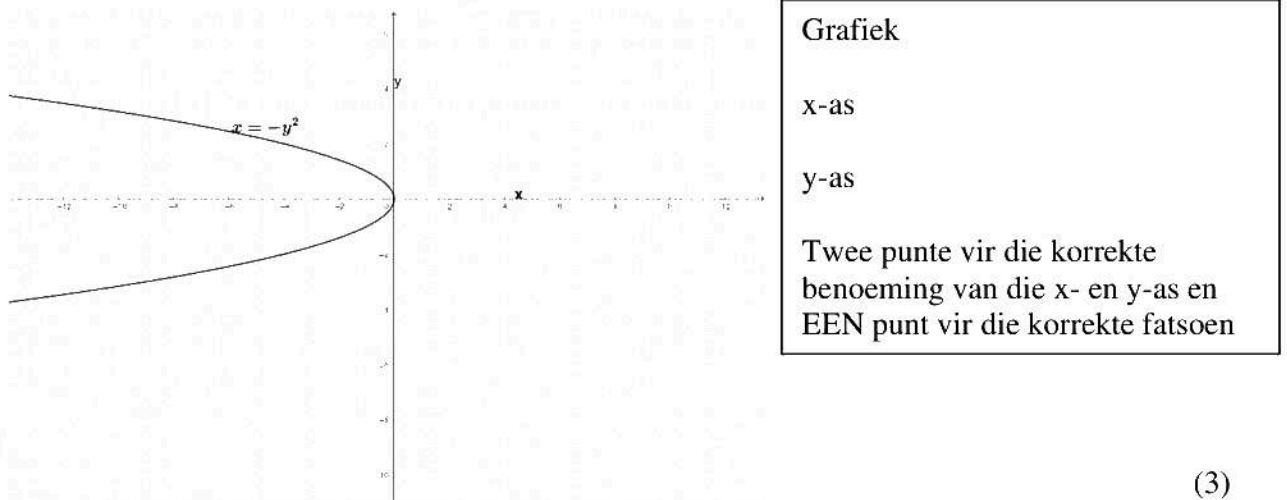
(3)

1.2

1.2.1 Relasie
1.2.2 Diskontinu

(2 × 1) (2)

1.3



(3)

1.3.1 Maksimumwaardegrafiek
1.3.2 x-as

(2 × 1) (2)

[10]

VRAAG 2

2.1

$$\begin{vmatrix} 5 & -6 & 3 \\ 2 & -3 & 2 \\ 3 & -7 & 5 \end{vmatrix}$$

$$\begin{aligned} \Delta_0 &= 5 \begin{vmatrix} -3 & 2 \\ -7 & 5 \end{vmatrix} + 6 \begin{vmatrix} 2 & 2 \\ 3 & 5 \end{vmatrix} + 3 \begin{vmatrix} 2 & -3 \\ 3 & -7 \end{vmatrix} \quad \checkmark \\ &= -5 + 24 - 15 \\ &= 4 \quad \checkmark \end{aligned}$$

$$\Delta_z = \begin{vmatrix} 5 & -6 & -9 \\ 2 & -3 & -5 \\ 3 & -7 & -16 \end{vmatrix}$$

$$\begin{aligned} &= 5 \begin{vmatrix} -3 & -5 \\ -7 & -16 \end{vmatrix} + 6 \begin{vmatrix} 2 & -5 \\ 3 & -16 \end{vmatrix} - 9 \begin{vmatrix} 2 & -3 \\ 3 & -7 \end{vmatrix} \quad \checkmark \\ &= 65 - 102 + 45 \\ &= 8 \quad \checkmark \end{aligned}$$

$$\therefore z = \frac{\Delta_z}{\Delta_0}$$

$$\begin{aligned} &= \frac{8}{4} \\ &= 2 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \Delta_0 &= \begin{vmatrix} -7 & -6 \\ 3 & 4 \end{vmatrix} \\ &= -28 + 18 \\ &= -10 \end{aligned}$$

(5)

2.2

$$\Delta_x = \begin{vmatrix} 27 & -6 \\ 5 & 4 \end{vmatrix}$$

$$\begin{aligned} &= 108 + 30 \quad \checkmark \\ &= 138 \end{aligned}$$

$$\Delta_y = \begin{vmatrix} -7 & 27 \\ 3 & 5 \end{vmatrix}$$

$$\begin{aligned} &= -35 - 81 \\ &= -116 \quad \checkmark \end{aligned}$$

$$\therefore x = \frac{\Delta_x}{\Delta_0} = \frac{138}{-10} = -13.8 \quad \checkmark$$

$$\therefore y = \frac{\Delta_y}{\Delta_0} = \frac{-116}{-10} = 11.6 \quad \checkmark$$

$$\begin{aligned}
 2.3 \quad 2.3.1 \quad |r| &= \sqrt{x^2 + y^2} \\
 &= \sqrt{(\sqrt{2})^2 + (-1)^2} \\
 &= \sqrt{3} \\
 \tan \theta &= \frac{y}{x} \quad \checkmark \\
 &= \frac{1}{\sqrt{2}} \\
 \text{ref } \angle &= 45^\circ \\
 &= 360^\circ - 45^\circ \\
 &= 315^\circ \quad \checkmark \\
 &= (\sqrt{3})^4 \angle 4 \times 315^\circ \\
 &= 9 \text{ cis } 1260^\circ \quad \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 2.3.2 \quad 2x^2 + 4x + 4 &= 0 \\
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \checkmark \\
 &= \frac{-2 \pm \sqrt{4 - 8}}{2} \quad \checkmark \\
 x &= -1 + j \text{ or } x = -1 - j \quad \checkmark
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 2.4 \quad \frac{2 \angle 30^\circ \quad 3 \angle 75^\circ}{4 \angle 36^\circ \quad 5 \angle 21^\circ} &\quad \checkmark \\
 &= \frac{6 \angle 45^\circ}{20 \angle 15^\circ} \quad \checkmark \\
 &= 0.3 \angle 30^\circ \quad \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 2.5 \quad \frac{-j-1}{-2+5j} \times \frac{-2-5j}{-2-5j} &\quad \checkmark \\
 &= \frac{2j+5j^2}{4+10j-10j-25j^2} \\
 &= \frac{2j}{9} - \frac{5}{29} \quad \checkmark
 \end{aligned} \tag{2}$$

[20]

VRAAG 3

$$\begin{aligned}
 3.1 \quad & (\cos 2x - 1)(\cos 2x + 1) = 0 \\
 & \cos 2x = \frac{1}{2} \quad \text{of} \quad \cos 2x = -1 \quad \checkmark \\
 & 2x = 60^\circ \quad \text{of} \quad 2x = 270^\circ \quad \checkmark \\
 & x = 30^\circ \quad \text{of} \quad x = 135^\circ \quad \checkmark \\
 & \therefore x = 30^\circ; 45^\circ; 315^\circ \quad \checkmark \\
 & \text{GS}[30^\circ; 45^\circ] \quad \checkmark
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 3.2 \quad & \frac{1}{\cos 75^\circ} = \sin 75^\circ \quad \checkmark \\
 & = \sin(45^\circ + 30^\circ) \\
 & = \sin 45^\circ \cdot \cos 30^\circ + \sin 30^\circ \cdot \cos 45^\circ \quad \checkmark \\
 & = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \\
 & = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \quad \checkmark \\
 & = \frac{\sqrt{3} + 1}{2\sqrt{2}} \quad \checkmark
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 3.3 \quad & = \cos x(-\cos x) + \frac{\sin x}{\cos x} \cdot \cos x(-\sin x) \quad \checkmark \\
 & = -\cos^2 x - \sin^2 x \\
 & = -(\cos^2 x + \sin^2 x) \quad \checkmark \\
 & = -1 \quad \checkmark
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 3.4 \quad & \text{LHS} = \frac{\sin 135^\circ \cdot \tan 210^\circ \cdot \sin 190^\circ}{\cos 100^\circ \sin 120^\circ} \\
 & = \frac{(-\sin 45^\circ)(\tan 30^\circ)(-\sin 10^\circ)}{(-\sin 10^\circ)(\sin 120^\circ)} \quad \checkmark \\
 & = \frac{-\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{2}} \quad \checkmark \\
 & = -\frac{\sqrt{2}}{3} \quad \checkmark
 \end{aligned} \tag{6}$$

[20]

VRAAG 4

4.1 Vir TP laat $f'(x) = 0$

$$\therefore 3x^2 - 2 = 0$$

$$x = \pm \sqrt{\frac{2}{3}} \quad \checkmark \checkmark$$

Vir ooreenstemmende y-waardes: indien $x = -\sqrt{\frac{2}{3}}$

$$y = \left(-\sqrt{\frac{2}{3}}\right)^3 - 2\left(-\sqrt{\frac{2}{3}}\right) - 4$$

$$= -2,91 \quad \checkmark$$

$$\text{if } x = \sqrt{\frac{2}{3}} \therefore y = \left(\sqrt{\frac{2}{3}}\right)^3 - 2\left(\sqrt{\frac{2}{3}}\right) - 4 = -5,089 \quad \checkmark$$

$$\frac{d^2y}{dx^2} = 6x = 6\left(\sqrt{\frac{2}{3}}\right) = +4,89 \quad \checkmark$$

$$\therefore \text{TP}_1 = \left[\sqrt{\frac{2}{3}}, -5,089\right] \text{ Minimum } \checkmark$$

$$\frac{d^2y}{dx^2} = 6x = 6\left(-\sqrt{\frac{2}{3}}\right) = -4,89 \quad \checkmark$$

$$\therefore \text{TP}_2 = \left[-\sqrt{\frac{2}{3}}, -2,91\right] \text{ Maksimum } \checkmark$$

Infleksiepunte:

$$\frac{d^2y}{dx} = 0 \quad \checkmark$$

$$6x = 0$$

$$\therefore x = 0$$

$$\therefore y = -4$$

Infleksiepunte: (0;-4) $\checkmark$

(10)

4.2 $y = \sqrt{\cot x}$

$$u = \cot x$$

$$y = \sqrt{u} = u^{\frac{1}{2}}$$

$$\frac{du}{dx} = -\cos ec^2 x \quad \checkmark$$

$$\frac{dy}{du} = \frac{1}{2\sqrt{u}} \quad \checkmark$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \frac{1}{2\sqrt{\cot x}} \times -\cos ec^2 x \quad \checkmark \checkmark$$

(4)

$$4.3 \quad \frac{dx}{dy} = -\frac{1}{3\sqrt[3]{y^4}} - 2 \cos 2y + \frac{1}{y} + \frac{1}{2} e^{-2y} + \frac{3\pi \cdot 5^{3\pi y} \cdot \ln 5}{2} - 0 + 6p \quad (7)$$

$$4.4 \quad y = \cot x = \frac{\cos x}{\sin x}$$

$$= \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{(\sin x)^2}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= -\operatorname{cosec}^2 x \quad (4)$$

[25]

VRAAG 5

$$5.1 \quad \frac{3 \sin 4x}{4} - \frac{7 \cdot 5^{\pi x}}{3\pi \cdot \ln 5} + 3e^{3x} - \frac{\pi}{2x^2} - 3\pi \ln x - \frac{1}{6} \cot \pi x + c \quad (7)$$

$$5.2 \quad V = [-\cos x - \sin x]_0^{\frac{\pi}{2}}$$

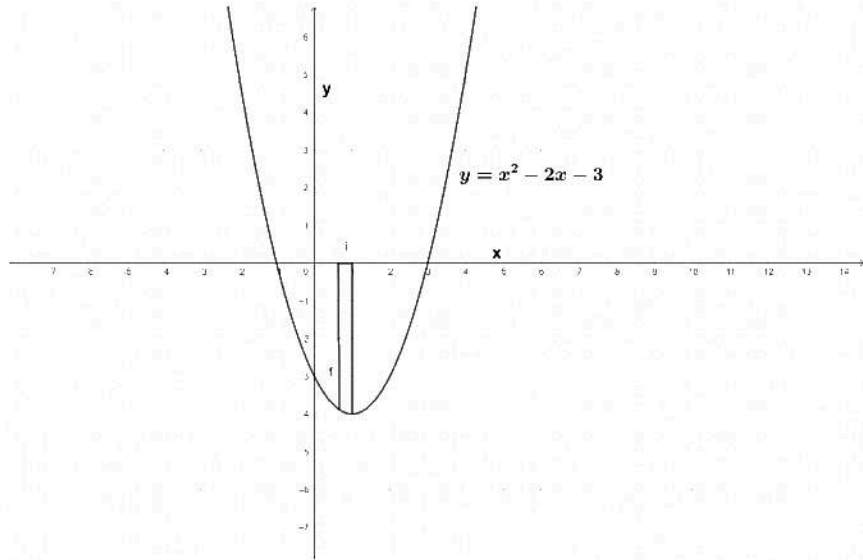
$$= -\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right) - (-\cos 0 - \sin 0)$$

$$= 0 - 1 - (-1 - 0)$$

$$= 0$$

(5)

5.3 5.3.1



Die grafiek	✓
Area	✓
Strook	✓

(3)

5.3.2

$$A = \int_{-1}^3 (x^2 - 2x - 3) dx$$

$$= \left[\frac{x^3}{3} - x^2 - 3x \right]_{-1}^3 \quad \checkmark$$

$$= \frac{3^3}{3} - 3^2 - 3(3) - \left(\frac{-1^3}{3} - (-1)^2 - 3(-1) \right) \quad \checkmark \checkmark$$

$$= |-10,666 \text{ u}^2| \quad \checkmark$$

$$= 10,666 \text{ u}^2 \quad \checkmark$$

(5)

5.4

$$\int \frac{3(x^3 - 27)}{2(x-3)} dx \quad \checkmark$$

$$= \frac{3}{2} \int \frac{(x-3)(x^2 + 3x + 9)}{(x-3)} dx \quad \checkmark \checkmark$$

$$= \frac{3}{2} \left(\frac{x^3}{3} + \frac{3x^2}{2} + 9x \right) + c \quad \checkmark$$

$$= \frac{x^3}{2} + \frac{9x^2}{4} + \frac{27x}{2} + c \quad +c \quad \checkmark$$

(5)

[25]

TOTAAL: 100