



**higher education
& training**

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N4

1 APRIL 2020

This marking guideline consists of 10 pages.

QUESTION 1

$$\begin{aligned}
 1.1 \quad & \frac{\cos x \cdot \cos x}{\tan x \cdot \cos x \cdot -\sec x} \quad \checkmark \quad \frac{\cos x \cdot \sec x}{\tan x \cdot \sin x - \csc x} \checkmark \\
 & = \frac{\cos x \cdot \frac{1}{\sin x}}{\tan x \cdot \cos x \cdot \frac{-1}{\cos x}} \quad \checkmark \quad \frac{\frac{\cos x}{1} \cdot \frac{1}{\cos x}}{\frac{-\sin x}{\cos x} \cdot \frac{\sin x}{1} \cdot \frac{1}{\sin x}} \checkmark \\
 & = \frac{\tan x}{\tan x \cdot -\cot x} \quad \checkmark \quad -\frac{1}{\tan x} \checkmark \\
 & = -\tan x \quad \checkmark \quad -\cot x \quad \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 1.2 \quad & 2\cos^2 \beta + \cos \beta - 1 = 0 ; -180^\circ \leq \beta \leq 180^\circ \\
 & (2\cos 2\beta - 1)(\cos 2\beta + 1) = 0 \quad \checkmark \\
 & \cos 2\beta = \frac{1}{2} \text{ or } \cos 2\beta = -1 \\
 & 2\beta = 60^\circ \text{ or } 2\beta = 270^\circ \\
 & \beta = 30^\circ \text{ or } \beta = 135^\circ \quad \checkmark \checkmark \\
 & GS = [30^\circ; 45^\circ] \quad \checkmark \checkmark
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 1.3 \quad 1.3.1 \quad & 1 - \sin x = 2\cos^2 \frac{x}{2} \\
 & 2\cos^2 \frac{x}{2} = 1 - \sin x \\
 & \cos^2 \frac{x}{2} = \frac{1 - \sin x}{2} \quad \checkmark \\
 & \cos \frac{x}{2} = \pm \sqrt{\frac{1 - \sin x}{2}} \quad \checkmark
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 1.3.2 \quad & \cos 15^\circ \rightarrow \cos \frac{30^\circ}{2} = \sqrt{\frac{1 - \sin 30^\circ}{2}} \quad \checkmark \\
 & = \sqrt{\frac{1 - \frac{1}{2}}{2}} \quad \checkmark \\
 & = \sqrt{\frac{1}{4}} \\
 & = \frac{1}{2} \quad \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 1.4 \quad & \frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x \\
 &= \frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} \\
 \text{LHS} \quad &= \frac{\sin x + 2\sin x \cos x}{1 + \cos x + \cos^2 x - \sin^2 x} \checkmark \checkmark \\
 &= \frac{\sin x (1 + 2\cos x)}{\cos x (1 + 2\cos x)} \checkmark \\
 &= \frac{\sin x}{\cos x} \\
 &= \tan x \checkmark \\
 &= \text{RHS}
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 1.5 \quad & \cos 75^\circ = \cos(45^\circ + 30^\circ) \\
 &= \cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \cdot \sin 30^\circ \checkmark \\
 &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\
 &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \checkmark \\
 &= \frac{\sqrt{3}-1}{2\sqrt{2}} \checkmark
 \end{aligned} \tag{3}$$

[20]

QUESTION 2

$$\begin{aligned}
 2.1 \quad 2.1.1 \quad & \text{Area} = \int_0^{\frac{\pi}{4}} 2\cos x dx \\
 &= 2 \int_0^{\frac{\pi}{4}} \cos x dx \\
 &= 2 \left[\sin x + c \right]_0^{\frac{\pi}{4}} \checkmark \\
 &= 2 \left[\sin\left(\frac{\pi}{4}\right) + c - (\sin 0^\circ + c) \right] \checkmark
 \end{aligned} \tag{4}$$

$$= 2 \left[\frac{1}{\sqrt{2}} + c - c \right] \checkmark$$

$$= 1,414 \text{ units}^2 \checkmark$$

2.1.2 $y = 2 \cos x$
 (i) For T.P let $x = 0$
 $-2 \sin x = 0$
 $x = \sin^{-1} 0$
 $x = 0^0$ or $x = 180^0 \checkmark \checkmark$
 y -values, if $x = 0^0$
 $y = 2 \cos 0^0$
 $y = 2 \checkmark$
 if $x = 180^0$
 $y = 2 \cos 180^0$
 $y = -2 \checkmark$
 $T.P_1 = (0^0; 2)$ and $T.P_2 = (180^0; -2) \checkmark \checkmark$

(6)

2.1.3 $\frac{d^2 y}{dx^2} = -2 \cos x$
 $= -2 \cos 0^0$
 $= -2$ maximum

$$\frac{d^2 y}{dx^2} = -2 \cos x \quad \checkmark$$

$$= -2 \cos 180^0$$

$$= 2 \text{ minimum}$$

$$T.P_1 = (0^0; 2) \text{ maximum } \checkmark$$

$$T.P_2 = (180^0; -2) \text{ minimum } \checkmark$$

(3)

2.2 $\frac{-1}{2x^2} + \frac{1}{2} \cos 3x + 2x^{\frac{3}{2}} - \frac{10 \cdot 10^x}{\ln 10} + \frac{1}{2} \sin 2x - 2x + c$

$\checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark \quad \checkmark$

(7)
[20]

QUESTION 3

3.1 $y = \sqrt{\tan x}$ $\ell et u = \tan x$
 $y = \sqrt{u} = u^{\frac{1}{2}}$ $\frac{dy}{du} = \sec^2 x$ ✓
 $\frac{dy}{du} = \frac{2u^{\frac{3}{2}}}{3}$
 $\therefore \frac{dy}{du} = \frac{dy}{du} \times \frac{dy}{du}$ ✓
 $= \frac{2u^{\frac{3}{2}}}{3} \times \sec^2 x$
 $= \frac{2}{3} \sqrt{u^3} \cdot \sec^2 x$ ✓
 $= \frac{2}{3} \sqrt{\tan^3 x} \cdot \sec^2 x$ ✓

(4)

3.2 $y = 2^x + \frac{1}{2} \ln x + \frac{1}{2} e^x + 6t$
 $\frac{dy}{dx} = 2^x \cdot \ln 2 + \frac{1}{2x} + \frac{1}{2} e^x + 0$
 ✓ ✓ ✓ ✓

(4)

3.3 $\lim_{x \rightarrow 2} \left(\frac{x^2 - 4}{x - 2} \right)$ (3)

$$= \frac{(x-2)(x+2)}{(x-2)}$$

✓

$$= (x+2)$$

✓

$$= 2+2$$

$$= 4 \checkmark$$

3.4

$$\begin{aligned}
 y &= -x^3 + 5x \\
 \lim_{x \rightarrow 0} \frac{(-x+h)^3 + 5(x+h) - (-x^3 + 5x)}{h} &\checkmark \\
 &= \frac{-x^3 - 3x^2 \cdot h - 3x \cdot h^2 + 5x + 5h + x^3 - 5x}{h} \checkmark \\
 &= \frac{h(-3x^2 - 3xh + 5)}{h} \checkmark \\
 &= -3x^2 + 5 \checkmark
 \end{aligned}$$

(4)

3.5

$$\begin{aligned}
 S &= \int_1^4 (\sqrt{t} - 5)^2 dt \\
 &= \int_1^4 (\sqrt{t} - 5)(\sqrt{t} - 5) dt \\
 &= \int_1^4 (t - 5\sqrt{t} - 5\sqrt{t} + 25) dt \checkmark \\
 &= \int_1^4 (t - 10\sqrt{t} + 25) dt \\
 &= \left[\frac{t^2}{2} - \frac{20t^{3/2}}{3} + 25t \right]_1^4 \\
 &= \frac{4^2}{2} - \frac{20(4)^{3/2}}{3} + 25(4) - \left(\frac{(1)^2}{2} - \frac{20(1)^{3/2}}{3} + 25(1) \right) \checkmark \\
 &= 8 - 53,333 + 100 - \frac{1}{2} + 6,667 - 25 \\
 &= 35,834 \checkmark
 \end{aligned}$$

(3)

3.6

$$\begin{aligned}
 &\int \left(\frac{4\sec^4 \theta - 9}{2\sec^2 \theta + 3} \right) d\theta \\
 &= \int \frac{(2\sec^2 \theta + 3)(2\sec^2 \theta - 3)}{(2\sec^2 \theta + 3)} d\theta \checkmark \\
 &= \int (2\sec^2 \theta - 3) d\theta \\
 &= 2 \tan \theta - 3\theta + c \checkmark
 \end{aligned}$$

(2)

[20]

QUESTION 4

4.1

$$(3 - 4j)^2 = \frac{-x - yj}{j^2}$$

$$j^2(3 - 4)(3 - 4j) = -x - yj$$

$$j^2(9 - 12j - 12j + 16j^2) = -x - yj \checkmark$$

$$j^2(9 - 24j + 16j^2) = -x - yj$$

$$-(9 - 24j - 16) = -x - yj \checkmark$$

$$-9 + 24j + 16 = -x - yj$$

$$24j + 7 = -x - yj$$

$$x = -7 \text{ and } y = -24 \checkmark \checkmark$$

(4)

4.2

$$(\sqrt{3} - j)^5$$

$$|r| = \sqrt{x^2 + y^2}$$

$$= \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$= \sqrt{4}$$

$$= 2 \quad \checkmark$$

$$\tan \theta = \frac{y}{x}$$

$$= \frac{-1}{\sqrt{3}}$$

$$\tan \theta = 0,578$$

$$\theta = 30,023$$

$$\therefore \theta = 360^\circ - 30,023$$

(4)

$$\begin{aligned}
 \theta &= 329,977 \quad \checkmark \\
 \therefore &= (r\theta)^2 & (r\theta)^5 \\
 &= (2\,329,977)^2 & (2\,329,977)^5 \\
 &= 4\,659,954 \quad \checkmark & 32\,1649,855 \\
 \therefore r(\cos\theta + i\sin\theta) & & \therefore r(\cos\theta + i\sin\theta) \\
 = 4(\cos 659,954 + i\sin 659,954) & & 32(\cos 1649,855 + i\sin 1649,855) \\
 = 1,997 - i3,466 \quad \checkmark & &
 \end{aligned}$$

$$\begin{aligned}
 4.3 \quad & \frac{2 \operatorname{cis}(-60^\circ) \cdot 5 \operatorname{cis} 45^\circ}{4 \operatorname{cis} 33^\circ \cdot 3 \operatorname{cis}(-21^\circ)} \\
 &= \frac{2 \cdot -60^\circ \cdot 5 \cdot 45^\circ}{4 \cdot 33^\circ \cdot 3 \cdot 21^\circ} \\
 &= \frac{10 \cdot 60^\circ + 45^\circ}{12 \cdot 33^\circ - 21^\circ} \quad \checkmark \\
 &= \frac{10 \cdot -15}{12 \cdot 12} \quad \checkmark \\
 &= 0,833 \cdot -27 \quad \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 4.4 \quad & 7a + 6b = 27 \\
 & 3a + 4b = 5 \\
 & D = \begin{vmatrix} -7 & 6 \\ 3 & 4 \end{vmatrix} \\
 & D = -28 - 18 \\
 & D = -46 \quad \checkmark \\
 & D_a = \begin{vmatrix} 27 & 6 \\ 5 & 4 \end{vmatrix} \quad \therefore a = \frac{Da}{D} = \frac{78}{-46} = -1,696 \quad \checkmark \\
 & = 108 - 30 \quad \checkmark \\
 & = 78 \\
 & D_b = \begin{vmatrix} -7 & 27 \\ 3 & 5 \end{vmatrix} \quad \therefore b = \frac{Db}{D} = \frac{-116}{-46} = 2,522 \quad \checkmark \\
 & = -35 - 81 \\
 & = -116 \quad \checkmark
 \end{aligned} \tag{5}$$

4.5 4.5.1

$$\begin{array}{r}
 \frac{2}{3} \quad -1 \quad -\frac{1}{2} \\
 |-\frac{1}{4} \quad 2 \quad 0| \\
 5 \quad -8 \quad \frac{-2}{3} \\
 D = \begin{vmatrix} \frac{-1}{4} & 0 & \frac{2}{3} & \frac{-1}{2} & \frac{2}{3} & \frac{-1}{2} \\ 5 & \frac{-2}{3} & 5 & \frac{-2}{3} & \frac{-1}{4} & 0 \end{vmatrix} \begin{array}{l} +2 \\ -8 \end{array} \\
 = 0,167 + 2(2,945) - 8(0 - 0,125) \checkmark \\
 = 0,167 + 5,89 + 1 \\
 = 7,059 \checkmark
 \end{array}$$

(3)

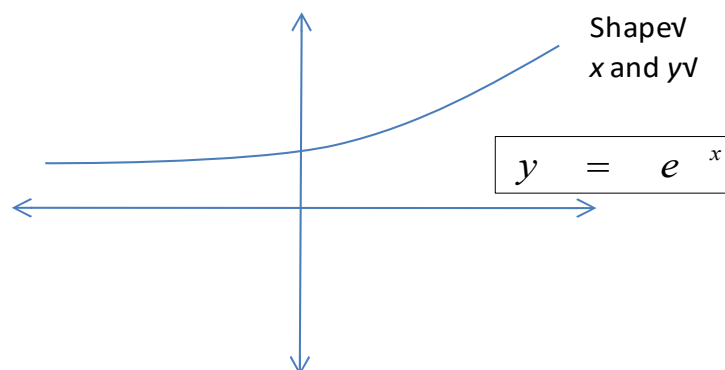
4.5.2

$$\begin{array}{l}
 = (-1)^3 \begin{vmatrix} \frac{-1}{4} & 0 \\ 5 & \frac{-2}{3} \end{vmatrix} \\
 = -(0,167 - 0) \\
 = -0,167 \checkmark
 \end{array}$$

(1)
[20]

QUESTION 5

5.1 5.1.1

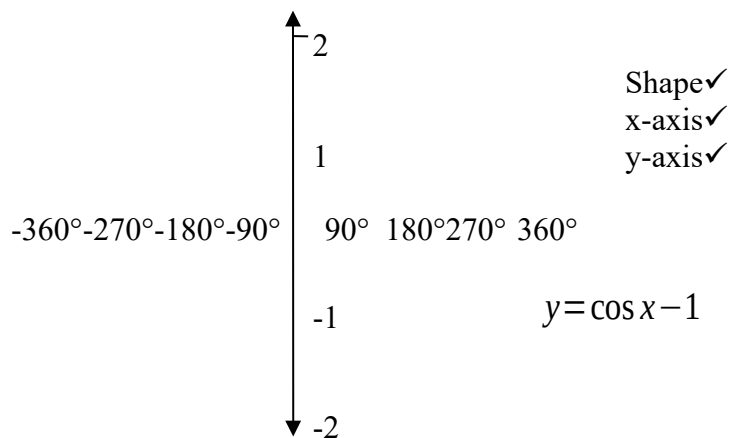


(2)

5.1.2 $= \{0 < y < \infty\} \checkmark$ (1)

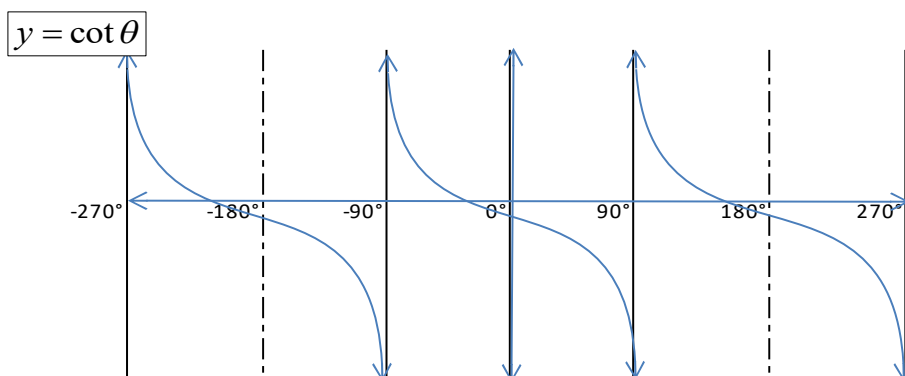
5.1.3 $x = e^y \checkmark$ (1)

5.2



(3)

5.3



(3)

5.4

$$\ln \sqrt{x-1} + \ln \sqrt{x+1} = a$$

$$\ln \sqrt{(x-1)(x+1)} = a \quad \checkmark$$

$$\frac{1}{2} \ln (x^2 - 1) = a$$

$$\ln (x^2 - 1) = 2a \quad \checkmark$$

$$x^2 - 1 = e^{2a}$$

$$x^2 = e^{2a} + 1 \quad \checkmark$$

$$x = \pm \sqrt{e^{2a} + 1} \quad \checkmark$$

(4)

5.5

$$27(k-1)^3 - 8(k+1)^3$$

$$\underbrace{[3(k-1) - 2(k+1)]}_{\checkmark} \underbrace{\left[(3(k-1))^2 + 6(k-1)(k+1) + (2(k+1))^2 \right]}_{\checkmark\checkmark}$$

(3)

5.6 $3^{2x-1} = \ln 5$
 $\ln 3(2x-1) = \ln 5$
 $2,197x - 1,099 = 1,609 \checkmark$
 $x = \frac{2,708}{2,197} \checkmark$
 $= 1,233 \checkmark$

(3)
[20]

TOTAL: 100