



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N3

22 November 2024

This marking guideline consists of 12 pages.

MARKING LEGEND	
SYMBOL	EXPLANATION OF SYMBOL

QUESTION 1

			<u>MARKING INSTRUCTIONS</u>	
1.1	$(3x^2 + 7x)^2 - 4(3x^2 + 7x) - 12$ $= (3x^2 + 7x - 6)(3x^2 + 7x + 2)$ $= (3x - 2)(x + 3)(3x + 1)(x + 2)$	<u>ALTERNATIVE SOLUTION</u> $(3x^2 + 7x)^2 - 4(3x^2 + 7x) - 12$ Let $a = 3x^2 + 7x$ Then $a^2 - 4a - 12$ $= (a - 6)(a + 2)$ $= (3x^2 + 7x - 6)(3x^2 + 7x + 2)$ $= (3x - 2)(x + 3)(3x + 1)(x + 2)$	1M for each correct factor 1 M for each correct factor when fully factorising the product.	(6)
1.2	$f(x) = x^3 - 2x^2 - 5x + 6$ $\therefore f(1) = (1)^3 - 2(1)^2 - 5(1) + 6$ $= 1 - 2 - 5 + 6$ $= 0 \quad \checkmark$ Therefore $x - 1$ is a factor of $f(x)$ $\begin{array}{r} \checkmark \qquad \qquad x^2 - x - 6 \quad \checkmark \\ x - 1 \overline{) x^3 - 2x^2 - 5x + 6} \\ \underline{x^3 - x^2} \\ -x^2 - 5x \\ \underline{-x^2 + x} \quad \checkmark \\ -6x + 6 \\ \underline{-6x + 6} \end{array}$ $f(x) = (x - 1)(x^2 - x - 6)$ $= (x - 1)(x - 3)(x + 2)$ Synthetic division can also be used in the place of long division $\begin{array}{r rrrr} 1 & 1 & -2 & -5 & 6 \\ & & 0 & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$	<u>ALTERNATIVE SOLUTION</u> $f(x) = x^3 - 2x^2 - 5x + 6$ $\therefore f(3) = (1)^3 - 2(1)^2 - 5(1) + 6$ $= 1 - 2 - 5 + 6$ $= 0 \quad \checkmark$ Therefore $x - 3$ is a factor of $f(x)$ $\begin{array}{r} \checkmark \qquad \qquad x^2 + x - 2 \quad \checkmark \\ x - 3 \overline{) x^3 - 2x^2 - 5x + 6} \\ \underline{x^3 - 3x^2} \\ x^2 - 5x \\ \underline{x^2 - 3x} \quad \checkmark \\ -2x + 6 \\ \underline{-2x + 6} \end{array}$ $f(x) = (x - 3)(x^2 + x - 2)$ $= (x - 3)(x + 2)(x - 1)$	1 mark for using the Remainder Theorem to find the first factor. 1 mark for first factor 2 marks for correctly using long division, synthetic division, or trial and error -1 M for dividend and 1 M for method. 2 marks for accurately factorising the dividend	(6)

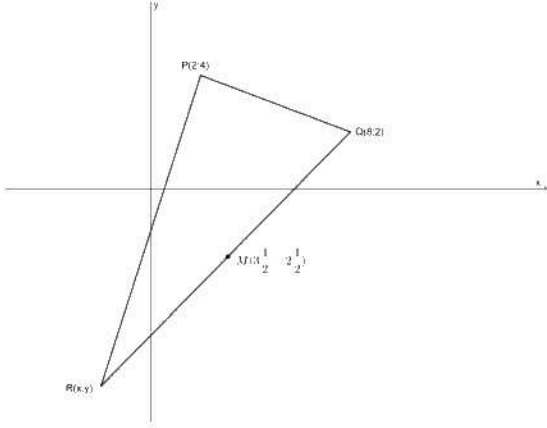
			MARKING INSTRUCTIONS	
1.3	$2x^2 - 3x = 5$ $\therefore x^2 - \frac{3}{2}x = \frac{5}{2} \quad \checkmark$ $\therefore x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{9}{16} + \frac{5}{2} \quad \checkmark$ $\therefore \left(x - \frac{3}{4}\right)^2 = \frac{9 + 40}{16} \quad \checkmark$ $\therefore \left(x - \frac{3}{4}\right)^2 = \frac{49}{16}$ $\therefore x - \frac{3}{4} = \pm \frac{7}{4} \quad \checkmark$ $\therefore x = +\frac{3}{4} + \frac{7}{4} \quad \text{or} \quad x = +\frac{3}{4} - \frac{7}{4}$ $\therefore x = \frac{10}{4} \quad \text{or} \quad x = -\frac{4}{4}$ $\therefore x = \frac{5}{2} \quad \checkmark \quad \text{or} \quad x = -1 \quad \checkmark$		1M divide both sides by coefficient of x^2 1M for adding $(\frac{1}{2} \times \text{coefficient of } x)^2$ to both sides of the equation 1 M for writing the LHS as a perfect square trinomial 1 M for determining the square root of both sides of the equation 1M for each correct x value	(6)
				[18]

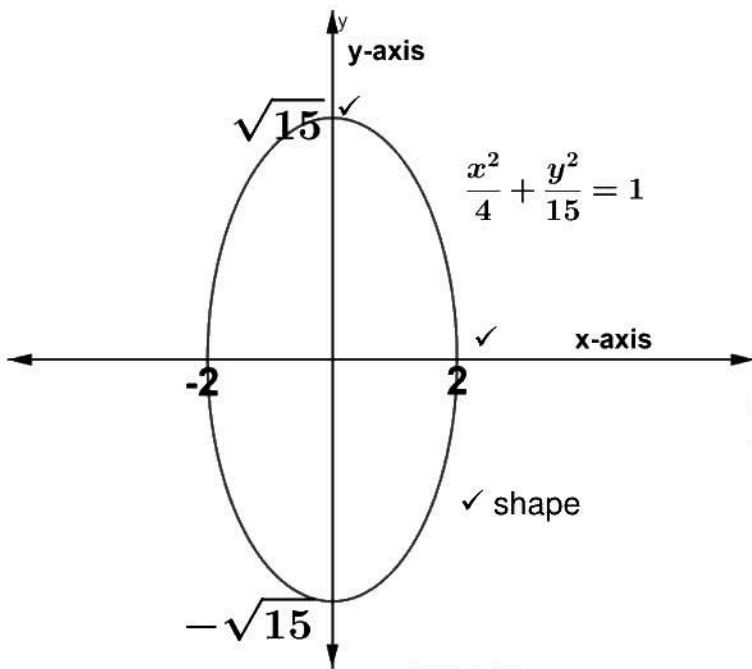
QUESTION 2

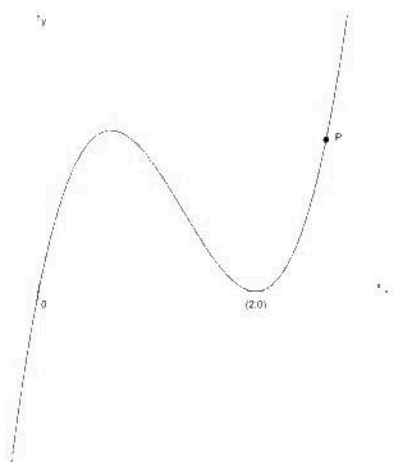
2.1	$\frac{x^{\frac{1}{2}} - 64x^{-\frac{1}{2}}}{x^{\frac{1}{2}} - 16 + 64x^{-\frac{1}{2}}}$ $= \frac{x^{\frac{1}{2}} - \frac{64}{x^{\frac{1}{2}}}}{x^{\frac{1}{2}} - 16 + \frac{64}{x^{\frac{1}{2}}}} \quad \checkmark$ $= \frac{x - 64}{x - 16x^{\frac{1}{2}} + 64} \quad \checkmark$ $= \frac{(x^{\frac{1}{2}} - 8)(x^{\frac{1}{2}} + 8)}{(x^{\frac{1}{2}} - 8)(x^{\frac{1}{2}} - 8)} \quad \checkmark$ $= \frac{x^{\frac{1}{2}} + 8}{x^{\frac{1}{2}} - 8} \quad \checkmark$	ALTERNATIVE SOLUTION $\frac{x^{\frac{1}{2}} - 64x^{-\frac{1}{2}}}{x^{\frac{1}{2}} - 16 + 64x^{-\frac{1}{2}}}$ $= \frac{\left(x^{\frac{1}{4}}\right)^2 - \left(8x^{-\frac{1}{4}}\right)^2}{\left(x^{\frac{1}{4}}\right)^2 - 16 + \left(8x^{-\frac{1}{4}}\right)^2} \quad \checkmark$ $= \frac{\left(x^{\frac{1}{4}} - 8x^{-\frac{1}{4}}\right)\left(x^{\frac{1}{4}} + 8x^{-\frac{1}{4}}\right)}{\left(x^{\frac{1}{4}} - 8x^{-\frac{1}{4}}\right)\left(x^{\frac{1}{4}} - 8x^{-\frac{1}{4}}\right)} \quad \checkmark$ $= \frac{x^{\frac{1}{4}} + 8x^{-\frac{1}{4}}}{x^{\frac{1}{4}} - 8x^{-\frac{1}{4}}} \quad \checkmark$	2 M for simplifying 2 M for factorising all numerators and denominators 1M for answer	(5)
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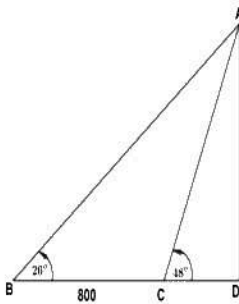
			<u>MARKING INSTRUCTIONS</u>	
2.2	$\log_{\frac{1}{2}}(3-x) = \log_2\left(\frac{1}{x^2}\right) + 2$ $\log_{\frac{1}{2}}(3-x) = \frac{\log_{\frac{1}{2}}\left(\frac{1}{x^2}\right)}{\log_{\frac{1}{2}}2} + 2 \quad \checkmark$ $\log_{\frac{1}{2}}(3-x) = \frac{\log_{\frac{1}{2}}\left(\frac{1}{x^2}\right)}{-1} + 2$ $\log_{\frac{1}{2}}(3-x) = -\log_{\frac{1}{2}}\left(\frac{1}{x^2}\right) + 2 \quad \checkmark$ $\log_{\frac{1}{2}}(3-x) + \log_{\frac{1}{2}}\left(\frac{1}{x^2}\right) = 2$ $\log_{\frac{1}{2}} \frac{(3-x)}{x^2} = +2 \quad \checkmark$ $\frac{(3-x)}{x^2} = \left(\frac{1}{2}\right)^2 \quad \checkmark$ $12 - 4x = x^2$ $x^2 + 4x - 12 = 0 \quad \checkmark$ $(x+6)(x-2) = 0$ $x \neq -6 \text{ or } x = 2 \quad \checkmark$	<u>ALTERNATIVE SOLUTION</u> $\log_{\frac{1}{2}}(3-x) = \log_2\left(\frac{1}{x^2}\right) + 2$ $\frac{\log_2(3-x)}{\log_2\left(\frac{1}{2}\right)} = \log_2\left(\frac{1}{x^2}\right) + 2$ $\frac{\log_2(3-x)}{\log_2 2^{-1}} = \log_2\left(\frac{1}{x^2}\right) + 2$ $-\log_2(3-x) = \log_2\left(\frac{1}{x^2}\right) + 2 \quad \checkmark$ $-\log_2(3-x) - \log_2(x^{-2}) = 2$ $-\log_2(3-x) + \log_2 x^2 = 2 \quad \checkmark$ $\log_2 \frac{x^2}{(3-x)} = +2 \quad \checkmark$ $2^2 = \frac{x^2}{(3-x)}$ $4 = \frac{x^2}{(3-x)} \quad \checkmark$ $12 - 4x = x^2$ $x^2 + 4x - 12 = 0 \quad \checkmark$ $(x+6)(x-2) = 0$ $x \neq -6 \text{ or } x = 2 \quad \checkmark$	1M -writing the logs to the base 1/2 or 2 by using the log law [$\log_b a = \frac{\log_c a}{\log_c b}$] 1M simplifying 1M -transpose log-term to LHS 1M-write the LHS as single logarithm 1M -rewrite in exponential form 1 M -write in general form 1M-valid root 1M-invalid root	(8)
				[13]

QUESTION 3				
			<u>MARKING INSTRUCTIONS</u>	
3.1	$A = De^{rt}$ $\frac{A}{D} = e^{rt} \quad \checkmark$ $\ln \frac{A}{D} = \ln e^{rt} \quad \checkmark$ $\ln \frac{A}{D} = rt \ln e$ $\ln \frac{A}{D} = rt \quad \checkmark$ $\frac{\ln \frac{A}{D}}{r} = t \quad \checkmark$ or $\frac{\ln A - \ln D}{r} = t$	<u>ALTERNATIVE SOLUTION</u> $A = De^{rt}$ $\frac{A}{D} = e^{rt} \quad \checkmark$ $\log \frac{A}{D} = \log e^{rt} \quad \checkmark$ $\log \frac{A}{D} = rt \log e \quad \checkmark$ $\frac{\log \frac{A}{D}}{r \log e} = t \quad \checkmark$ or $\frac{\log A - \log D}{r \log e} = t$	1M -divide both sides by D 1M write the logarithm ln on both sides 1M simplification 1M final answer	(4)
3.2	Let the number be x . $\therefore 3x^2 + 7x = 20 \quad \checkmark$ $\therefore 3x^2 + 7x - 20 = 0$ $\therefore (3x - 5)(x + 4) = 0 \quad \checkmark$ $\therefore 3x = 5 \text{ or } x = -4$ $\therefore x = \frac{5}{3} \checkmark \text{ or } x = -4 \quad \checkmark$		1M -set up an equation 1M -Solve the equation 1M for each correct root	(4)
3.3	$\frac{3}{x-1} - \frac{1}{x^2-4x+3} - \frac{1}{1-x} = 1$ $\therefore \frac{3}{x-1} - \frac{1}{(x-1)(x-3)} + \frac{1}{x-1} = 1$ $\therefore 3(x-3) - 1 + (x-3) = (x-3)(x-1)$ $\therefore 3x - 9 - 1 + x - 3 = x^2 - 4x + 3$ $\therefore x^2 - 8x + 16 = 0 \quad \checkmark$ $\therefore (x-4)(x-4) = 0$ $\therefore x = 4 \text{ or } x = 4 \quad \checkmark$		2M -factorising 2 nd and 3 rd fraction ½ M for multiplying each term by the LCD 1M write it in general form 1M for writing solution	(6)
				[14]

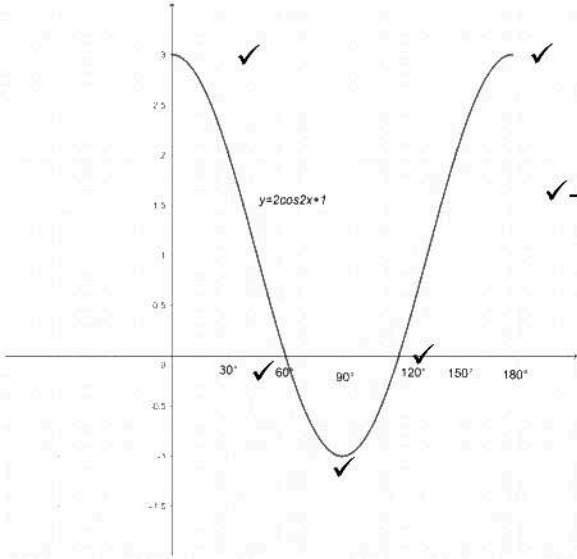
QUESTION 4		
	<u>MARKING INSTRUCTIONS</u>	
		
4.1	$M_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{4-2}{2-8} \quad \checkmark$ $= \frac{2}{-6} \quad \checkmark$ $= -\frac{1}{3}$ $\theta = \tan^{-1}\left(-\frac{1}{3}\right) = 180^\circ - 18,43^\circ \quad \checkmark$ $\theta = 161,6^\circ \quad \checkmark$	1M SF (substituting in the correct formula) 1M answer 1M CA(follow up from gradient) 1M answer (4)
4.2	$x = \frac{x_1+x_2}{2} \quad y = \frac{y_1+y_2}{2}$ $\frac{7}{2} = \frac{8+x}{2} \quad \checkmark \quad -\frac{5}{2} = \frac{2+y}{2} \quad \checkmark$ $x = -1 \quad \checkmark \quad y = -7 \quad \checkmark$ $\therefore$ The coordinates of R = (-1; -7)	1M for x and one marks for y, substituting in the correct formula 1 M each for answers (4)
4.3	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $PQ = \sqrt{(8 - 2)^2 + (2 - 4)^2} \quad \checkmark$ $= \sqrt{36 + 4} = \sqrt{40} = 6,325 \text{ or } 2\sqrt{5} \quad \checkmark$	1M SF 1 M for answer (2)

QUESTION 5		MARKING INSTRUCTIONS	
5.1	$\frac{x^2}{4} + \frac{y^2}{15} = 1$ 	1M : x-intercepts at (2; 0), (-2; 0) 1M: y-intercept at (0; $\sqrt{15}$) ; (0; $-\sqrt{15}$) 1M for shape	(3)
5.2	$y = \frac{\sqrt{x}}{2} + \frac{2}{\sqrt[4]{x}}$ $y = \frac{x^{\frac{1}{2}}}{2} + \frac{2}{x^{\frac{1}{4}}}$ $= \frac{1}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{4}}$ $\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{2}x^{-\frac{1}{2}} + 2 \times \left(-\frac{1}{4}\right)x^{-\frac{5}{4}}$ $= \frac{1}{4x^{\frac{1}{2}}} - \frac{1}{2x^{\frac{5}{4}}}$ $= \frac{1}{4\sqrt{x}} - \frac{1}{2\sqrt[4]{x^5}}$	½ marks for each term , converting the radical to fractional form(the preparation step) 1 Mark for differentiating each term, using the power rule Deduct 1 mark if the derivative notation is placed incorrectly or is missing ½ mark for each correct derivative in radical form	

		<u>MARKING INSTRUCTIONS</u>	
5.3			
5.3.1	$f(x) = x(x-2)(x-2) \quad \checkmark$ $f(x) = x(x^2 - 4x + 4)$ $f(x) = x^3 - 4x^2 + 4x \quad \checkmark$ From $f(x) = x^3 + bx^2 + cx$ follows that $b = -4$ and $c = 4 \quad \checkmark$	1M SF 1M simplify 1 mark for correctly reading off the values of b and c .	(3)
5.3.2	$f(x) = x^3 - 4x^2 + 4x$ $\therefore f'(x) = 3x^2 - 8x + 4$ $\therefore 20 = 3x^2 - 8x + 4$ $\therefore 3x^2 - 8x - 16 = 0$ $\therefore (3x + 4)(x - 4) = 0 \quad \checkmark$ $\therefore 3x = -4 \quad \text{or } x = 4$ $\therefore x \neq -\frac{4}{3} \quad x = 4 \quad \checkmark$ If $x = 4$ then $f(4) = (4)^3 - 4(4)^2 + 4(4)$ $\quad \quad \quad = 64 - 64 + 16$ $\quad \quad \quad = 16 \quad \checkmark$ The coordinates of P are (4; 16)	1M for derivative and 1 mark for gradient 1M factorisation 1M for valid x 1M for y	(5)

QUESTION 6			
		<u>MARKING INSTRUCTIONS</u>	
6.1	$\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$ $\text{LHS} = \sin^4 x - \cos^4 x$ $= (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) \quad \checkmark$ $= 1 \times (\sin^2 x - \cos^2 x)$ $= 1 - \cos^2 x - \cos^2 x \quad \checkmark$ $= 1 - 2\cos^2 x \quad \checkmark$ $= \text{RHS}$	1M factorisation 1M for applying the square identities 2M simplification	(4)
6.2	$5\sin\theta = 8\cos\theta$ $\therefore \frac{\sin\theta}{\cos\theta} = \frac{8}{5} \quad \checkmark$ $\therefore \tan\theta = 1,6 \quad \checkmark$ $\theta = 57,995^\circ \text{ or } \theta = 180^\circ + 57,995^\circ$ $\theta = 57,995^\circ \checkmark \text{ or } \theta = 237,995^\circ \quad \checkmark$	1M cross multiply 1M convert to one ratio 2M for correct angles	(4)
6.3	6.3.1 In $\triangle ABC$: $\widehat{BAC} = 48^\circ - 26^\circ = 22^\circ \quad \checkmark$ $\therefore \frac{AC}{\sin 26^\circ} = \frac{800}{\sin 22^\circ} \quad \checkmark$ $\therefore AC = \frac{800 \sin 26^\circ}{\sin 22^\circ}$ $\therefore AC = 936,174 \quad \checkmark$ In $\triangle ACD$: $\frac{AD}{AC} = \sin 48^\circ \quad \checkmark$ $AD = 936,174 \times \sin 48^\circ$ $= 695,713m \quad \checkmark$	 1M SF 1 mark for calculating angle $\angle BAC$ either in the formula or in the initial steps. 1M value of A 1M SF 1M value of AD	(5)

				<u>MARKING INSTRUCTIONS</u>	
	6.3.2	In ΔACD: $\frac{CD}{AD} = \cot 48^\circ$ $CD = 695,713 \times \cot 48^\circ \quad \checkmark \quad \therefore CD = 936,174 \times \cos 48^\circ \quad \checkmark$ $= 626,423m \quad \checkmark$ <u>ALTERNATIVE SOLUTION 2</u> In ΔABD $\tan 26^\circ = \frac{AD}{BD}$ $\therefore \tan 26^\circ = \frac{695,713}{BD}$ $\therefore BD = \frac{695,713}{\tan 26^\circ}$ $\therefore BD = 1426,423 \quad \checkmark$ $\therefore CD = 1426,423 - 800$ $= 626,423m \quad \checkmark$	<u>ALTERNATIVE SOLUTION 1</u> $\frac{CD}{AC} = \cos 48^\circ$ $= 626,423m \quad \checkmark$	1 M SF 1M answer	(2)

6.4	$y = 2\cos 2x + 1$ 	2 Marks for x-intercepts $(60^\circ; 0)$ and $(120^\circ; 0)$ 1 Mark for y-intercept $(0^\circ; 3)$ 2 Marks for turning points $(90^\circ; -1)$ and $(180^\circ; 3)$ The other turning point is already described under the y-intercept (do not count it again) 1M shape- The graph is drawn over the interval 0° and 180° The graph have a minimum TP at $(90^\circ; -1)$ and maximum TPs at $(0^\circ; 3)$ and $(180^\circ; 3)$ and x-intercepts at $(60^\circ; 0)$ and $(120^\circ; 0)$	(6)
		TOTAL:	[21]

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