



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N3

3 August 2023

This marking guideline consists of 15 pages.

MARKING LEGEND	
SYMBOL	EXPLANATION OF SYMBOL
A	Accuracy
M	Method
CA	Consistent accuracy (follow up)
B/D	Breakdown
S	Statement
R	Reason
S/R	Statement and reason
F	Choosing correct formula
SF	Substitution into correct formula

QUESTION 1

		<u>SOLUTION</u>		<u>EXPLANATION</u>	
1.1	1.1.1	$ab^2 - b + m(1 - ab)$ $= b(ab - 1) - m(ab - 1) \checkmark$ $= (ab - 1)(b - m) \checkmark$ <u>ALTERNATIVE SOLUTION 1</u> $ab^2 - b + m(1 - ab)$ $= ab^2 - b + m - mab$ $= ab(b - m) - (b - m) \checkmark$ $= (ab - 1)(b - m) \checkmark$	<u>ALTERNATIVE SOLUTION 2</u> $ab^2 - b + m(1 - ab)$ $= m(1 - ab) - b(1 - ab) \checkmark$ $= (1 - ab)(m - b) \checkmark$	1M each taking out CF 1 factorising 1M each CF 1M A	(3)
	1.1.2	$x^4 - 2x^3 - 3x^2 + 4x + 2$ $= (x^4 - 3x^2 + 2) - 2x^3 + 4x \checkmark$ $= (x^2 - 2)(x^2 - 1) - 2x(x^2 - 2) \checkmark$ $= (x^2 - 2)[(x^2 - 1) - 2x] \checkmark$ $= (x^2 - 2)[x^2 - 2x - 1] \checkmark$		½ mark for grouping 1M for factorising a trinomial and 1 mark for taking out the CF 1M for taking out the CF and ½ mark for simplifying the bracket	(4)

	<u>SOLUTION</u>		<u>EXPLANATION</u>	
1.2	$f(x) = 6x^3 + 4x^2 + px + 3$ $2x + 1 = 0$ $x = -\frac{1}{2} \checkmark$ $R = f(-\frac{1}{2}) = 6(-\frac{1}{2})^3 + 4(-\frac{1}{2})^2 + p(-\frac{1}{2}) + 3 \checkmark$ $\therefore 10 = 6(-\frac{1}{2})^3 + 4(-\frac{1}{2})^2 + p(-\frac{1}{2}) + 3$ $\therefore 10 = 6(-\frac{1}{8}) + 1 - \frac{p}{2} + 3 \checkmark$ $\therefore 10 = -\frac{3}{4} + 4 - \frac{p}{2}$ $\therefore 40 = -3 + 16 - 2p \checkmark$ $\therefore 2p = -27$ $\therefore p = -\frac{27}{2} \checkmark$ or $p = -13\frac{1}{2}$ or $p = -13,5$		1M determining value of x 1M substitution 1M simplifying (any step) 1M further simplification 1M A Any one of the last 3 answers should be taken as correct.	(5)
1.3	$\frac{b^2 + 3b + 2(b+2)}{2b^2 + b - 3} \times \frac{4b^2 - 9}{9b + 36} \times \frac{1 - 2b + b^2}{2b^2 - 5b + 3}$ $= \frac{b^2 + 5b + 4 \checkmark}{(2b+3)(b-1)} \times \frac{(2b+3)(2b-3)}{9(b+4)} \times \frac{(b-1)(b-1)}{(2b-3)(b-1)}$ $= \frac{(b+4)(b+1) \checkmark}{(2b+3)(b-1) \checkmark} \times \frac{(2b+3)(2b-3) \checkmark}{9(b+4) \checkmark} \times \frac{(b-1)(b-1) \checkmark}{(2b-3)(b-1) \checkmark}$ $= \frac{b+1}{9} \checkmark$		½ M-simplifying 1M for factorising quadratic trinomials and ½ M for taking out the CF - for factorising each numerator and denominator 1M A	(7)
				[19]

QUESTION 2

		<u>SOLUTION</u>		<u>EXPLANATION</u>	
2.1	2.1.1	$\frac{\sqrt{2x+1} + 2\sqrt{2x-1}}{\sqrt{2x+1} + \sqrt{2x-1} + x} = 1$ $\therefore \sqrt{2x+1} + 2\sqrt{2x-1} = \sqrt{2x+1} + \sqrt{2x-1} + x$ $\therefore \sqrt{2x+1} - \sqrt{2x+1} + 2\sqrt{2x-1} - \sqrt{2x-1} = x$ $\therefore \sqrt{2x-1} = x \quad \checkmark$ $2x-1 = x^2$ $x^2 - 2x + 1 = 0 \quad \checkmark$ $\therefore (x-1)(x-1) = 0$ $\therefore x-1 = 0 \text{ or } x-1 = 0$ $x = 1 \quad \checkmark \quad x = 1$ $TEST : LHS = \frac{\sqrt{2(1)+1} + 2\sqrt{2(1)-1}}{\sqrt{2(1)+1} + \sqrt{2(1)-1} + (1)} = 1$ Hence, $x = 1$ is the solution	✓	1 M cross multiplication 1 M simplifying 1M applying exponential /surd rules 1M A The students do not need to show the last part (where we prove that $x = 1$ is a valid solution)	(4)
	2.1.2	$\log_3 x = 2 + 15 \log_x 3$ $\therefore \log_3 x = 2 + 15 \times \frac{1}{\log_3 x} \quad \checkmark$ $\therefore (\log_3 x)^2 = 2 \log_3 x + 15 \quad \checkmark$ $\therefore (\log_3 x)^2 - 2 \log_3 x - 15 = 0$ $\therefore (\log_3 x - 5)(\log_3 x + 3) = 0 \quad \checkmark$ $\therefore \log_3 x - 5 = 0 \text{ or } \log_3 x + 3 = 0$ $\therefore \log_3 x = 5 \quad \log_3 x = -3$ $\therefore x = 3^5 \quad \text{or} \quad x = 3^{-3}$ $\therefore x = 243 \quad \checkmark \quad \therefore x = \frac{1}{3^3}$ $x = \frac{1}{27} \quad \checkmark$		1M applying rule on changing the base 1M cross-multiplication 1M factorisation 1M each for solutions of x	(5)

	<u>SOLUTION</u>		<u>EXPLANATION</u>	
2.2	$\frac{16x - 9y}{4x^{\frac{1}{2}} + 3y^{\frac{1}{2}}}$ $= \frac{\left(4x^{\frac{1}{2}} + 3y^{\frac{1}{2}}\right)\left(4x^{\frac{1}{2}} - 3y^{\frac{1}{2}}\right)}{4x^{\frac{1}{2}} + 3y^{\frac{1}{2}}} \quad \checkmark$ $= 4x^{\frac{1}{2}} - 3y^{\frac{1}{2}} \quad \text{or} \quad 4\sqrt{x} - 3\sqrt{y} \quad \checkmark$		1M factorisation 1M A	(2)
				[11]

QUESTION 3

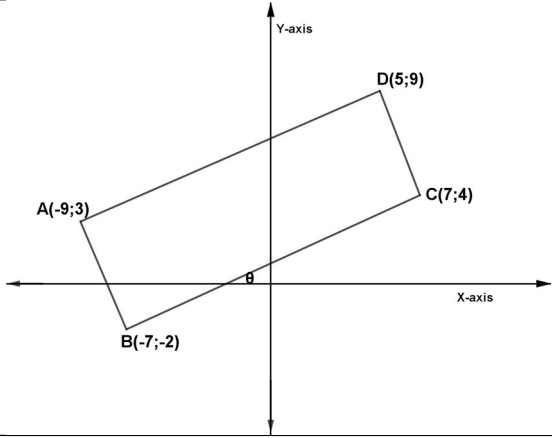
	<u>SOLUTION</u>		<u>EXPLANATION</u>	
3.1	Let one side of the first(smaller) square be x. $\checkmark$ $\therefore$ then the side of the second(larger) square will be $x + \frac{x}{2} = \frac{3x}{2} \quad \checkmark$ The perimeter of the first square is $4 \times$ length or $4x$ $\therefore$ Perimeter of the first(smaller) square $= 4x \quad \checkmark$ Perimeter of the second(larger) square $\therefore 4\left(\frac{3x}{2}\right) = 6x \quad \checkmark$ $\therefore 6x - 4x = 12 \quad \checkmark$ $2x = 12$ $x = 6$ The length of the side of the smaller square $= 6$ m and the length of the larger square $= 9$ m. $\checkmark$		1M [$\frac{1}{2}$ mark each for expressing the sides in terms of x] 2M writing perimeters in terms of x(reasoning) 1M equating (reasoning) 2M answers	

	<u>SOLUTION</u>		<u>EXPLANATION</u>	
	<u>ALTERNATIVE SOLUTION</u> Let the side of the (first)larger square = x ✓ then the side of the (second) smaller square = $\frac{2}{3}x$ ✓ $\therefore 4x - 4 \times \frac{2}{3}x = 12$ ✓ $\therefore 12x - 8x = 36$ $\therefore 4x = 36$ ✓ $\therefore x = 9$ $\therefore$ side of (first)larger square = $9cm$ ✓ side of (second) smaller square = $\frac{2}{3}(9) = 6cm$ ✓		1M each for writing the sides in terms of x 1M to equating the difference of the perimeters in terms of x (reasoning) 1M simplifying 2M answers	(6)
3.2	$A = (a+b)^{2x}$ $\log A = \log (a+b)^{2x}$ ✓ $\log A = 2x \log(a+b)$ ✓ $\frac{\log A}{2 \log(a+b)} = x$ ✓	<u>ALTERNATIVE SOLUTION</u> $A = (a+b)^{2x}$ $\ln A = \ln (a+b)^{2x}$ ✓ $\ln A = 2x \ln(a+b)$ ✓ $\frac{\ln A}{2 \ln(a+b)} = x$ ✓	1M introducing log 1M applying log law 1M A	(3)

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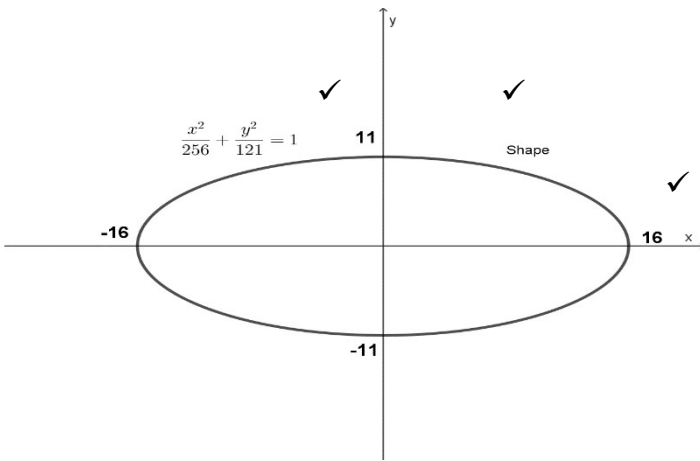
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QUESTION 4

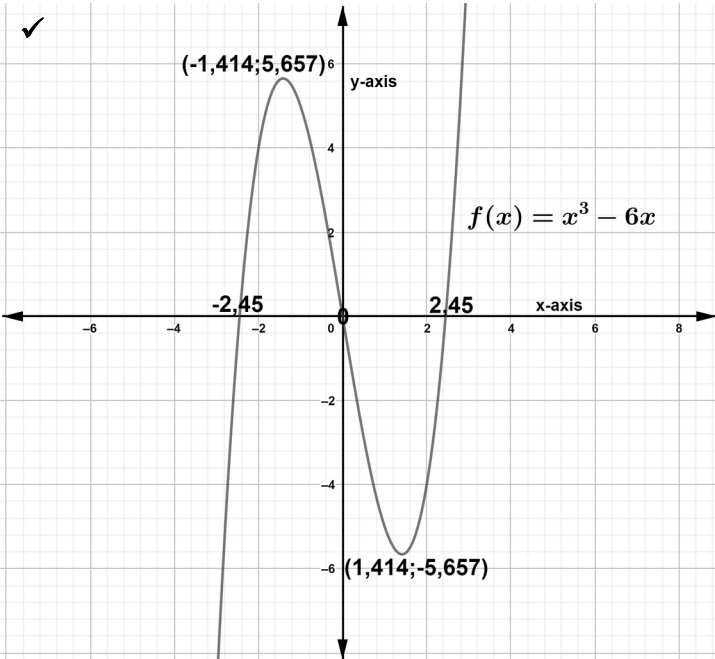
		<u>SOLUTION</u>		<u>EXPLANATION</u>	
4.1					
4.1.1		$M_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{9 - 3}{5 - (-9)} \checkmark$ $= \frac{6}{14}$ $= \frac{3}{7} \checkmark$		1M substitution 1M A	(2)
4.1.2		$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5 - (-9))^2 + (9 - 3)^2} \checkmark$ $= \sqrt{(14)^2 + (6)^2}$ $= \sqrt{196 + 36}$ $= \sqrt{232}$ $= 2\sqrt{58} \checkmark$		1M substitution 1M A	(2)
4.1.3		$M_{BC} = M_{AB} = \frac{3}{7} \checkmark \text{ (lines } \square \text{)}$ $y - y_1 = m(x - x_1)$ $y + 2 = \frac{3}{7}(x + 7) \checkmark$ $y = \frac{3}{7}x + 3 - 2$ $y = \frac{3}{7}x + 1 \checkmark$		1M for gradient, NOTE: Carry over the mistake in QUESTION 4.1.1) [no marks for reason] 1M substitution 1M A	(3)

		<u>SOLUTION</u>		<u>EXPLANATION</u>	
	4.1.4	$\theta = \tan^{-1} m$ $= \tan^{-1} \left(\frac{3}{7} \right) \checkmark$ $= 23,199^\circ \checkmark$		1M substitution 1M CA	(2)
	4.1.5	$y - y_1 = m(x - x_1)$ $M_{AD} = \frac{3}{7}$ $\therefore \frac{3}{7} \times m = -1 \quad (\text{lines } \perp)$ $\therefore m = -\frac{7}{3} \quad \checkmark$ $\therefore y - 3 = -\frac{7}{3}(x + 9) \quad \checkmark$ $\therefore y - 3 = -\frac{7}{3}x - \frac{63}{3}$ $\therefore y = -\frac{7}{3}x - 21 + 3 \quad \checkmark$ $\therefore y = -\frac{7}{3}x - 18 \quad \checkmark$		1M determining gradient NOTE: Carry over mistake from 4.1.1 (if gradient is wrongly calculated). 1M substitution 1M simplifying 1M writing answer in the form $y=mx+c$	(4)
4.2		Midpoint $AC = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $= \left(\frac{-9 + 7}{2}, \frac{3 + 4}{2} \right) \checkmark$ $= \left(-1; \frac{7}{2} \right) \text{ or } (-1; 3,5) \quad \checkmark$ Midpoint $BD = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $= \left(\frac{-7 + 5}{2}, \frac{-2 + 9}{2} \right) \checkmark$ $= \left(-1; \frac{7}{2} \right) \text{ or } (-1; 3,5) \quad \checkmark$ The midpoints of AC and BD are common and at the same point $\left(-1; \frac{7}{2} \right)$ which means they divide each other into two equal parts (bisect each other).		1M substitution 1M CA 1M substitution 1M CA 1M reasoning	(5)
					[18]

QUESTION 5

	<u>SOLUTION</u>		<u>EXPLANATION</u>	
5.1	$\frac{x^2}{256} + \frac{y^2}{121} = 1$ $\frac{x^2}{(16)^2} + \frac{y^2}{(11)^2} = 1$ 		1M for the shape If the shape is not correct-no marks, not even for the intercepts) 1M each for the x- and y- intercept if the shape is correct	(3)
5.2	$y = \frac{1}{x} + 2\sqrt{x} - \frac{1}{\sqrt{x}}$ $y = x^{-1} + 2x^{\frac{1}{2}} - x^{-\frac{1}{2}}$ $\frac{dy}{dx} = -x^{-2} + 2 \times \frac{1}{2} x^{-\frac{1}{2}} - \left(-\frac{1}{2} x^{-\frac{3}{2}} \right)$ $= -x^{-2} + x^{-\frac{1}{2}} + \frac{1}{2} x^{-\frac{3}{2}}$ $= -\frac{1}{x^2} + \frac{1}{x^{\frac{1}{2}}} + \frac{1}{2x^{\frac{3}{2}}}$ $= -\frac{\sqrt{1}}{x^2} + \frac{1}{\sqrt{x}} + \frac{\sqrt{1}}{2\sqrt{x^3}}$		1½ M standard form 1M for each correct derivative 1½ M for writing it in the correct form Penalise the students with 1 mark if $\frac{dy}{dx}$ was omitted	(6)

		<u>SOLUTION</u>		<u>EXPLANATION</u>	
5.3	5.3.1	$f(x) = x^3 - 6x$ $f'(x) = 3x^2 - 6$ ✓ For TP's, let $f'(x) = 0$ ✓ $\therefore 3x^2 - 6 = 0$ $3x^2 = 6$ $x = \pm\sqrt{2}$ $\therefore x = 1,414$ ✓ or $x = -1,414$ ✓ If $x = 1,414$, then $f(x) = (1,414)^3 - 6(1,414)$ $= -5,657$ ✓ If $x = -1,414$, then $f(x) = (-1,414)^3 - 6(-1,414)$ $= 5,657$ ✓ $\therefore$ TP's are $(1,414; -5,657)$ and $(-1,414; 5,657)$	<u>ALTERNATIVE SOLUTION</u> If $x = \sqrt{2}$, then $f(x) = (\sqrt{2})^3 - 6(\sqrt{2})$ $= 2\sqrt{2} - 6\sqrt{2}$ $= -4\sqrt{2}$ ✓ $\approx -5,657$ If $x = -\sqrt{2}$, then $f(x) = (-\sqrt{2})^3 - 6(-\sqrt{2})$ $= -2\sqrt{2} + 6\sqrt{2}$ $= 4\sqrt{2}$ ✓ $\approx 5,657$ $\therefore$ TP's are $(\sqrt{2}; -4\sqrt{2})$ and $(-\sqrt{2}; 4\sqrt{2})$	1M differentiation 1M for applying knowledge on TP's 2 M for x-solutions 2M for y-solutions	(6)

	<u>SOLUTION</u>	<u>EXPLANATION</u>	
5.3.2	x-intercepts: let $y = 0$ Then $x^3 - 6x = 0$ $\therefore x(x^2 - 6) = 0$ $\therefore x = 0$ or $x^2 = 6$ $\therefore x = 0$ or $x = \pm\sqrt{6}$ $\therefore x = 0$ or $x = \pm 2,45$ y-intercept: let $x = 0$ then $y = 0$ ✓ 	4 M -1 marks each for the 2 points of intercepts and 2 turning points	(4)
			[19]

QUESTION 6

	<u>SOLUTION</u>		<u>EXPLANATION</u>	
6.1	$\left(\tan x + \frac{1}{\cos x} \right)^2 = \frac{1 + \sin x}{1 - \sin x}$ $LHS = \left(\tan x + \frac{1}{\cos x} \right)^2$ $= \left(\frac{\sin x}{\cos x} + \frac{1}{\cos x} \right)^2 \quad \checkmark$ $= \left(\frac{\sin x + 1}{\cos x} \right)^2 \quad \checkmark$ $= \frac{(\sin x + 1)^2}{\cos^2 x} \quad \checkmark$ $= \frac{(\sin x + 1)^2}{1 - \sin^2 x} \quad \checkmark$ $= \frac{(\sin x + 1)(\sin x + 1)}{(1 + \sin x)(1 - \sin x)}$ $= \frac{\sin x + 1}{1 - \sin x} \quad \checkmark$ $= RHS$		1 M changing tan x 1M adding over LCD 1M applying exponent and surd rules 1M applying square trigonometrical law 1M Simplify	(5)

	<u>SOLUTION</u>	<u>ALTERNATIVE SOLUTION</u>	<u>EXPLANATION</u>	
6.2	$5 \cos \beta = 2 \sin \beta$ $\therefore \frac{5 \cos \beta}{\cos \beta} = \frac{2 \sin \beta}{\cos \beta} \quad \checkmark$ $\therefore 5 = \frac{2 \sin \beta}{\cos \beta}$ $\therefore 2 \tan \beta = 5 \quad \checkmark$ $\therefore \tan \beta = \frac{5}{2}$ $\therefore \beta = \tan^{-1}\left(\frac{5}{2}\right) \quad \checkmark$ $\quad \quad \quad = 68,199^\circ \quad \checkmark$ <i>or</i> $\beta = 180^\circ + 68,199^\circ$ $\quad \quad \quad = 248,199^\circ \quad \checkmark$	$5 \cos \beta = 2 \sin \beta$ $\therefore \frac{5 \cos \beta}{2 \sin \beta} = \frac{2 \sin \beta}{2 \sin \beta} \quad \checkmark$ $\therefore \frac{5 \cos \beta}{2 \sin \beta} = 1$ $\therefore \frac{5}{2} \cot \beta = 1$ $\therefore \cot \beta = \frac{2}{5} \quad \checkmark$ $\therefore \tan \beta = \frac{5}{2}$ $\therefore \beta = \tan^{-1}\left(\frac{5}{2}\right) \quad \checkmark$ $\quad \quad \quad = 68,199^\circ \quad \checkmark$ <i>or</i> $\beta = 180^\circ + 68,199^\circ$ $\quad \quad \quad = 248,199^\circ \quad \checkmark$	1M dividing both sides by $\cos \beta / \sin \beta$ 1M applying trigonometric laws (finding equation) 1M make B the subject of the formula 2M for solutions	(5)

	<u>SOLUTION</u>		<u>EXPLANATION</u>	
6.3	$\hat{ACB} = 180^\circ - 67^\circ$ $= 113^\circ$ $\hat{CAB} = 180^\circ - 54^\circ - 113^\circ$ $= 13^\circ$ <i>In $\triangle ADC$</i> $\frac{\sin 13^\circ}{1,8} = \frac{\sin 113^\circ}{AB}$ ✓ $\therefore AB = \frac{1,8 \sin 113^\circ}{\sin 13^\circ}$ $\therefore AB = 7,366m$ ✓ <i>In $\triangle ABD$</i> $\frac{AD}{AB} = \sin 54^\circ$ ✓ $\therefore \frac{AD}{7,366} = \sin 54^\circ$ ✓ $\therefore AD = 7,366 \sin 54^\circ$ ✓ $\therefore AD = 5,959m$	<u>ALTERNATIVE SOLUTION</u> $\hat{ACB} = 180^\circ - 67^\circ$ $= 113^\circ$ $\hat{CAB} = 180^\circ - 54^\circ - 113^\circ$ $= 13^\circ$ <i>In $\triangle ABC$</i> $\frac{AC}{\sin 54^\circ} = \frac{1,8}{\sin 13^\circ}$ ✓ $\therefore AC = \frac{1,8 \sin 54^\circ}{\sin 13^\circ}$ $\therefore AC = 6,475m$ ✓ <i>In $\triangle ADC$</i> $\frac{AD}{AC} = \sin 67^\circ$ ✓ $\therefore \frac{AD}{6,474} = \sin 67^\circ$ ✓ $\therefore AD = 6,474 \sin 67^\circ$ ✓ $\therefore AD = 5,959m$	1M for $\hat{CAB}$ 1M for $\hat{ACB}$ 1M finding one of the sides 1M drawing up an equation 1M method 1M CA	(6)
6.4	$a = 1$ ✓ $b = 1$ ✓ $c = 1$ ✓		Full marks answers only	(3)
				[19]

	TOTAL:	100
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