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NASIENRIGLYNE

EKSAMEN		NASIONALE SENIOR SERTIFIKAAT	
GRAAD		12	
DATUM		JUNIE 2024	
VAK		WISKUNDE	
VRAESTEL		2	
PUNTE TOTAAL		150	
TYDSDUUR (URE)		3	
AANTAL BLADSYE		29	



VRAAG 1

Om bloedskenking te bevorder is 'n bloedskenkersentrum by 'n winkelsentrum gevestig. Die tabel hieronder verskaf die daaglikse telling van bloedeenhede wat in TIEN dae deur die mense by die winkelsentrum geskenk is.

Dae	1	2	3	4	5	6	7	8	9	10
Eenhede bloed	45	59	65	73	79	85	91	99	101	110

- 1.1.1 Bereken die gemiddelde van die eenhede bloed wat per dag oor die tydperk van 10 dae geskenk is. (2)

$$\bar{x} = \frac{807}{10} = 80,7$$

✓✓ Antwoord

- 1.1.2 Bepaal die standaardafwyking van die data. (2)

$$\sigma = 19,45$$

✓✓ Antwoord

- 1.1.3 Hoeveel dae was die aantal eenhede bloed wat by die winkelsentrum geskenk is buite een standaardafwyking van die gemiddelde? (3)

$$\text{Boonste limiet} = 80,7 + 19,45 = 100,15$$

✓ gem+ 1SA

$$\text{Onderste limiet} = 80,7 - 19,45 = 61,25$$

✓ gem-1SA

✓ Antwoord

$$(61,25 ; 100,15)$$

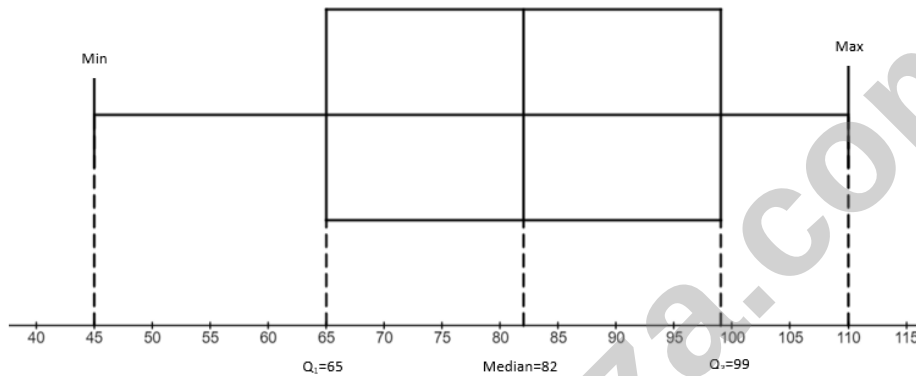
$$\text{Aantal dae} = 5$$

Slegs korrekte antwoord: volpunte, mits 1.1.1 en 1.1.2 albei korrek is

- 1.2.1 Teken 'n mond- en snordigram om die aantal eenhede bloed voor te stel wat deur mense by die winkelsentrum geskenk is. (3)

Min = 45
 $Q_1 = 65$
 Median = 82
 $Q_3 = 99$
 Max = 110

- ✓ Min & Maks waardes
- ✓ Q_1 & Q_3
- ✓ Mediaan & skaal



- 1.2.2 Beskryf die skeefheid van die data. Motiveer jou antwoord. (2)

Data is effens negatief skeef.
 Die gemiddelde is minder as die mediaan; $80,7 < 82$

- ✓ Antwoord
- ✓ Motivering

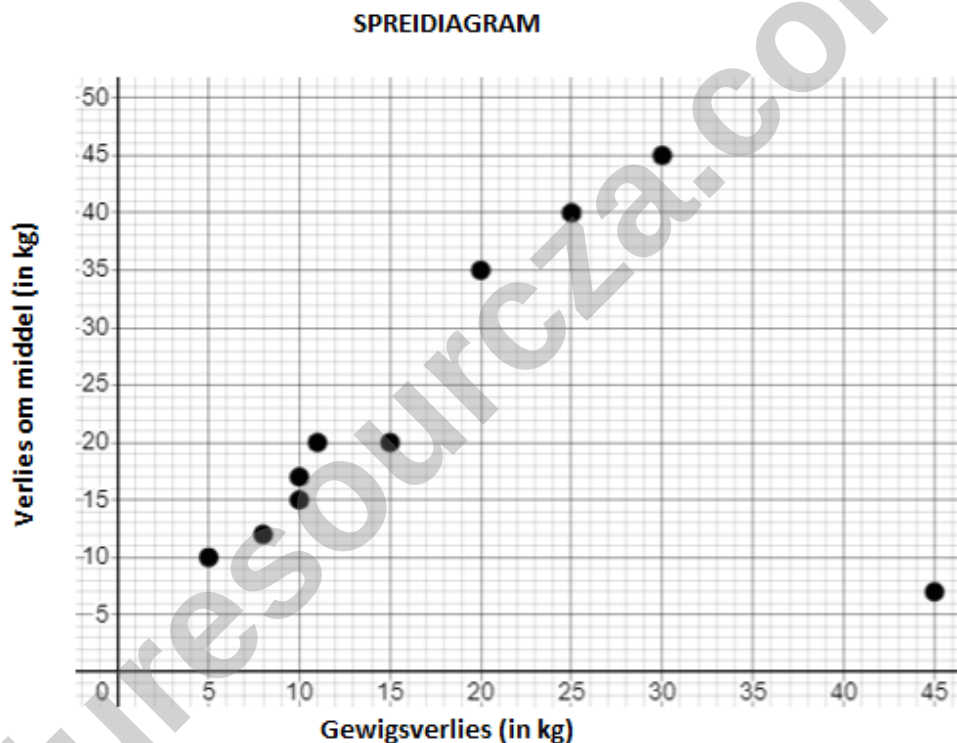
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VRAAG 2

Die tabel hieronder verteenwoordig die gewigsverlies en middellyfverlies van 10 deelnemers wat aan 'n gewigsverlieskompetisie deelgeneem het.

Gewigsverlies (in kg)	5	10	45	8	10	15	25	30	11	20
Verlies om middel (in kg)	10	15	7	12	17	20	40	45	20	35

'n Spreidiagram van bogenoemde resultate word hieronder getoon.



2.1 Identifiseer 'n uitskieter in die bostaande data.

(1)

(45; 7)

✓ Antwoord

2.2 Bepaal die vergelyking van die kleinste-kwadrates-regressielyn vir die data.

(2)

$$A = 16,98$$

✓ $A = 16,98$

$$B = 0,29$$

✓ $B = 0,29$

$$y = 0,29x + 16,98$$

- 2.3 Indien 'n deelnemer in dieselfde tyd 21,2 kg verloor het. Voorspel die middellyfverlies van hierdie deelnemer, korrek tot twee desimale plekke. (2)

$$y = 0,29x + 16,98$$

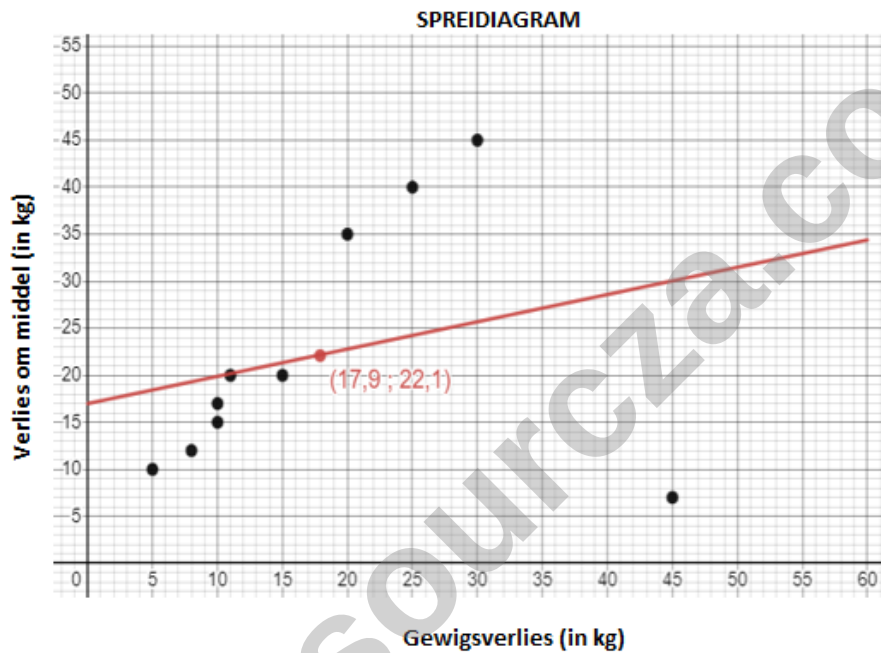
✓ Subst $x = 21,2$

$$y = 0,29(21,2) + 16,98$$

$$y = 23,13 \text{ mm}$$

✓ Antwoord

- 2.4 Trek die kleinste vierkantige regressielyn op die spreidingsdiagram hierbo verskaf. (3)



✓ y-afsnit
(0;16,98)

✓ (17,9 ; 22,1)

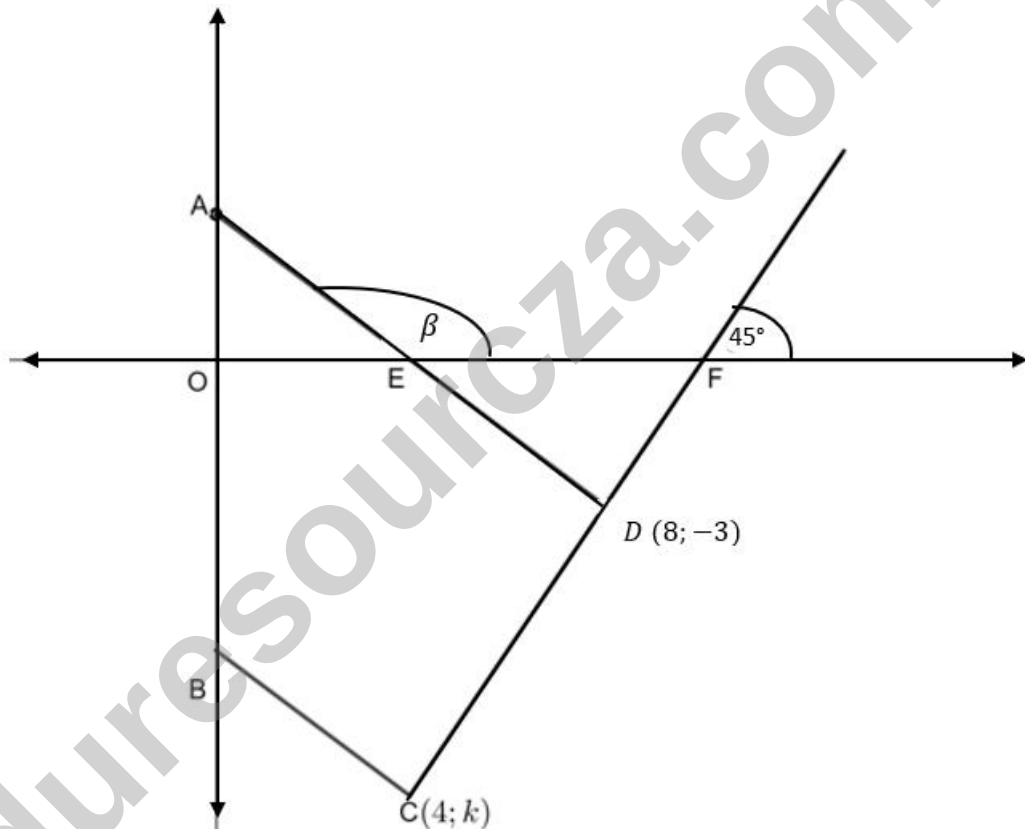
✓ geen x-afsnit

[8]

VRAAG 3

In die diagram hieronder:

- AD is 'n reguitlyn met die vergelyking $y = -\frac{3}{4}x + 3$.
- E en A is die x – en y –afsnitte respektiewelik van AD .
- Die inklinasiehoek van AD is β .
- Vanaf $C(4; k)$ word 'n lyn, ewewydig aan AD word getrek, wat die y –as by B sny.
- Lyn CD word getrek deur $D(8; -3)$ en sny die x -as by F .
- Die inklinasiehoek van CD is 45° .



3.1 Bepaal die koördinate van A en E .

(4)

$$A(0; 3)$$

$$E\text{-}x\text{-afsnit}; y = 0$$

$$0 = -\frac{3}{4}x + 3$$

$$x = 4$$

$$E(4; 0)$$

$$\checkmark A(0; 3)$$

$$\checkmark y = 0$$

$$\checkmark \text{Vereen-voudig}$$

$$\checkmark E(4; 0)$$



3.2 Toon dat $k = -7$. (3)

$$m_{CD} = \tan 45^\circ = 1 \quad \checkmark m_{CD}=1$$

$$m_{DF} = \frac{k - (-3)}{4 - 8} = 1 \quad \checkmark \text{Sub } ((8;-3)$$

$$\checkmark \text{Sub } (4;k)$$

$$k + 3 = -4$$

$$k = -7$$

OF:

OF

$$m_{CD} = \tan 45^\circ = 1$$

$$\checkmark m_{CD}=1$$

$$\therefore y = mx + c \quad \text{OR}$$

$$y - y_1 = m(x - x_1)$$

$$-3 = (1)(8) + c$$

$$y + 3 = (1)(x - 8)$$

$$c = -11$$

$$y = x - 11$$

$$y = x - 11$$

$$\checkmark \text{Subst } (8;-3)$$

$$\therefore k = x - 11$$

$$\checkmark \text{Subst } (4;k)$$

$$k = 4 - 11$$

$$\therefore k = -7$$

3.3 Bepaal die vergelyking van BC. (3)

$$\text{Gegee } AD \parallel BC, m_{AD} = m_{BC}$$

$$\checkmark m_{BC} = -\frac{3}{4}$$

$$\therefore m_{BC} = -\frac{3}{4}$$

$$\checkmark \text{Subst } (4;-7)$$

$$y = mx + c$$

OF

$$y - y_1 = m(x - x_1)$$

$$-7 = \left(-\frac{3}{4}\right)(4) + c$$

$$y + 7 = \left(-\frac{3}{4}\right)(x - 4)$$

$$c = -4$$

$$\checkmark \text{Antwoord}$$

$$\therefore y = -\frac{3}{4}x - 4$$

$$y = -\frac{3}{4}x - 4$$



3.4 Bepaal of $\triangle DEF$ reghoekig is. Wys al jou berekeninge. (4)

$$m_{DF} = \tan 45^\circ = 1$$

✓ $m_{DF}=1$

$$m_{AD} = -\frac{3}{4}$$

✓ $m_{AD} \times m_{DF}$

$$m_{AD} \times m_{DF} = -\frac{3}{4} \times 1$$

✓ $\neq -1$

$$m_{AD} \times m_{DF} = -\frac{3}{4} \neq -1$$

✓ Gevolg-
trekking

$\therefore \triangle DEF$ is nie reghoekig nie.

3.5 Bereken die oppervlakte van $\triangle DEF$. (5)

CF:

$$y = mx + c$$

OR

$$y - y_1 = m(x - x_1)$$

$$-3 = (8) + c$$

$$y - (-3) = x - 8$$

$$c = -11$$

$$y = x - 11$$

$$y = x - 11$$

✓ F(11;0)

$$F(11;0)$$

✓ $EF = 7$

$$EF = 11 - 4 = 7$$

$$DF = \sqrt{(11 - 8)^2 + (0 - (-3))^2}$$

✓ $DF = 3\sqrt{2}$

$$DF = 3\sqrt{2}$$

✓ Korr. Subst in
oppv reël

$$Area = \frac{1}{2} EF \cdot DF \sin 45^\circ$$

✓ Antwoord

$$Area = \frac{1}{2} (7)(3\sqrt{2}) \left(\frac{1}{\sqrt{2}}\right)$$

$$Area = \frac{21}{2} \text{ or } 10,5 \text{ eenheid}^2$$



- 3.6 Laat G 'n punt in die vierde kwadrant wees sodat CEDG 'n parallelogram vorm.
Bereken die koördinate van G. (4)

$$E(4; 0)$$

CA vanaf 3.1

Hoeklyne van parm halveer mekaar.

✓ midpt CD
waarde

$$\text{Midpt CD: } \left(\frac{8+4}{2} ; \frac{-3-7}{2} \right) = (6 ; -5)$$

✓ midpt EG
waarde

$$\therefore \text{midpt EG} = (6; -5) = \left(\frac{x+4}{2} ; \frac{y+0}{2} \right) \dots$$

$$\therefore \frac{x+4}{2} = 6 \quad \text{en} \quad \frac{y+0}{2} = -5$$

$$x = 8$$

$$y = -10$$

$$\therefore G(8; -10)$$

OF:

OF

Translasie:

Vanaf E na D x-waardes: 4

$$\checkmark 4 + 4$$

Vanaf D na G y-waardes: 7 af

$$\checkmark -3 - 7$$

$$G : (4 + 4 ; -3 - 7)$$

$$\checkmark x = 8$$

$$\therefore G(8; -10)$$

$$\checkmark y = -10$$

OF:

C na E het dieselfde x -waardes, $\therefore EC \perp x - as$

D en E behoort dieselfde x -waardes te hê omdat $EC \parallel DG$,

$$x_G = 8$$

Vanaf D na G y-waardes: 7 afwaarts

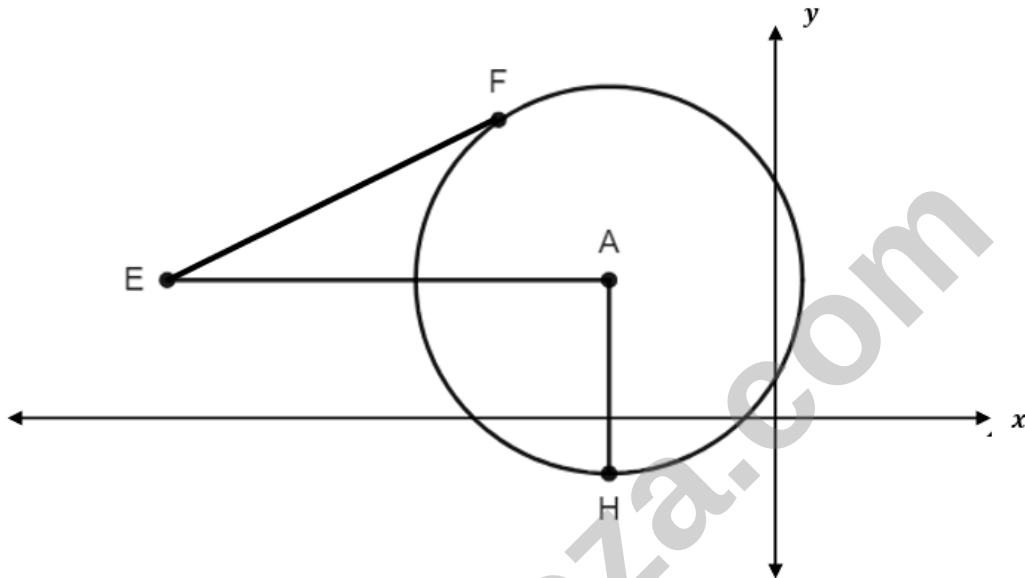
$$\therefore G(8; -10)$$

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VRAAG 4

In die diagram word sirkel met middelpunt A en vergelyking $x^2 + y^2 + 12x - 10y = -12$ gegee. AE is ewewydig aan die x -as en AH is loodreg op die x -as.

EF is 'n raaklyn aan die sirkel sodat $EF = \sqrt{95}$.



- 4.1 Bepaal die koördinate van A en die radius van die sirkel. (5)

$$x^2 + 12x + (6)^2 + y^2 - 10y + (5)^2 = -12 + 36 + 25$$

✓ $(x + 6)$

$$(x + 6)^2 + (y - 5)^2 = 49$$

✓ $(y - 5)$

✓ 49

$$A = (-6; 5)$$

✓ $A(-6;5)$

$$\text{Radius} = 7$$

✓ $r=7$

- 4.2 Bepaal die vergelyking van die raaklyn aan die sirkel by punt H. (1)

$$y = -2$$

✓ Antwoord

- 4.3 Bepaal die koördinate van E. (3)

$$EA = \sqrt{(\sqrt{95})^2 + (7)^2} \dots \text{Pythagoras}$$

✓ Gebruik
Pythagoras

$$EA = 12$$

✓ $EA=12$

✓ $E(-18;5)$

$$E(-6 - 12; 5)$$

$$= E(-18; 5)$$

OF:

OF



$$EA = \sqrt{(\sqrt{95})^2 + (7)^2} \dots \text{Pythagoras}$$

$$EA = 12$$

$$(12)^2 = (x + 6)^2 + (5 - 5)^2$$

$$144 = x^2 + 12x + 36$$

$$x^2 + 12x - 108 = 0$$

$$(x - 6)(x + 18) = 0$$

$$x = 6 \quad \text{or} \quad x = -18$$

$$\therefore E(-18; 5)$$

- ✓ Gebruik Pythagoras
- ✓ afstand-formule
- ✓ E(-18;5)

4.4 Bereken die grootte van $\hat{F}EA$.

(2)

$$\tan \hat{F}EA = \frac{7}{\sqrt{95}}$$

- ✓ tan verhouding

$$\hat{F}EA = 35,69^\circ$$

- ✓ Antwoord

OF

OF

$$\sin \hat{F}EA = \frac{7}{12}$$

- ✓ sin verhouding
- ✓ Antwoord

$$\hat{F}EA = 35,69^\circ$$

OF

OF

$$\cos \hat{F}EA = \frac{\sqrt{95}}{12}$$

- ✓ cos verhouding
- ✓ Antwoord

$$\hat{F}EA = 35,69^\circ$$



- 4.5 'n Sirkel met middelpunt B en met die vergelyking $(x + 10)^2 + (y - 5)^2 = k$, raak intern aan sirkel A. Bereken die waarde van k . (2)

$$A (-6 ; 5)$$

$$B (-10; 5)$$

$$AB = \sqrt{(-10 + 6)^2 + (5 - 5)^2}$$

$$AB = 4 \text{ eenhede}$$

$$\checkmark AB=4$$

$$r_A - r_B = AB$$

$$7 - r = 4$$

$$r = 3$$

$$\therefore k = r^2 = 9$$

$$\checkmark k=9$$

- 4.6 Sirkel A word om die x -as gereflekteer en 2 eenhede na regs transleer om 'n nuwe sirkel met middelpunt P te vorm. Skryf die vergelyking van sirkel P neer. (2)

$$(x + 6 - 2)^2 + (y + 5)^2 = 49$$

$$(x + 4)^2 + (y + 5)^2 = 49$$

$$\checkmark (x+4)^2$$

$$\checkmark (y+5)^2$$

[15]



VRAAG 5

5.1 Indien $17 \cos \theta = -8$ en $0^\circ < \theta < 180^\circ$, bereken, **sonder die gebruik van 'n sakrekenaar**:

5.1.1 $\tan(360^\circ - \theta)$. (3)

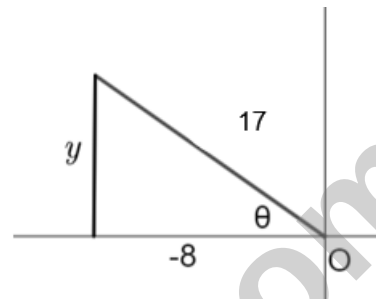
$$\cos \theta = -\frac{8}{17}$$

$$y = \sqrt{(17)^2 - (8)^2}$$

$$y = 15$$

$$\tan(360^\circ - \theta) = -\tan \theta$$

$$= -\left(\frac{15}{8}\right)$$



$$\checkmark \cos \theta = -\frac{8}{17}$$

$$\checkmark y = 15$$

✓ Antwoord

5.1.2 $\sin 2\theta$. (3)

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left(\frac{15}{17}\right) \left(-\frac{8}{17}\right)$$

$$= -\frac{240}{289}$$

CA vanaf 6.1.1

✓ Identiteit

✓ $\sin \theta$

✓ Antwoord

(nie desimaal)

5.1.3 $\sin(810^\circ - \theta)$. (3)

$$\sin(810^\circ - \theta) = \sin(90^\circ - \theta)$$

$$= \cos \theta$$

$$= -\frac{8}{17}$$

✓ $\sin(90^\circ - \theta)$

✓ $\cos \alpha$

✓ Antwoord



- 5.2 Druk die volgende uit as 'n enkele trigonometriese verhouding. **Die gebruik van 'n sakrekenaar is nie toegelaat nie.**

$$\begin{aligned} & \frac{2}{\sin(30^\circ + x) \cos x - \cos(30^\circ + x) \sin x} \times \frac{\sin(138^\circ) \cdot \sin(48^\circ)}{1 - 2 \sin^2 318^\circ} \quad (7) \\ &= \frac{2}{\sin(30^\circ + x - x)} \times \frac{(\sin 42^\circ)(\sin 48^\circ)}{1 - 2(-\sin 42^\circ)^2} \quad \begin{array}{l} \checkmark -\sin 42^\circ \\ \checkmark -\sin 48^\circ \\ \checkmark \sin 30^\circ \\ \checkmark \cos 42^\circ \\ \checkmark \sin 84^\circ \end{array} \\ &= \frac{2}{\sin 30^\circ} \times \frac{\sin 42^\circ \cdot \cos 42^\circ}{1 - 2(\sin^2 42^\circ)} \quad \checkmark \frac{1}{2} \\ &= \frac{\sin 84^\circ}{\frac{1}{2} \cdot \cos 84^\circ} \quad \checkmark 2 \tan 84^\circ \\ &= 2 \tan 84^\circ \end{aligned}$$

OF

OF

$$\begin{aligned} &= \frac{2}{\sin(30^\circ + x - x)} \times \frac{(\sin 42^\circ)(\sin 48^\circ)}{\cos^2 318^\circ} \quad \begin{array}{l} \checkmark -\sin 42^\circ \\ \checkmark -\sin 48^\circ \\ \checkmark \cos 42^\circ \\ \checkmark \sin 84^\circ \end{array} \\ &= \frac{2}{\sin 30^\circ} \times \frac{\sin 42^\circ \cdot \cos 42^\circ}{\cos 84^\circ} \quad \checkmark \frac{1}{2} \\ &= \frac{\sin 84^\circ}{\frac{1}{2} \cdot \cos 84^\circ} \quad \checkmark \cos 84^\circ \\ &= 2 \tan 84^\circ \quad \checkmark 2 \tan 84^\circ \end{aligned}$$

- 5.3 Gegee:

$$\frac{2 \sin 2x - \cos^2 x - \sin^2 x + \cos 2x}{2 \cos x - \sin x} = 2 \sin x$$

- 5.3.1 Vir watter waarde(s) van x is die identiteit ongedefinieerd? (4)

$$\begin{aligned} & \text{Identity is ongedefinieerd indien } 2 \cos x - \sin x = 0 \quad \checkmark 2 \cos x - \sin x = 0 \\ & 2 \cos x = \sin x \\ & 2 = \frac{\sin x}{\cos x} \quad \checkmark \tan x = 2 \\ & \tan x = 2 \quad \checkmark \checkmark \text{Antwoord} \end{aligned}$$



$$x = 63,43^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$$

of

$$x = 180^\circ + 63,43^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$$

$$x = 243,43^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$$

5.3.2 Bewys die identiteit volledig.

(5)

$$LK = \frac{2(2 \sin x \cos x) - (\cos^2 x + \sin^2 x) + (1 - 2 \sin^2 x)}{2 \cos x - \sin x}$$

$$= \frac{4 \sin x \cos x - 1 + 1 - 2 \sin^2 x}{2 \cos x - \sin x}$$

$$= \frac{2 \sin x (\cos x - \sin x)}{2 \cos x - \sin x}$$

$$= 2 \sin x$$

$$= RK$$

OF

$$LK = \frac{2(2 \sin x \cos x) - \cos^2 x - \sin^2 x + (\cos^2 x + \sin^2 x) - 2 \sin^2 x}{2 \cos x - \sin x}$$

$$= \frac{4 \sin x \cos x - 2 \sin^2 x}{2 \cos x - \sin x}$$

$$= \frac{2 \sin x (\cos x - \sin x)}{2 \cos x - \sin x}$$

$$= 2 \sin x$$

$$= RK$$

OF

$$LK = \frac{2(2 \sin x \cos x) - \cos^2 x - \sin^2 x + (2 \cos^2 x - 1)}{2 \cos x - \sin x}$$

$$= \frac{4 \sin x \cos x - \cos^2 x - \sin^2 x + 2 \cos^2 x - (\cos^2 x + \sin^2 x)}{2 \cos x - \sin x}$$

$$= \frac{2 \sin x (\cos x - \sin x)}{2 \cos x - \sin x}$$

$$= 2 \sin x$$

$$\begin{aligned} &\checkmark 2 \sin x \cos x \\ &\checkmark (\cos^2 x + \sin^2 x) \\ &\checkmark 1 - 2 \sin^2 x \\ &\checkmark 1 \\ &\checkmark \\ &2 \sin x (\cos x - \sin x) \end{aligned}$$

OF

$$\begin{aligned} &\checkmark (\cos^2 x + \sin^2 x) \\ &\checkmark 4 \sin x \cos x \\ &\checkmark 4 \sin x \cos x - 2 \sin^2 x \\ &\checkmark 2 \sin x \\ &\checkmark (\cos x - \sin x) \end{aligned}$$

OF

$$\begin{aligned} &\checkmark 2 \cos^2 x - 1 \\ &\checkmark 4 \sin x \cos x \\ &\checkmark (\cos^2 x + \sin^2 x) \\ &\checkmark 2 \sin x \\ &\checkmark (\cos x - \sin x) \end{aligned}$$



= RK

5.3.3 Vervolgens, of andersins, bepaal die waarde(s) van x sodanig dat

$$2 \sin 2x - \cos^2 x - \sin^2 x + \cos 2x = 4 \cos x - 2 \sin x, \text{ as } x \in [-360^\circ; 360^\circ] \quad (4)$$

$$2 \sin 2x - \cos^2 x - \sin^2 x + \cos 2x = 2(2 \cos x - \sin x)$$

$$\frac{2 \sin 2x - \cos^2 x - \sin^2 x + \cos 2x}{2 \cos x - \sin x} = 2$$

$$\therefore 2 \sin x = 2$$

$$\sin x = 1$$

$$x \in \{-270^\circ; 90^\circ\}$$

✓ Gemene faktor
 $2(2 \cos x - \sin x)$

✓ ÷ beide kante

$$2 \cos x - \sin x$$

$$\checkmark 2 \sin x = 2$$

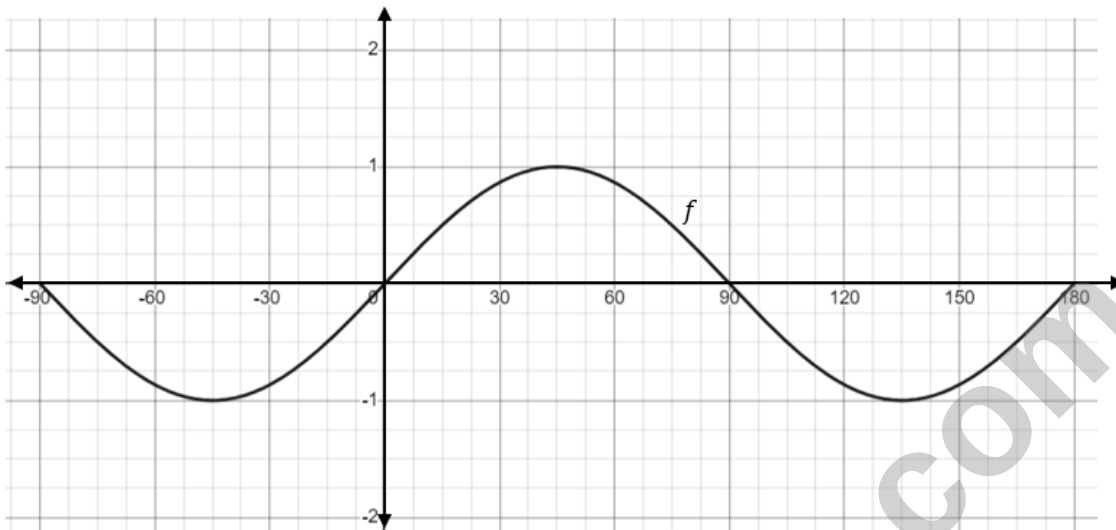
$$\checkmark -270^\circ$$

$$\text{EN } 90^\circ$$

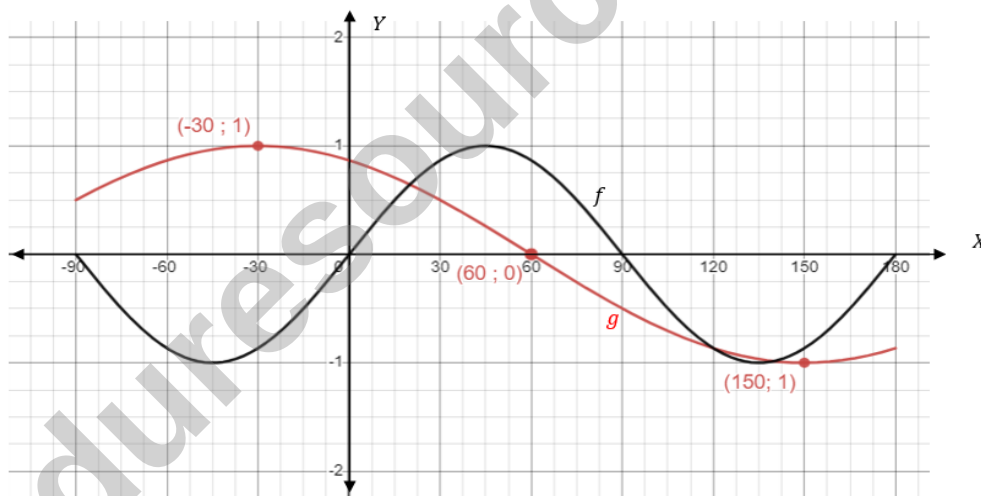
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VRAAG 6

Die onderstaande diagram toon die grafiek van $f(x) = \sin 2x$, waar $x \in [-90^\circ; 180^\circ]$.



- 6.1 Skets die grafiek van $g(x) = \cos(x + 30^\circ)$ op dieselfde assestelsel as hierbo vir $x \in [-90^\circ; 180^\circ]$. (4)

✓ $(-30^\circ; 1)$ ✓ $(60^\circ; 0)$ ✓ $(150^\circ; -1)$

✓ vorm

- 6.2 Bepaal die x -waarde(s) vir die snydingspunt van f en g , indien $x \in [-90^\circ; 180^\circ]$. (4)

$$f(x) = g(x)$$

$$\sin 2x = \cos(x + 30^\circ)$$

$$\sin 2x = \sin[90^\circ - (x + 30^\circ)]$$

$$V.H. = 60^\circ - x$$

$$2x = 60^\circ - x + k \cdot 360^\circ; k \in \mathbb{Z} \text{ of } 2x = 180^\circ - (60^\circ - x) + k \cdot 360^\circ; k \in \mathbb{Z}$$

$$3x = 60^\circ + k \cdot 360^\circ$$

$$x = 120^\circ + k \cdot 360^\circ$$

$$x = 20^\circ + k \cdot 120^\circ; k \in \mathbb{Z}$$

$$x \in \{20^\circ; 120^\circ; 140^\circ\}$$

✓ $\sin(60^\circ - x)$ ✓ $20^\circ + k \cdot 120^\circ$

✓✓ Al DRIE

 x -waardes



6.3 Gebruik jou grafiek en die waarde(s) bereken in 6.1 en bepaal die waarde(s) van x so dat:

6.3.1 $f'(x) = 0$, if $x > 0$ (2)

$$x = 45^\circ \quad \text{or} \quad x = 135^\circ$$

$$\checkmark x = 45^\circ$$

$$\checkmark x = 135^\circ$$

6.3.2 $\sin x \cos x > \frac{1}{2} \cos(x + 30^\circ)$ (5)

CA vanaf 6.1

$$2 \sin x \cos x > \cos(x + 30^\circ)$$

$$\sin 2x > \cos(x + 30^\circ)$$

$$\checkmark \sin 2x >$$

$$\cos(x + 30^\circ)$$

$$\checkmark (20^\circ; \checkmark 120^\circ)$$

$$x \in (20^\circ; 120^\circ) \cup (140^\circ; 180^\circ]$$

$$\checkmark (140^\circ$$

$$\checkmark 180^\circ]$$

OF

$$20^\circ < x < 120^\circ \quad \text{or} \quad 140^\circ < x \leq 180^\circ$$

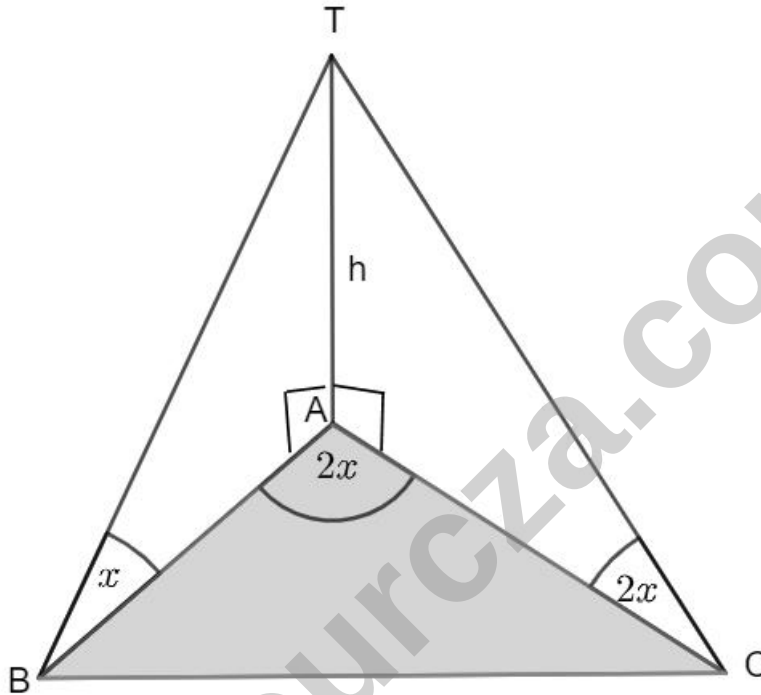
[15]

VRAAG 7

In die diagram is AT 'n vertikale toring.

A, B en C lê in dieselfde horisontale vlak. Die hoogtehoek na die bokant van die toring vanaf B is x en die hoogtehoek na die bokant van die toring vanaf C is $2x$.

$$\hat{BAC} = 2x.$$



7.1 Druk AC en AB uit in terme van x en h .

(3)

$$\tan \hat{ACT} = \frac{AT}{AC}$$

✓ Tan verhouding

$$AC = \frac{h}{\tan 2x}$$

$$AC = \frac{h}{\tan 2x}$$

$$\tan \hat{ABT} = \frac{AT}{AB}$$

$$AB = \frac{h}{\tan x}$$

$$AB = \frac{h}{\tan x}$$

7.2 Vervolgens, bewys dat $Oppervlakte \Delta ABC = \frac{h^2 \cos 2x}{2 \tan x}$ (4)

$$Oppv \Delta ABC = \frac{1}{2} \cdot AB \cdot AC \sin B\hat{A}C$$

$$Oppv \Delta ABC = \frac{1}{2} \left(\frac{h}{\tan x} \right) \left(\frac{h}{\tan 2x} \right) \sin 2x$$

✓ Korr subst in
koo formule

$$Oppv \Delta ABC = \frac{h^2 \sin 2x}{2 \tan x \cdot \frac{\sin 2x}{\cos 2x}}$$

✓ tan 2x ident.

$$Oppv \Delta ABC = \frac{h^2 \sin 2x}{2 \tan x} \times \frac{\cos 2x}{\sin 2x}$$

✓ x omgekeerd

$$Oppv \Delta ABC = \frac{h^2 \cos 2x}{2 \tan x}$$

7.3 Bepaal die oppervlakte van ΔABC indien $h = 12$ en $x = 32^\circ$. (2)

$$Oppv \Delta ABC = \frac{(12)^2 \cos 2(32)}{2 \tan(32)}$$

✓ Subst.

$$Oppv \Delta ABC = 50,51 \text{ eenhede}^2$$

✓ Antwoord

[9]

VRAAG 8

8.1 Voltooi die volgende stelling korrek:

Die hoek tussen 'n raaklyn aan 'n sirkel en 'n koord getrek vanaf die kontakpunt is . . .

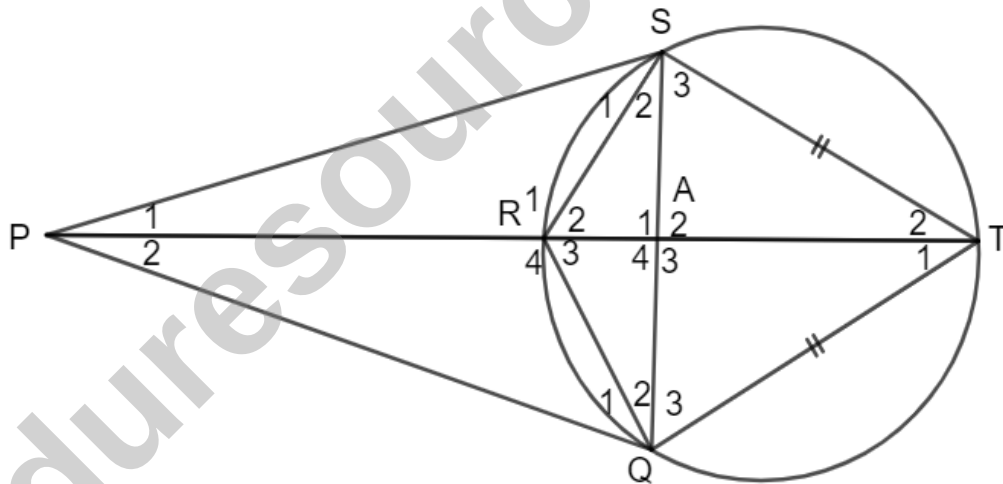
(2)

. . . gelyk aan 'n hoek in die teenoorgestelde sirkelsegment van die sirkel.

✓✓ Antwoord

8.2 In die diagram:

- Q, R, S en T lê op die omtrek van die sirkel.
- $ST = QT$
- PS en PQ is raaklyne aan die sirkel by S en Q respektiewelik.
- PRAT is 'n reguitlyn en sny die reguitlyn SQ in A.



8.2.1 Gee 'n rede waarom $\angle PSQ = \angle PQT$.

(2)

$$PS = PQ$$

✓✓ R

$$\therefore \angle PSQ = \angle PQT \quad \dots \quad \angle \text{'e teenoor} = \text{sye}$$

OF

$\angle PSQ = \angle PQT \quad \dots \quad$ Raaklyne vanaf gemeenskaplike punt buite sirkel.

8.2.2 Bewys dat $Q\hat{R}S = 2\hat{S}_3$.

(3)

$\hat{S}_3 = \hat{Q}_3 \dots \angle$ 'e teenoor = sye

✓ S & R

$\hat{R}_3 = \hat{S}_3 \dots \angle$ 'e in dieselfde segment

✓ S & R

$\hat{Q}_3 = \hat{R}_2 \dots \angle$ 'e in dieselfde segment

✓ S & R

$Q\hat{R}S = \hat{Q}_3 + \hat{Q}_3 = 2\hat{S}_3$

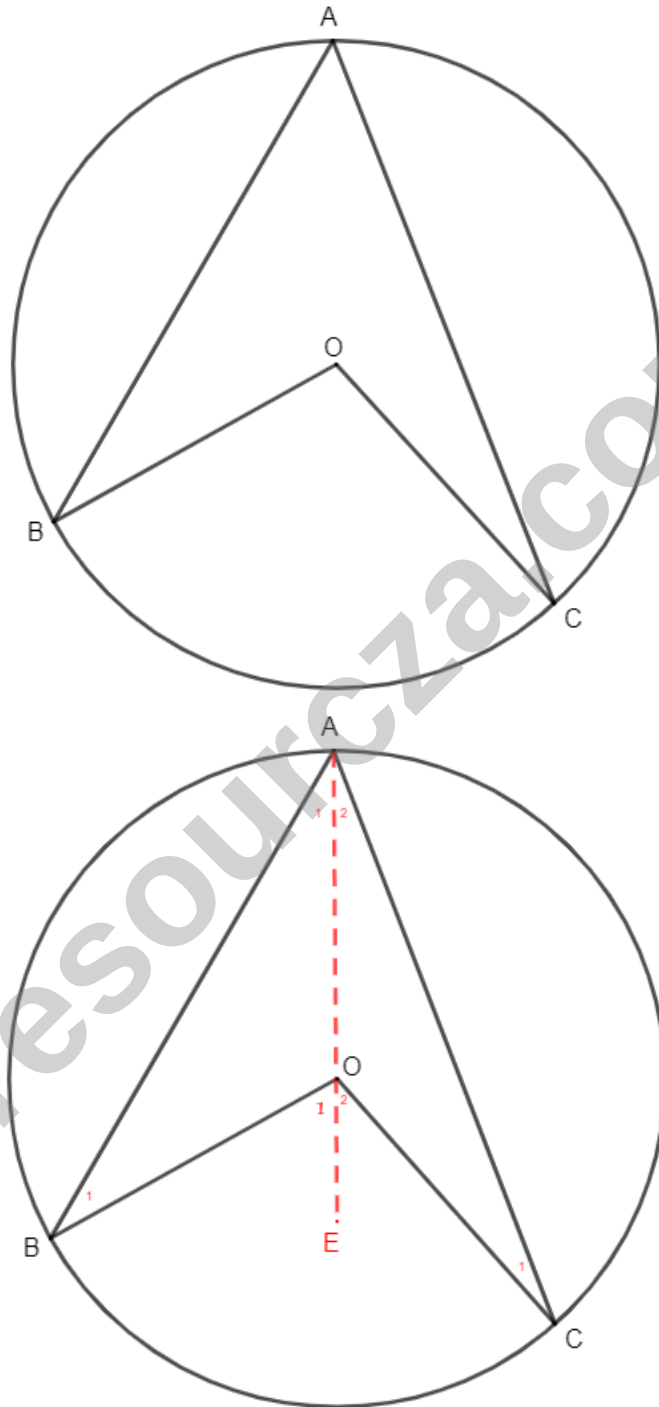
[7]

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VRAAG 9

9.1 Gebruik die diagram om te bewys dat $\widehat{BOC} = 2\widehat{BAC}$.

(5)



Konstruksie: Trek AO en verleng na D:

✓ Konstruksie

$\widehat{A_1} = \widehat{B} \dots \angle$ 'e teenoor = sye AO = OB (radii)

✓ $\angle$ 'e teenoor = sye

And $\widehat{A_2} = \widehat{C} \dots \angle$ 'e teenoor = sye AO = OC (radii)

✓ buite $\angle$ = som t.o.

$\widehat{O_1} = \widehat{A_1} + \widehat{B} \dots$ buite $\angle$ = som teenoorst binne $\angle$ 'e van $\triangle AOB$

$\therefore \widehat{O_1} = 2\widehat{A_1}$

$\hat{O}_2 = \hat{A}_2 + \hat{C} \dots$ buite $\angle$ = som teenoorst. binne $\angle$ 'e van $\triangle AOC$

$$\therefore \hat{O}_2 = 2\hat{A}_2$$

$$\checkmark \hat{B}\hat{O}\hat{C} = \hat{O}_1 + \hat{O}_2$$

$$\hat{B}\hat{O}\hat{C} = \hat{O}_1 + \hat{O}_2$$

$$\hat{B}\hat{O}\hat{C} = 2\hat{A}_1 + 2\hat{A}_2$$

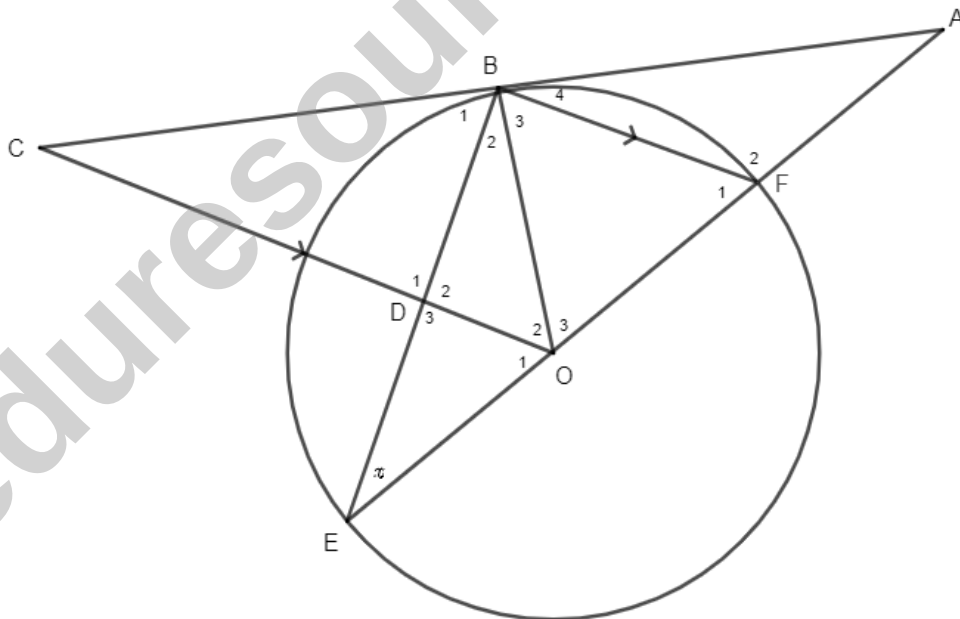
$$\hat{B}\hat{O}\hat{C} = 2(\hat{A}_1 + \hat{A}_2)$$

$$\hat{B}\hat{O}\hat{C} = 2\hat{B}\hat{A}\hat{C}$$

$$\checkmark \hat{B}\hat{O}\hat{C} = 2(\hat{A}_1 + \hat{A}_2)$$

9.2 In die diagram:

- O is die middelpunt van die sirkel.
- EF, die middellyn van die sirkel is verlang na A.
- ABC is 'n raaklyn aan die sirkel by B.
- $CO \parallel BF$
- $\hat{E} = x$



9.2.1 Stel, met redes, DRIE ander hoeke gelyk aan x .

(3)

$\hat{B}_4 = x \dots$ taaklyn-koord

✓ S&R

$\hat{B}_2 = x \dots$ $\angle$ 'e t.o.= sye ($BO = EO$ radii)

✓ S&R

$\hat{C} = \hat{B}_4 = x \dots$ ooreenk. $\angle$ 'e, $BF \parallel CO$

✓ S&R



9.2.2 Bepaal, met redes, $\hat{O}_3$ in terme van x . (2)

$$\hat{O}_3 = 2x \dots \angle \text{ by mdp} = 2x \angle \text{ by omtrek}$$

✓ 2x

✓ R

OF

$$\hat{O}_3 = 2x \dots \text{Buite } \angle = \text{som t.o. binne } \angle \text{'e}$$

9.2.3 Bewys dat D die middelpunt van BE is. (4)

$$\hat{E}\hat{B}\hat{F} = 90^\circ \dots \angle \text{ in semi-sirkel}$$

✓ S

✓ R

$$\therefore \hat{D}_2 = 90^\circ \dots \text{Ko-binne } \angle \text{'e} ; BF \parallel CO$$

✓ S & R

$$\therefore OD \perp BE$$

$$\therefore BD = DE \dots \text{lyn vanaf middelpunt } \perp \text{ op koord}$$

✓ R

9.2.4 Bewys dat BOEC 'n koordevierhoek is. (2)

$$\hat{C} = \hat{E} = x \dots \text{bewys}$$

$$\checkmark \hat{C} = \hat{E} = x$$

BOEC is koordevierhoek. . . omgekeerde $\angle$ 'e in dieselfde segment

✓ R

9.2.5 Bewys dat $\triangle ABF \parallel \triangle AEB$.

(3)

In $\triangle ABF$ en $\triangle AEB$:

$$\hat{A} = \hat{A} \dots \text{gemeen}$$

✓ gemeen $\angle$

$$\hat{B}_4 = \hat{E} = x \dots \text{bewys}$$

✓ 2^{de} $\angle$

$$\therefore \hat{A}\hat{F}B = \hat{A}\hat{B}E \dots \text{binne } \angle\text{'e van } \triangle\text{'e}$$

✓ R

$$\therefore \triangle ABF \parallel \triangle AEB \dots \text{H,H,H}$$

9.2.6 Bewys dat $2ED \cdot AB = AE \cdot BF$.

(3)

$$\frac{EB}{BF} = \frac{AE}{AB} \dots \triangle ABF \parallel \triangle AEB$$

✓ Verhou-ding

$$EB \cdot AB = BF \cdot AE$$

✓ Vereenv. y

$$\checkmark EB = 2ED$$

But $EB = 2 \cdot ED \dots D$ is mdpt van EB

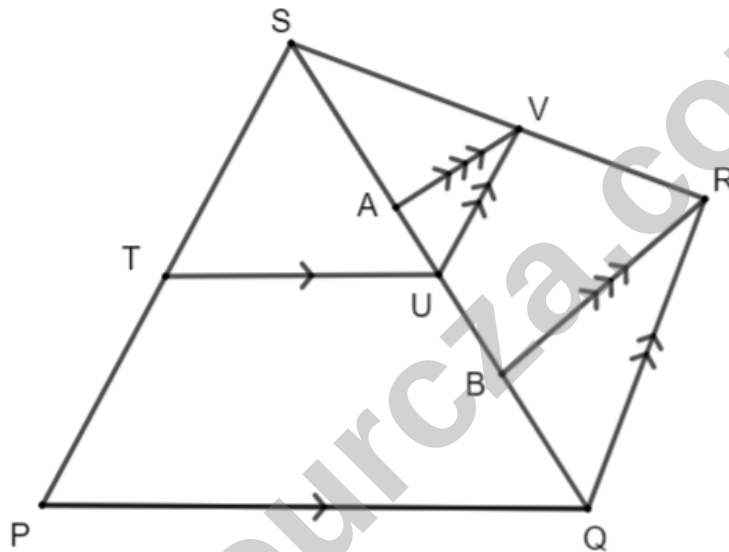
$$\therefore 2ED \cdot AB = BF \cdot AE$$

[22]

VRAAG 10

In die diagram is ΔSPQ en ΔSQR gegee:

- T lê op SP en U op SQ met $TU \parallel PQ$.
- V lê op SR met $UV \parallel QR$.
- A en B lê op SQ met $AV \parallel BR$.
- $SP = 14$ eenhede.
- $ST = 8$ eenhede.
- B is die middelpunt van UQ.



10.1 Bepaal, met redes, die verhouding van $SV : VR$.

(4)

$$TP = 14 - 8 = 6 \text{ eenhede}$$

✓ $TP=6$

$$\frac{SV}{VR} = \frac{SU}{UQ} \dots \text{eweredigheidstelling, } UV \parallel QR \text{ OF lyn } \parallel \text{een kant van } \Delta$$

✓ S & R

$$\frac{SU}{UQ} = \frac{ST}{TP} \dots \text{eweredigheidstelling } TU \parallel PQ \text{ OR lyn } \parallel \text{een kant van } \Delta$$

✓ S & R

$$\therefore \frac{SV}{VR} = \frac{ST}{TP} = \frac{8}{6} = \frac{4}{3}$$

✓ $\frac{4}{3}$



10.2 Bepaal, met redes, die verhouding van BR : AV. (3)

$\therefore \frac{RB}{VA} = \frac{SR}{SV}$... $\Delta SVA ||| \Delta SRB$ **OF** eweredigheidst. ; AV || BR **OR** lyn ||
een sy van Δ

✓ S & R

✓ SV + VR

$$\frac{RB}{VA} = \frac{SV + VR}{SV} = \frac{8 + 6}{8} = \frac{14}{8} = \frac{7}{4}$$

✓ $\frac{7}{4}$

10.3 Toon dat $8UB = 3SU$. (3)

$\frac{SU}{UQ} = \frac{SV}{VR} = \frac{4}{3}$... eweredigheidst, UV || QR **OR** lyn || een sy van Δ

✓ S & R

UB = BQ . . . gegee B midpt van UQ

✓ UB = BQ

$$\therefore \frac{SU}{UB} = \frac{4}{1,5} = \frac{8}{3}$$

✓ $\frac{SU}{UB} = \frac{8}{3}$

$$3SU = 8UB$$

[10]

GROOT TOTAAL: [150]

**INFORMATION SHEET: MATHEMATICS**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; \quad -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$