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MARKING GUIDELINES

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SOUTH AFRICAN COMPREHENSIVE ASSESSMENT INSTITUTE
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QUESTION 1

Pamela recorded the amount of data in MB that she used on each of the first 15 days in May. The information is shown in the table below:

26	13	3	18	12	34	24	58	16	10	15	69	20	17	40
----	----	---	----	----	----	----	----	----	----	----	----	----	----	----

1.1	Calculate the mean for the data set.	(1)
	$\bar{x} = \frac{375}{15} = 25$	✓ Answer (1)
1.2	Determine the standard deviation for the data set.	(1)
	$\sigma = 17,65$	✓ Answer (1)
1.3	Determine the number of days for which the data used was greater than one standard deviation above the mean.	(2)
	$Upper\ value = \bar{x} + \sigma$ $Upper\ value = 25 + 17,65$ $Upper\ value = 42,65$ $\therefore 2\ values\ above\ one\ standard\ deviation.$	✓ 42,65 ✓ 2 values (2)
		[4]

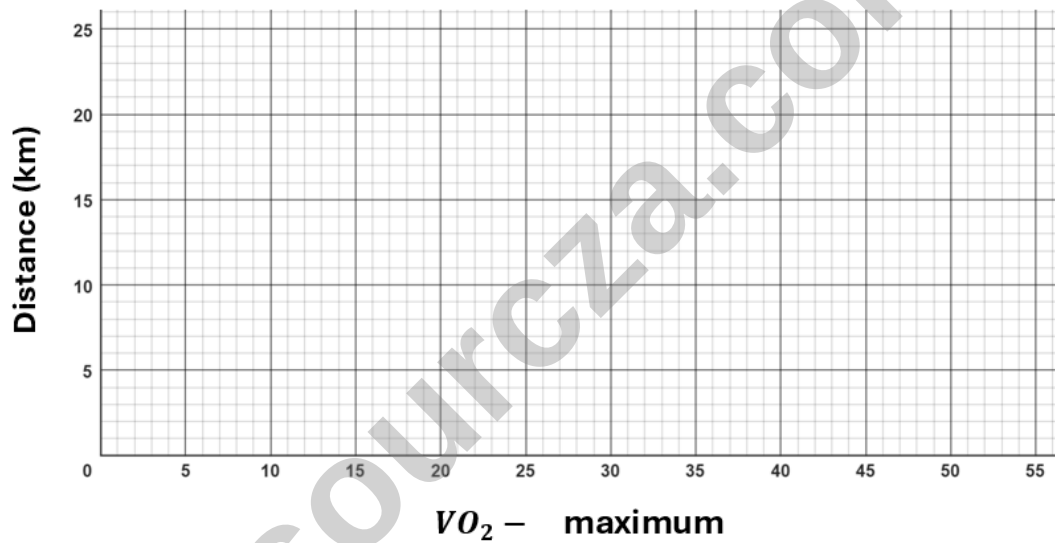
QUESTION 2

An athlete's ability to take in and use oxygen is known as their VO_2 -maximum.
The table below shows eleven athletes' VO_2 maximum and the distance they ran in an hour.

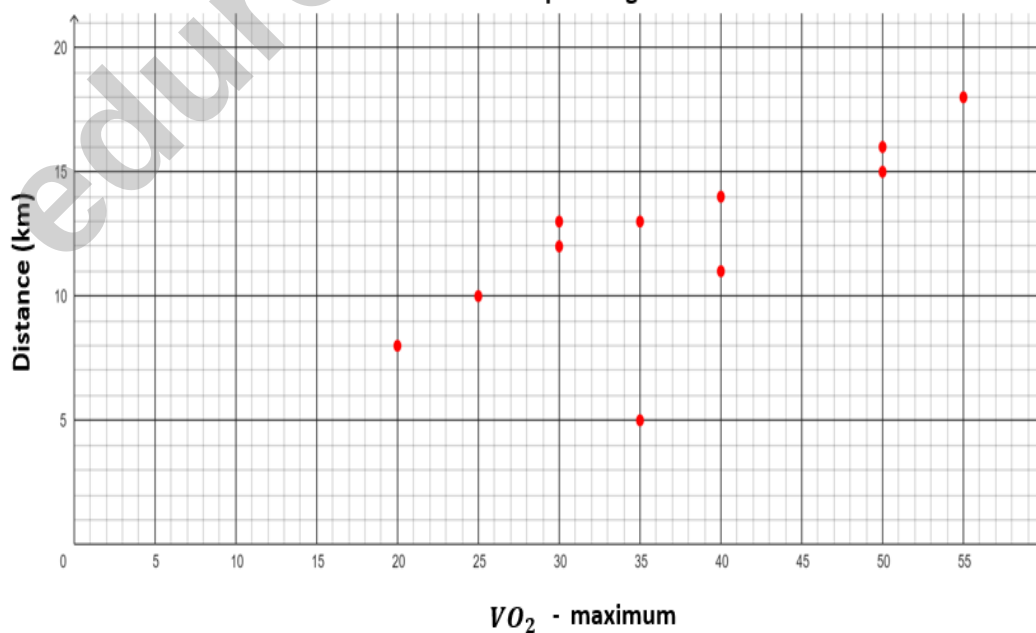
VO_2 -max	50	55	20	30	40	25	30	50	40	35	35
Distance (km)	15	18	8	13	14	10	12	16	11	13	5

2.1 Draw a scatter plot of the above data on the below set of axes. (2)

Scatter plot diagram.



Scatter plot diagram



✓ (20;8)
and (55;18)
✓ other
points
plotted
correctly.

(2)



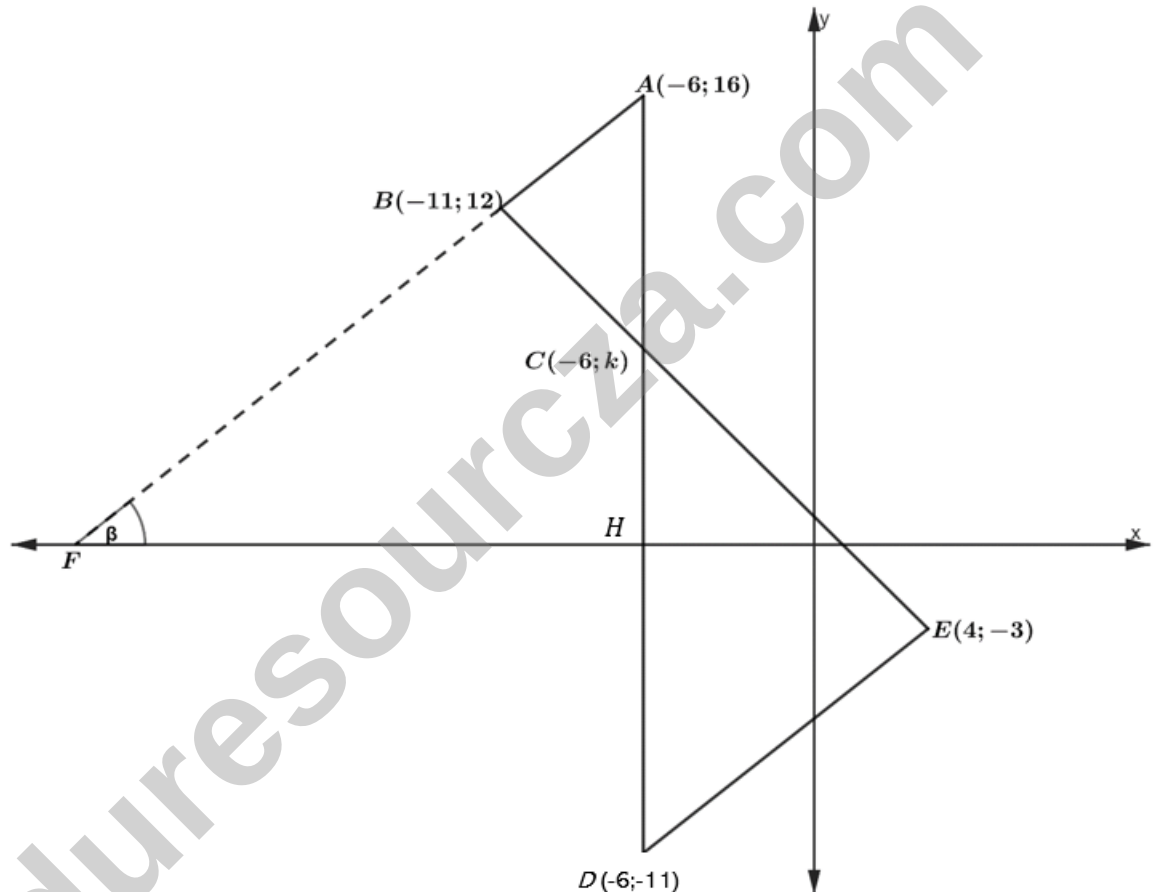
2.2	Calculate the equation of the least squares regression line of the data.	(3)
	$A = 3,15$ $B = 0,24$ $y = 0,24x + 3,15$	✓ $A = 3,15$ ✓ $B = 0,24$ ✓ Correct equation (3)
2.3	Draw the least square regression line on the scatter plot drawn in QUESTION 2.1 above.	(3)
	Scatter plot diagram VO₂ - maximum	✓ y-intercept ✓ ONE value (37,27;12,27) OR (20;8) OR (55;16,4) ✓ Correct line (3)
2.4	Identify if there are any outliers in the data. Provide a reason for your answer.	(2)
	$(35;5)$ is an outlier. This value deviates from all the other data in this set.	✓ $(35;5)$ ✓ Reason
2.5	Predict the VO ₂ maximum of an athlete who ran 19 km.	(2)
	$19 = 0,24x + 3,15$ $x = 66,04$	✓ Sub $y = 19$ ✓ $x = 66,04$ (2)
2.6	Determine the correlation coefficient of the data and comment on the correlation.	(2)
	$r = 0,73$ Moderate positive correlation	✓ $r = 0,73$ ✓ moderate positive (2)
		[14]



QUESTION 3

In the diagram below:

- $A(-6; 16); B(-11; 12); D(-6; -11)$ and $E(4; -3)$ are given.
- AD intersects BE at point $C(-6; k)$.
- AB produced meets the x -axis in F .
- $\widehat{HFA} = \beta$.
- AD intersects the x -axis at H .
- ACD and BCE are straight lines.



3.1	Show that the value of $k = 7$.	(4)
	$m_{BE} = \frac{12 - (-3)}{-11 - 4}$ $m_{BE} = -1$ $\therefore m_{BC} = \frac{12 - k}{-11 - (-6)}$ $m_{BC} = -1$ $12 - k = 5$ $k = 7$	✓ Subst in corr. Formula ✓ $m_{BE} = -1$ ✓ Subst $(-6; k)$ ✓ Simplify



	OR $m_{BE} = \frac{12 - (-3)}{-11 - 4}$ $m_{BE} = -1$ $y = mx + c$ $-3 = -(4) + c$ $c = 1$ $\therefore y = -x + 1$ or $y - y_1 = m(x - x_1)$ $(y - (-3)) = -(x - 4)$ $y + 3 = -x + 4$ $y = -x + 1$ $C(-6; k)$: $k = -(-6) + 1 = 7$ OR $DE = \sqrt{(4 - (-6))^2 + (-3 - (-11))^2}$ $DE = \sqrt{164}$ $DE = 2\sqrt{41} \text{ units}$ $\frac{AC}{CD} = \frac{AB}{ED} \dots \text{line } \parallel \text{ one side of } \triangle$ $\frac{16 - k}{k - (-11)} = \frac{\sqrt{41}}{2\sqrt{41}} = \frac{1}{2}$ $2(16 - k) = k + 11$ $3k = 21$ $k = 7$	OR ✓ Subst in corr. Formula ✓ $m_{BE} = -1$ ✓ $y = -x + 1$ ✓ Subst $(-6; k)$ OR ✓ Subst in corr. Formula ✓ $CE = 2\sqrt{41}$ ✓ $\frac{AC}{CD} = \frac{AB}{ED}$ ✓ $\frac{16 - k}{k - (-11)} = \frac{\sqrt{41}}{2\sqrt{41}}$
3.2	Calculate the length of BC.	(2)
	$BC^2 = (12 - 7)^2 + (-11 + 6)^2$ $BC = 5\sqrt{2}$	✓ subst in corr formula ✓ Answer



3.3	Show that C is NOT the midpoint of BE.	(2)
	$\text{Midpoint } BE = \left(\frac{-11 + 4}{2} ; \frac{12 - 3}{2} \right)$ $\text{Midpoint } BE = \left(-\frac{7}{2} ; \frac{9}{2} \right)$ $C = (-6;7), \therefore \text{NOT midpoint of BE}$	✓ Subst in correct formula ✓ $\left(-\frac{7}{2} ; \frac{9}{2} \right)$ (2)
3.4	Calculate the size of $\hat{B}AD$.	(5)
	$m_{AB} = \frac{16 - 12}{-6 + 11} = \frac{4}{5}$ $\tan \hat{B}FX = m_{AB} = \frac{4}{5}$ $\hat{B}FX = \tan^{-1} \left(\frac{4}{5} \right) = 38,6598 \dots \approx 38,66^\circ$ $\beta = 180^\circ - 38,66^\circ = 141,34^\circ$ $\hat{B}AD = 141,34 - 90^\circ \dots \text{ext } \angle \text{ of } \Delta = \text{sum opp int } \angle \text{'s}$ $\hat{B}AD = 51,34^\circ$ OR $AD \perp x\text{-axis } m_{AD} \text{ is undefined.}$ $\therefore \hat{F}HA = 90^\circ$ $m_{AB} = \frac{4}{5} \text{ from 3.2}$ $\therefore \hat{A}FH = 38,66^\circ$ $\therefore \hat{B}AD = 180^\circ - 90^\circ - 38,66^\circ = 51,34^\circ$	✓ m_{AB} ✓ $\tan \hat{B}FX = \frac{4}{5}$ $\checkmark \hat{B}FX = 38,66^\circ$ $\checkmark \alpha = 141,34^\circ$ $\checkmark \hat{B}AD = 51,34^\circ$ OR $\checkmark \hat{F}HA = 90^\circ$ $\checkmark m_{AB} = \frac{4}{5}$ $\checkmark \hat{A}FH = 38,66^\circ$ $\checkmark 180^\circ - 90^\circ - 38,66^\circ$ $\checkmark \hat{B}AD = 51,34^\circ$



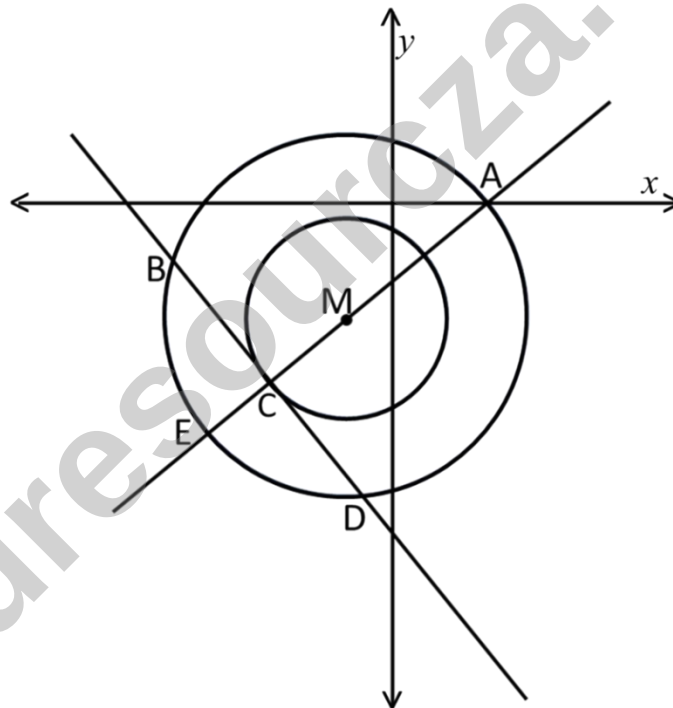
3.5	Calculate the numerical value of $\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEC}$	(5)
	$AC = 16 - 7 = 9 \text{ units (k-value from 3.1)}$ $DC = 7 + 11 = 18 \text{ units}$ $BC = 5\sqrt{2}$ $EC = \sqrt{(-6 - 4)^2 + (7 + 3)^2} = 10\sqrt{2}$ $\hat{BCA} = \hat{ECD} \dots \text{vertically opposite angles}$ $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle DEC} = \frac{\frac{1}{2} \times BC \times AC \times \sin \hat{BCA}}{\frac{1}{2} \times DC \times CE \times \sin \hat{DCE}}$ $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle DEC} = \frac{(5\sqrt{2})(9)}{(18)(10\sqrt{2})}$ $\frac{\text{Area } \triangle ABC}{\text{Area } \triangle DEC} = \frac{1}{4}$	$\checkmark DC = 18$ $\checkmark$ $EC = 10\sqrt{2}$ $\checkmark \hat{BCA} = \hat{ECD}$ $\checkmark$ area-rule subst. $\checkmark$ Answer
		[18]



QUESTION 4

In the diagram below:

- The small circle (P_1) and the big circle (P_2) have the same center M .
- A and E are points on circle P_2 .
- EM intersects circle P_1 at point C .
- The tangent BD to circle P_1 intersects circle P_2 at B and D .
- The equation of circle P_1 is given by $x^2 + 2x + y^2 + 10y + 6 = 0$.
- The equation of line EM is $y = x - 4$.



4.1	Give the coordinates of centre M .	(3)
	$(x + 1)^2 + (y + 5)^2 = -6 + 1 + 25$ $(x + 1)^2 + (y + 5)^2 = 20$ $M (-1; -5)$	✓ $(x + 1)^2$ ✓ $(y + 5)^2$ ✓ $(-1; -5)$
4.2	Determine the radius of circle P_1 .	(1)
	$r = \sqrt{20}$ (accept $2\sqrt{5}$ or 4,47)	✓ Answer



4.3	C is the point where line EM and circle P ₁ intersect. Determine the coordinates of C, correct to two decimal places. Show all workings.	(6)
	Given: $y = x - 4$ $(x + 1)^2 + (x - 4 + 5)^2 = 20$ $x^2 + 2x + 1 + x^2 + 2x + 1 = 20$ $2x^2 + 4x + 2 - 20 = 0$ $2x^2 + 4x - 18 = 0$ $x^2 + 2x - 9 = 0$ $x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-9)}}{2(1)}$ $x = \frac{-2 \pm \sqrt{40}}{2}$ $x = -1 + \sqrt{10} \quad \text{or} \quad x = -1 - \sqrt{10}$ $x = 2,16 \quad \text{or} \quad x = -4,16$ N/A $y = (-4,16) - 4$ $y = -8,16$ C(-4,16 ; -8,16)	✓ Substitution $2x^2 + 4x - 18 = 0$ subst in quadratic eq $x = -4,16$ ✓ N/A $x = 2,16$ ✓ C(-4,16 ; -8,16) (y -value)
4.4	Determine the gradient of the tangent to circle P ₂ at point E.	(3)
	$M\hat{C}D = 90^\circ \dots \text{radius} \perp \text{tangent}$ $m_{ME} = 1$ (given) $\therefore m_{\text{tangent}} = -1 \dots \text{radius} \perp \text{tangent}$ OR $m_{ME} = \frac{-8,16 + 5}{-4,16 + 1}$ $m_{ME} = 1$ $m_{BC} \times m_{ME} = -1 \dots \text{radius} \perp \text{tangent}$ $\therefore m_{\text{tangent}} = -1$	✓ $M\hat{C}D = 90^\circ$ ✓ $m_{ME} = 1$ ✓ Answer OR ✓ $m_{ME} = 1$ ✓ $m_{BC} \times m_{ME} = -1$ ✓ Answer
4.5	If it is given that $CD = 5\sqrt{2}$, determine MD, the radius of circle P ₂ .	(4)
	$MD^2 = CD^2 + MC^2 \dots \text{Pythagoras}$ $MD^2 = (5\sqrt{2})^2 + (\sqrt{20})^2$ $MD = \sqrt{70} \text{ units}$	✓ S $(5\sqrt{2})^2$ $(\sqrt{20})^2$ $MD = \sqrt{70}$ Accept 8,37 units

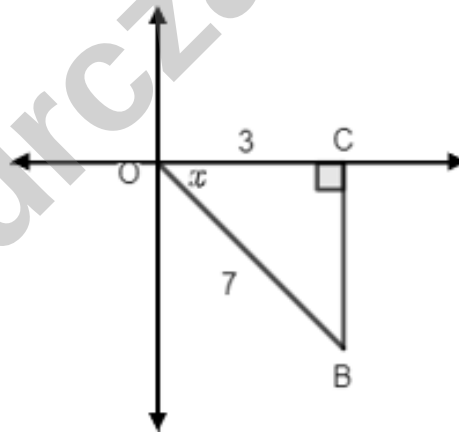


4.6	Write down the equation of P_2 in the form $(x - a)^2 + (y - b)^2 = r^2$.	(1)
	$(x + 1)^2 + (y + 5)^2 = 70$	✓ Answer
4.7	Prove that point $q(\sqrt{5}; 0)$ lies inside circle P_2 . Show all your calculations.	(4)
	Distance from q to centre: $= \sqrt{(\sqrt{5} + 1)^2 + (0 + 5)^2}$ $= 5,96$ $\sqrt{70} = 8,37$ $5,96 < 8,37 (\sqrt{70})$ $\therefore q(\sqrt{5}; 0) \text{ lies inside the circle } C_2.$	✓ correct subst ✓ 5,96 ✓ 8,37 ✓ $5,96 < 8,37 (\sqrt{70})$
		[22]



QUESTION 5

5.1	If $7 \cos x = 3$ and $x \in [90^\circ; 360^\circ]$, calculate the value of $3 \cos 2x$ without the use of a calculator.	(4)
	$\cos x = \frac{3}{7}$ $3 \cos 2x = 3(2 \cos^2 x - 1)$ $= 3 \left[2 \left(\frac{3}{7} \right)^2 - 1 \right]$ $= -\frac{93}{49}$ OR $\cos x = \frac{3}{7}$ $y = \sqrt{(7)^2 - (3)^2}$ $y = 2\sqrt{10}$ $3 \cos 2x = 3(1 - 2 \sin^2 x)$ $= 3 \left[1 - 2 \left(\frac{2\sqrt{10}}{7} \right)^2 \right]$ $= -\frac{93}{49}$ OR $\cos x = \frac{3}{7}$ $3 \cos 2x = 3(\cos^2 x - \sin^2 x)$ $= 3 \left[\left(\frac{3}{7} \right)^2 - \left(\frac{2\sqrt{10}}{7} \right)^2 \right]$ $= -\frac{93}{49}$	✓ $\cos x = \frac{3}{7}$ ✓ Identity ✓ Sub corr ratio ✓ Answer OR ✓ $\cos x = \frac{3}{7}$ ✓ Identity ✓ Sub corr ratio ✓ Answer OR ✓ $\cos x = \frac{3}{7}$ ✓ Identity ✓ Sub corr ratio ✓ Answer (4)





5.2	Prove that $2 \sin x - \cos^2 x$ can be written as $(\sin x + 1)^2 - 2$.	(3)
	$(\sin x + 1)^2 - 2$ $= \sin^2 x + 2 \sin x + 1 - 2$ $= 2 \sin x - 1 + \sin^2 x$ $= 2 \sin x - (1 - \sin^2 x)$ $= 2 \sin x - \cos^2 x$ OR $2 \sin x - \cos^2 x$ $= 2 \sin x - (1 - \sin^2 x)$ $= 2 \sin x - 1 + \sin^2 x$ $= \sin^2 x + 2 \sin x - 1$ $= \sin^2 x + 2 \sin x + 1 - 2$ $= (\sin x + 1)^2 - 2$	✓ quadratic trinomial ✓ Simplify ✓ Identity OR ✓ Identity ✓ quadratic trinomial ✓ (+1)(-2)

5.3.1	Determine the general solution for the value(s) of x if: $5 \sin 2x - \cos 2x = 0$	(5)
	$\frac{5 \sin 2x}{\cos 2x} - \frac{\cos 2x}{\cos 2x} = 0$ $5 \tan 2x - 1 = 0$ $\tan 2x = \frac{1}{5}$ $RA = 11,31^\circ$ $2x = 11,31^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$ $x = 5,66^\circ + k \cdot 90^\circ; k \in \mathbb{Z}$ $\therefore x = 5,66^\circ + k \cdot 90^\circ; k \in \mathbb{Z}$ (Accept 5,65)	✓ $\div \cos 2x$ ✓ $\tan 2x = \frac{1}{5}$ ✓ $RA = 11,31^\circ$ ✓ 1 mark for $x = 5,66^\circ + k \cdot 90^\circ$ ✓ 1 mark for $k \in \mathbb{Z}$



5.3.2	For which value(s) of x will $5 \tan 2x - 1 = 0$ and $x \in [-180^\circ; 180^\circ]$ be undefined?	(2)
	$x \in \{-135^\circ; -45^\circ; 45^\circ; 135^\circ\}$	✓ 1 mark TWO values ✓ 1 mark OTHER TWO values
5.4	Simplify to a single trigonometric function, without the use of a calculator : $-1 + \cos(180^\circ - \theta) \cdot \sin(\theta - 90^\circ)$	(3)
	$ \begin{aligned} & -1 + \cos(180^\circ - \theta) \cdot \sin(\theta - 90^\circ) \\ & = -1 + (-\cos \theta) \cdot (-\cos \theta) \\ & = -1 + \cos^2 \theta \\ & = -(1 - \cos^2 \theta) \\ & = -\sin^2 \theta \end{aligned} $	✓ $-\cos \theta$ ✓ $-\cos \theta$ ✓ Answer
5.5	If $\cos 44^\circ = \sqrt{p}$, determine $\sin^2 68^\circ$ in terms of p , without the use of a calculator .	(4)
	$ \begin{aligned} \cos(2(22^\circ)) &= \sqrt{p} \\ 2 \cos^2 22^\circ - 1 &= \sqrt{p} \\ 2 \cos^2 (90^\circ - 68^\circ) - 1 &= \sqrt{p} \\ 2 \sin^2 68^\circ - 1 &= \sqrt{p} \\ \sin^2 68^\circ &= \frac{\sqrt{p} + 1}{2} \end{aligned} $ OR $ \begin{aligned} \cos 136^\circ &= 1 - 2 \sin^2 68^\circ \\ 2 \sin^2 68^\circ &= 1 - \cos 136^\circ \\ &= 1 + \cos 44^\circ \\ \sin^2 68^\circ &= \frac{\sqrt{p} + 1}{2} \end{aligned} $	✓ $\cos(2(22^\circ))$ ✓ Identity ✓ Reduction (90-68) ✓ Answer OR ✓ $1 - 2 \sin^2 68^\circ$ ✓ Simplify ✓ Acute angle ✓ Answer



5.6	Prove the following identity:	
	$\frac{\sin 7x \cdot \cos 5x - \cos 7x \cdot \sin 5x}{\tan 2x} - 1 = -2 \sin^2 x$	(4)
	$LS = \frac{\sin(7x - 5x)}{\tan 2x} - 1$ $LS = \frac{\sin 2x}{\frac{\sin 2x}{\cos 2x}} - 1$ $LS = \sin 2x \times \frac{\cos 2x}{\sin 2x} - 1$ $LS = \cos 2x - 1$ $LS = 1 - 2 \sin^2 x - 1$ $LS = -2 \sin^2 x = RS$	✓ $\sin(7x - 5x)$ ✓ $\frac{\sin 2x}{\cos 2x}$ ✓ $\cos 2x - 1$ ✓ $1 - 2 \sin^2 x$
5.7	Given: $\tan x = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots}}}}$ Prove that: $\sin x = \sqrt{\cos x(\cos^2 x + \sin x)}$	(5)
	$(\tan x)^2 = \left(\sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots}}}} \right)^2$ $\tan^2 x = \cos x + \tan x$ $\frac{\sin^2 x}{\cos^2 x} = \cos x + \frac{\sin x}{\cos x}$ $\sin^2 x = \cos^3 x + \sin x \cos x$ $\sin^2 x = \cos x(\cos^2 x + \sin x)$ $\sin x = \sqrt{\cos x(\cos^2 x + \sin x)}$	✓ square both sides ✓ Subst $\tan x$ ✓ tan identity ✓ LCD ✓ $\sin^2 x = \dots$
		[30]



QUESTION 6

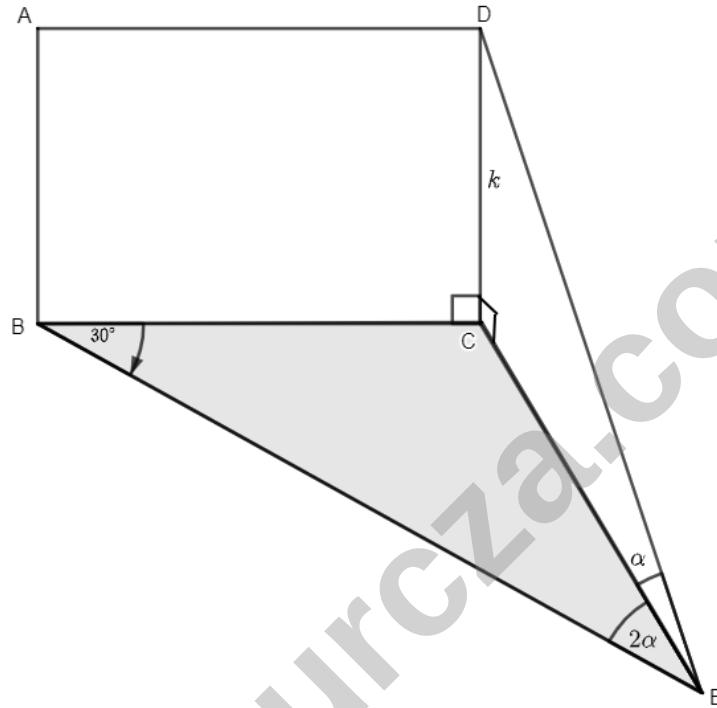
6.1	Determine the value(s) of x without the use of a calculator , if $\sin(x - 30^\circ) + 2 \cos x = 0$; $x \in [-150^\circ; 210^\circ]$ and $\cos x \neq 0$.	(6)
	$\sin x \cdot \cos 30^\circ - \cos x \cdot \sin 30^\circ + 2 \cos x = 0$ $\sin x \cdot \left(\frac{\sqrt{3}}{2}\right) - \cos x \cdot \left(\frac{1}{2}\right) + 2 \cos x = 0$ $\frac{\sqrt{3}}{2} \sin x + \frac{3}{2} \cos x = 0$ $\frac{\sqrt{3} \sin x}{2 \cos x} + \frac{3 \cos x}{2 \cos x} = \frac{0}{\cos x}$ $\frac{\sqrt{3}}{2} \tan x = -\frac{3}{2}$ $\tan x = -\frac{3}{\sqrt{3}}$ $RA = 60^\circ$ $x = 180^\circ - 60^\circ + k \cdot 180^\circ; k \in \mathbb{Z} \quad \text{OR} \quad x = -60^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$ $x \in \{-60^\circ; 120^\circ\}$	✓ compound id. ✓ $\frac{\sqrt{3}}{2}$ ✓ $\frac{1}{2}$ ✓ $\div \cos x$ ✓ $\tan x = -\frac{3}{\sqrt{3}}$ ✓ -60° AND 120°
6.2	The diagram below shows the graph of $f(x) = -2 \cos x$, where $x \in [-150^\circ; 210^\circ]$.	



6.2.1	Sketch the graph of $g(x) = \sin(x - 30^\circ)$ on the same diagram given above. Clearly show all intercepts with the axes as well as the turning points.	(4)
		✓ shape ✓ y- intercept ✓ All x- intercepts ✓ T/P $(-60; -1)$ and $(120; 1)$
6.2.2	Write down the amplitude of f .	(1)
	Amplitude = 2	✓ Answer
6.3	Use your graphs to determine the value(s) of x , in the interval $x \in [-150^\circ; 210^\circ]$ such that:	
6.3.1	$g(x) - f(x) = 0$.	(2)
	At -60°, $g(x) - f(x) = -1 - (-1) = 0$ At 120°, $g(x) - f(x) = 1 - 1 = 0$ $x \in \{-60^\circ; 120^\circ\}$ OR $x = -60^\circ$ or $x = 120^\circ$	✓ -60° ✓ 120°
6.3.2	Indicate, using the symbols A and B, your answer to question 6.3.1 on the graph drawn in question 6.2.1	(1)
		✓ Answer
		[14]

QUESTION 7

In the diagram B, C and E lie in the same horizontal plane. ABCD is a rectangular cardboard piece and CDE is a triangular cardboard piece with a right angle at C, and $DC = k$. The cardboard pieces are placed perpendicular to the horizontal plane as shown in the diagram. The angle of elevation from E to D is α . $\hat{CEB} = 2\alpha$ and $\hat{EBC} = 30^\circ$.



7.1	Write CE in terms of k and α .	(2)
	$\frac{k}{CE} = \tan \alpha$ $CE = \frac{k}{\tan \alpha}$ OR $\frac{CE}{\sin(90^\circ - \alpha)} = \frac{k}{\sin \alpha}$ $CE = \frac{k \cos \alpha}{\sin \alpha}$	✓ tan ratio ✓ Answer OR ✓ sin rule ✓ Answer

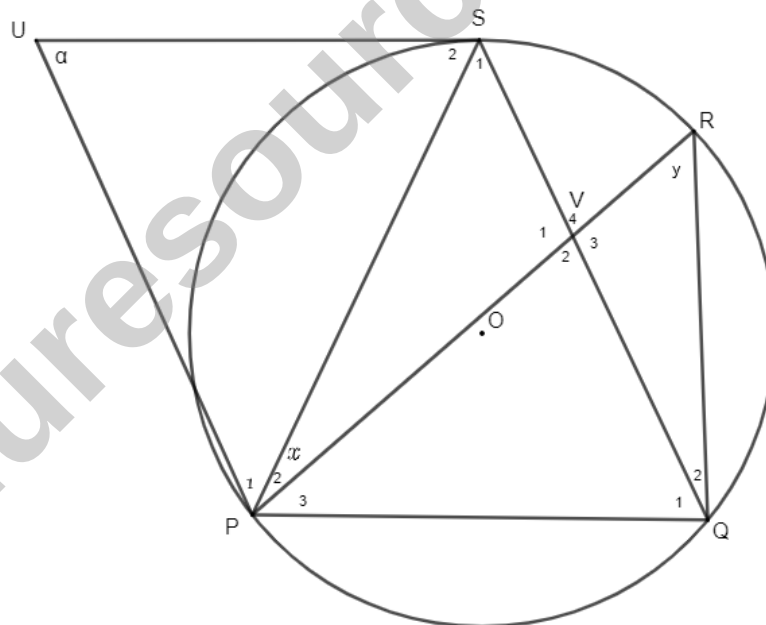


7.2	Prove that $BC = 4k \cos^2 \alpha$	(4)
	$\frac{BC}{\sin 2\alpha} = \frac{CE}{\sin 30^\circ}$ $BC = \frac{\frac{k}{\tan \alpha} \cdot 2 \sin \alpha \cos \alpha}{\frac{1}{2}}$ $BC = \frac{4k \cdot \sin \alpha \cos \alpha}{\frac{\sin \alpha}{\cos \alpha}}$ $BC = 4k \cos^2 \alpha$	✓ Sin rule ✓ double angle identity ✓ $\frac{1}{2}$ ✓ $\frac{\sin \alpha}{\cos \alpha}$
7.3	Show that the area of $\triangle BCE$ is equal to: $\frac{2k^2 \cos^3 \alpha \cdot \sin(30^\circ + 2\alpha)}{\sin \alpha}$	(3)
	$Area \triangle BCE = \frac{1}{2} \cdot BC \cdot EC \cdot \sin(180^\circ - (30^\circ + 2\alpha))$ $Area \triangle BCE = \frac{1}{2} \cdot (4k \cos^2 \alpha) \left(\frac{k}{\tan \alpha} \right) (\sin(30^\circ + 2\alpha))$ $Area \triangle BCE = \frac{2k^2 \cos^2 \alpha \cdot \sin(30^\circ + 2\alpha)}{\tan \alpha}$ $Area \triangle BCE = \frac{2k^2 \cos^2 \alpha \cdot \sin(30^\circ + 2\alpha)}{\frac{\sin \alpha}{\cos \alpha}}$ $Area \triangle BCE = \frac{2k^2 \cos^2 \alpha \cdot \sin(30^\circ + 2\alpha)}{1} \times \frac{\cos \alpha}{\sin \alpha}$ $Area \triangle BCE = \frac{2k^2 \cos^3 \alpha \cdot \sin(30^\circ + 2\alpha)}{\sin \alpha}$	✓ area rule ✓ Substitution ✓ $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$
		[9]

QUESTION 8

In the diagram:

- O is the centre of the circle.
- The circle passes through P, Q, R, and S.
- SQ and PR intersect at V.
- SU is a tangent to the circle at S.
- UP is drawn.
- $\hat{U} = \alpha$.
- $\hat{R} = y$.
- $\hat{P}_2 = x$.
- $\alpha = x + y$.



8.1	Give, with a reason, the size of:	(2)
8.1.1	$\hat{Q}_2$	
	$\hat{Q}_2 = x \dots \angle$'s in same circle segment	✓ S ✓ R

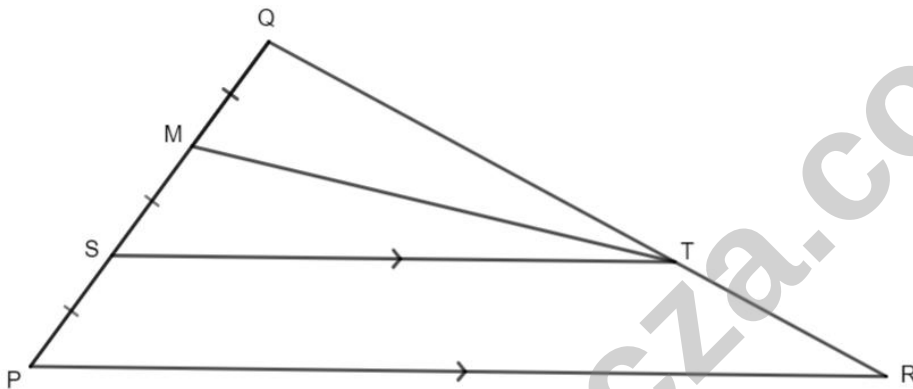


8.1.2	$\hat{S}_1$	(1)
	$\hat{S}_1 = y \dots \angle$'s in same circle segment	✓ S & R
8.2	Prove that VSUP is a cyclic quadrilateral.	(3)
	$\hat{V}_2 = x + y \dots$ Ext $\angle \triangle SPV =$ sum opp int $\angle$'s $\hat{U} = \alpha = x + y \dots$ given $\therefore \hat{V}_2 = \hat{U}$ $\therefore$ VSUP is cyclic quadrilateral $\dots$ Converse ext $\angle =$ opp. Int $\angle$ OR $\hat{V}_2 = x + y \dots$ Ext $\angle \triangle SPV =$ sum opp int $\angle$'s $\hat{V}_1 = 180^\circ - (x + y) \dots \angle$'s on straight line $\hat{V}_1 + \hat{V}_2 = 180^\circ - (x + y) + (x + y)$ $\hat{V}_1 + \hat{V}_2 = 180^\circ$ $\therefore$ VSUP is cyclic quadrilateral $\dots$ Converse Opp $\angle$'s supplementary	✓ S & R ✓ S ✓ R OR ✓ S & R ✓ S ✓ R
8.3	If $PQ = QR$, prove that PQ is a tangent to the circle through V, S, U and P.	(3)
	$\hat{P}_3 = y \dots \angle$'s opposite = sides $\therefore \hat{S}_1 = \hat{P}_3 = y$ PQ is a tangent $\dots$ converse tan-chord theorem	✓ S & R ✓ S ✓ R
		[9]

QUESTION 9

In the diagram $\triangle PQR$ is drawn with S and T points on PQ and QR respectively.

- M is a point on QS.
- $ST \parallel PR$
- $PS = SM = MQ$



9.1	Let $SP = k$. Calculate $\frac{TR}{QT}$.	(2)
	$\frac{TR}{QT} = \frac{SP}{QS}$. . . line parallel to one side of $\triangle$ $\frac{TR}{QT} = \frac{k}{2k} = \frac{1}{2}$	✓ S&R ✓ Answer
9.2	Determine the numerical value of $\frac{\text{Area of } \triangle QMT}{\text{Area of } \triangle QPR}$.	(4)
	$\frac{\text{Area of } \triangle QMT}{\text{Area of } \triangle QPR} = \frac{\frac{1}{2} QM \cdot QT \sin Q}{\frac{1}{2} QP \cdot QR \sin Q}$ $\frac{\text{Area of } \triangle QMT}{\text{Area of } \triangle QPR} = \frac{\frac{1}{2} (k)(2m) \sin Q}{\frac{1}{2} (3k)(3m) \sin Q}$ $\frac{\text{Area of } \triangle QMT}{\text{Area of } \triangle QPR} = \frac{2}{9}$	✓ area rule ✓ $\frac{1}{2}(k)(2m) \sin Q$ ✓ $\frac{1}{2}(3k)(3m) \sin Q$ ✓ Answer

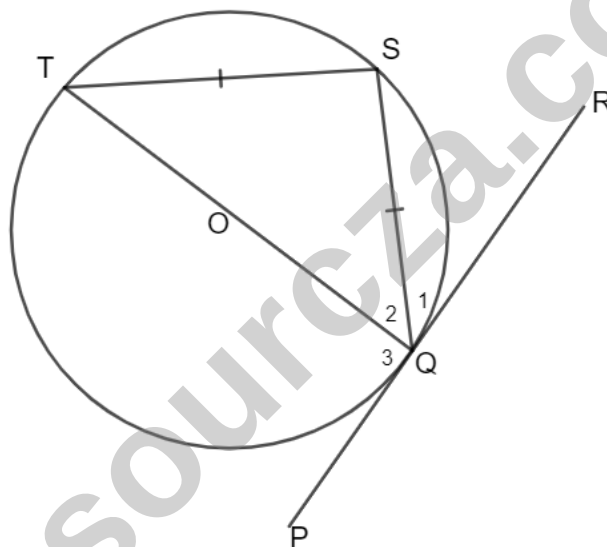


9.3	If $ST = 22$ cm, determine the length of PR .	(4)
	In $\triangle PQR$ and $\triangle SQT$: $\hat{Q} = \hat{Q} \dots$ common angle $\hat{QPR} = \hat{QST} \dots$ corresponding $\angle$'s ; $ST \parallel PR$ $\hat{QRP} = \hat{QTS} \dots$ corresponding $\angle$'s ; $ST \parallel PR$ OR Sum int. $\angle$'s of $\triangle$ $\triangle PQR \parallel \triangle SQT \dots A;A;A$ $\therefore \frac{PR}{ST} = \frac{PQ}{SQ} \dots \triangle PQR \parallel \triangle SQT$ $\frac{PR}{22} = \frac{3k}{2k}$ $PR = 33 \text{ cm}$	✓ common ✓ corresp. $\angle ST \parallel PR$ ✓ S&R $\parallel$ ✓ Answer
		[10]

QUESTION 10

In the diagram:

- TOQ is a diameter of the circle with centre O.
- T, S and Q lie on the circumference of the circle.
- PR is a tangent to the circle at Q.
- TS = SQ.



Determine, with reasons, the size of $\hat{Q}_1$.

(4)

$$\hat{Q}_2 = \hat{T} \dots \angle\text{'s opposite} = \text{sides}$$

$$\hat{T} = \hat{Q}_1 \dots \text{tan-chord}$$

$$\therefore \hat{Q}_1 = \hat{Q}_2$$

$$\hat{Q}_1 + \hat{Q}_2 = 90^\circ \dots \text{radius} \perp \text{tangent}$$

$$\hat{Q}_1 = 45^\circ$$

OR

$$\hat{S} = 90^\circ \dots \angle \text{in semi-circle}$$

$$\hat{Q}_2 = \hat{T} \dots \angle\text{'s opposite} = \text{sides}$$

$$\hat{Q}_2 = 45^\circ$$

$$\hat{T} = \hat{Q}_1 \dots \text{tan-chord}$$

$$\therefore \hat{Q}_1 = 45^\circ$$

✓ S & R

✓ S & R

✓ S & R

✓ Answer

OR

✓ S & R

✓ S & R

✓ S & R

✓ Answer

[4]

QUESTION 11

11.1	In the diagram $\triangle ABC$ and $\triangle DEF$ is given with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. Use this diagram to prove that $\frac{AB}{ED} = \frac{BC}{EF}$.	(6)
	Construction: Draw GH such that $AB=DG$ and $AC=DH$. In $\triangle ABC$ and $\triangle DGH$: $AB = DG$ Construction $AC = DH$ Construction $\hat{A} = \hat{D}$ Given $\therefore \triangle ABC \equiv \triangle DGH$ S; A; S In $\triangle DEF$: $D\hat{G}H = \hat{B}$ $\triangle ABC \equiv \triangle DGH$ But $\hat{B} = \hat{E}$ Given $D\hat{G}H = \hat{E}$ $\therefore GH \parallel EF$ Corresponding angles are equal $\frac{DG}{DE} = \frac{DH}{DF}$ Line parallel to one side of $\triangle$ But $DG = AB$ And $DH = AC$ Construction $\therefore \frac{AB}{DE} = \frac{AC}{DF}$	✓ Construction ✓ $\triangle ABC \equiv \triangle DGH$ ✓ S;A;S ✓ $D\hat{G}H = \hat{E}$ ✓ $GH \parallel EF$ & R ✓ R



11.2	In the diagram: • POS is the diameter of the circle with centre O. • QAS and PAR are straight lines. • P, Q, R, S and T lie on the circumference of the circle. • $QA = AS$. • $\hat{S}_1 = \hat{S}_2$. • SA produced meets the circle in Q. • QR, PT and ST are drawn.	
11.2.1	Write down, with a reason, the size of $\hat{T}$.	(2)
	$\hat{T} = 90^\circ \dots \angle$ in semicircle.	✓ 90° ✓ R



11.2.2	Prove that $\triangle PTS \parallel \triangle OAS$.	(4)
	$\hat{A}_2 = 90^\circ \dots$ line from midpoint of circle to midpoint of chord is $\perp$ on chord. In $\triangle PTS$ and $\triangle OAS$: $\hat{T} = \hat{A}_2 = 90^\circ \dots$ proven $\hat{S}_1 = \hat{S}_2 \dots$ given $\therefore \hat{P}_1 = \hat{O}_2 \dots$ int $\angle$'s $\triangle$'s $\therefore \triangle PTS \parallel \triangle OAS \dots$ A;A;A	✓ S&R ✓ S&R ✓ S&R ✓ R
11.2.3	Prove that $TS \cdot OP = QA \cdot PS$	(4)
	$\frac{TS}{AS} = \frac{PS}{OS} \dots \triangle PTS \parallel \triangle OAS$ But $QA = AS \dots$ given And $OS = OP \dots$ radii $\therefore \frac{TS}{QA} = \frac{PS}{OP}$ $\therefore TS \cdot OP = QA \cdot PS$	✓ S & R ✓ S ✓ S & R ✓ Ratio (4)
		[16]
GRAND TOTAL: [150]		