



eduresourceza.com



KINGSWOOD COLLEGE
MATHEMATICS: PAPER 2

GRADE 12

26 Sept 2023

150 Marks

Time: 3 hours

EXAMINER: LP
MODERATOR: AT, JS

Name:

Memo V.2.

Teacher: LP AT

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. Write your name in the space provided and circle your Mathematics teacher's initials.
2. This question paper consists of **19 pages** and **9 questions**. Page 20 is extra space to write.
Check that your paper is complete.
3. Read the questions carefully.
4. All necessary working must be shown.
5. Approved non-programmable calculators may be used.
6. Answers must be rounded off to one decimal digit, unless otherwise stated.
7. Diagrams must be neat and clearly labelled.
8. Diagrams are not drawn to scale.
9. It is in your own interests to write legibly and present your work neatly.
10. A formula sheet is provided.

QUESTION	1	2	3	4	5	6	7	8	9	TOTAL
ACHIEVED										
TOT	18	16	3	13	22	18	6	25	19	150
	stat	A geom	E geom	trig	AT trig	A geom	trig	E geom	Trig 5 A ge7 Mea 7	

- a) The following data represents the number of people that recovered from Covid-19 in 10 African countries as of 20 July 2020.

RECOVERED									
8	39	63	69	104	141	183	191	301	592
Source: https://www.worldometers.info/coronavirus/?utm_campaign=homeAdvegas1?#countries									

- 1) Determine the standard deviation.

L1
(1)

$$\sigma = 163,1 \checkmark \textcircled{1} \rightarrow \text{calculator}$$

- 2) How many countries lie within ONE standard deviation of the mean?

L2
(4)

$$\bar{x} = 169,1 \checkmark$$

$$(5,97; 332,2) \quad (1\bar{x} - \sigma; 1\bar{x} + \sigma) \quad \text{upper fence}$$

9 countries $\textcircled{4}$

- 3) When the total number of recoveries from Mozambique was added, the mean recoveries changed to 209,72. Determine how many people recovered in Mozambique.

L3
(3)

$$\Sigma = 1691 \checkmark$$

$$\bar{x} = \frac{\Sigma x}{n}$$

$$209,72 \times 11 = 2306,9 \checkmark$$

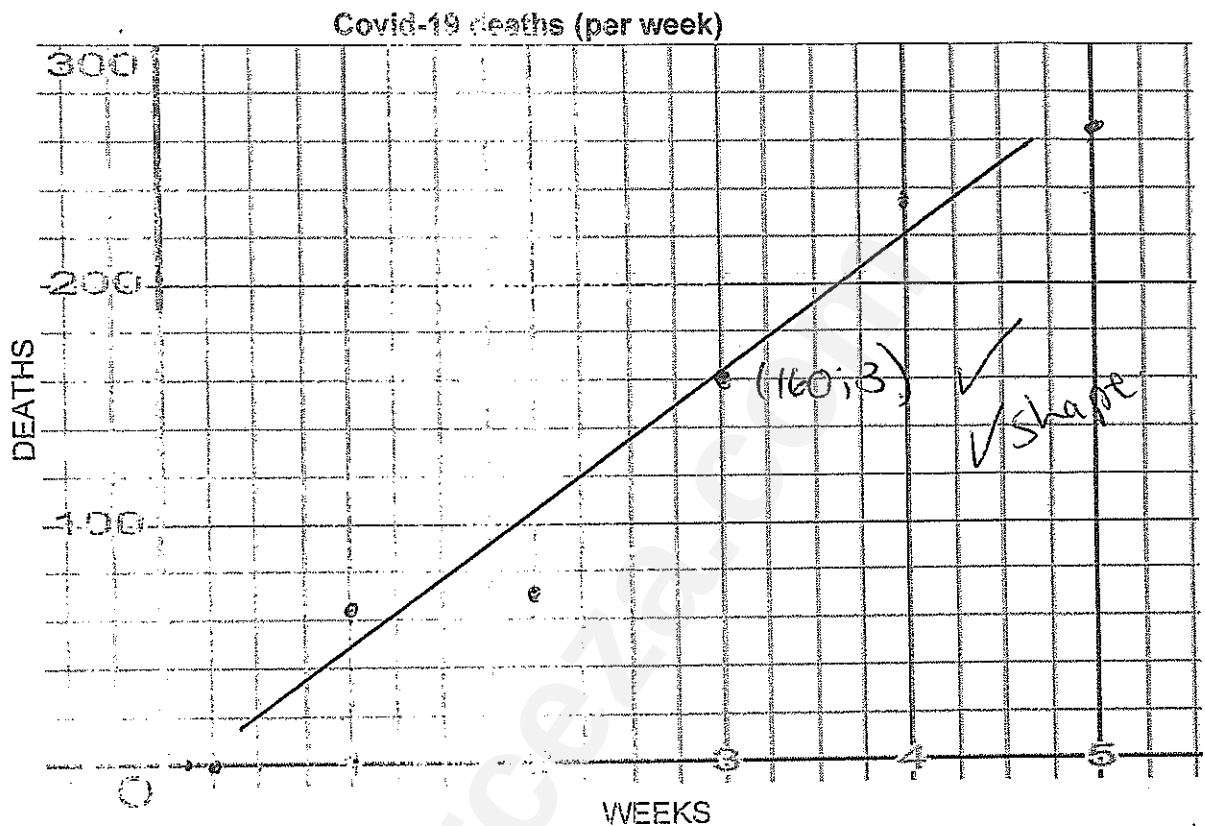
$$2306,9 - 1691 = 615,9 \checkmark \textcircled{3}$$

$$\approx 616$$

- b) The table below represents the number of Covid-19 deaths (per week) recorded, for 5 weeks, in South Africa, since 01 May 2020.

Week	1	2	3	4	5
No of Deaths	62	68	160	236	265
Source: https://www.worldometers.info/coronavirus/country/south-africa/					

- 1) Represent the information above in a scatter plot on the grid provided below. (2)



- 2) Determine the equation of the least squares regression line (2)

$$y = -13,5 + 57,3x \quad (2)$$

- 3) Draw the least squares regression line on the scatter plot on the graph above. (2)

Handwritten notes: $(\bar{x}; \bar{y}) \rightarrow (3; 160)$ and $(1; 43,8)$, $(2; 101,1)$, $(4; 215,7)$, $(5; 273)$.

- 4) Determine the correlation coefficient of the above data and describe the relation between the weeks and the number of deaths. Round answer off to two decimal places (2)

Handwritten notes: $r = 0,97$ strong positive (2)

- 5) Estimate the number of deaths in week 12. (1)

Handwritten notes: $\hat{y} = 674,1$ and ≈ 674 (1)

- 6) The actual number of deaths in week 12 was 1747. Give a possible reason for the difference. (1)

Handwritten notes: $\checkmark$ reason (1) Humankind will be wiped out.

Handwritten notes: No cure found.

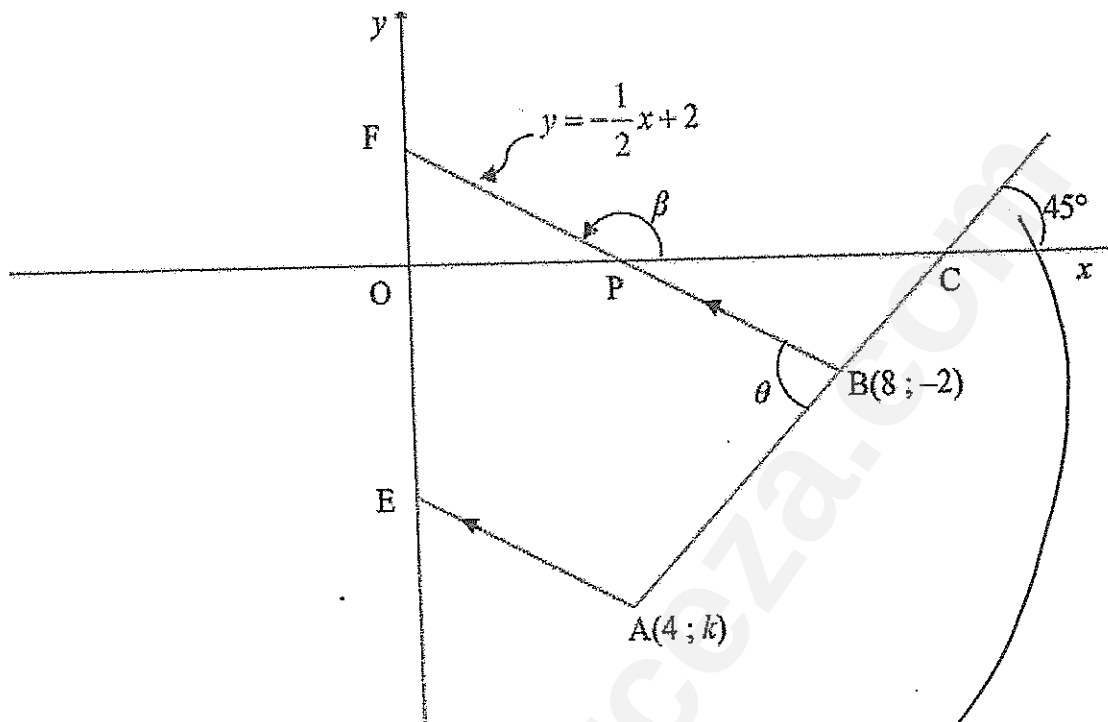
Handwritten notes: No safety measures in place.

Handwritten notes: Disease spread faster than data suggests.

QUESTION 2

Downloaded from eduresourceza.com

In the diagram below, the line BF is drawn from B(8 ; -2) to cut the x-axis at P and the y-axis at F. The inclination of BF is β and the equation of BF is $y = -\frac{1}{2}x + 2$. From A(4 ; k), another line is drawn parallel to BF and cuts the y-axis at E. The line passing through A and B has an inclination of 45° and cuts the x-axis at C. $\widehat{ABF} = \theta$



- a) Determine the gradient of AB

$$\tan 45^\circ = m$$

$$m = 1 \checkmark$$

$$\tan \theta = m$$

L1
(1)

- b) Show that the value of k is -6.

$$1 = \frac{-2 - k}{8 - 4} \checkmark$$

$$k = -6 \checkmark$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

L2
(2)

Downloaded from eduresourceza.com

- c) Determine the equation of EA.

L2
(3)

$$m = -\frac{1}{2} \quad \checkmark \quad FB \parallel EA \quad m_{FB} = m_{EA}$$

$$(4; -6) \quad y + 6 = -\frac{1}{2}(x - 4) \quad \checkmark$$

$$y = -\frac{1}{2}x - 4 \quad \checkmark \quad (3)$$

- d) Calculate the size of
- β

L1
(1)

$$\tan \beta = \frac{1}{2}$$

$$\beta = 153,4^\circ \quad \checkmark \quad (1)$$

- e) Calculate the size of
- θ

L2
(3)

CA
Construct AP:

$$AB = 4\sqrt{2}$$

$$PB = 2\sqrt{5}$$

$$6^2 = (4\sqrt{2})^2 + (2\sqrt{5})^2 - 2(4\sqrt{2})(2\sqrt{5}) \cos \theta$$

- f) Determine the length of BF

L1
(3)

$$d_{BF} = \sqrt{(0-8)^2 + (2+2)^2} \quad \checkmark \quad F(0; 2)$$

$$= 8,9 = 4\sqrt{5} \quad \checkmark$$

(3)

- e) Area of
- $\triangle EFB$
- .

CA
EFB = 63,4 sum LA

$$\text{Area} = \frac{1}{2} \times 4\sqrt{5} \times 6 \sin 63,4$$

$$= 24 \text{ units}^2 \quad \checkmark \quad (3)$$

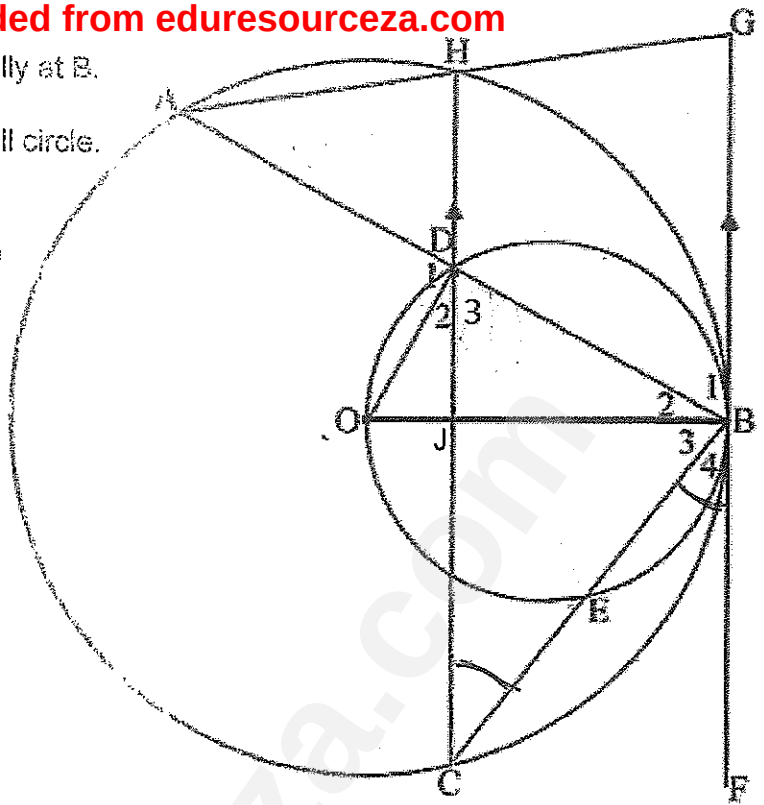
$$\text{Area} = \frac{1}{2} (6)(8) \quad \checkmark \quad (3)$$

$$= 24 \text{ units}^2$$

QUESTION 3

Downloaded from eduresourceza.com

Two circles touch each other internally at B.
O, the centre of the bigger circle,
lies on the circumference of the small circle.
FBG is a common tangent.
AB and BC are chords of the bigger circle and intersect the smaller circle at D and E respectively.
AHG forms straight line.
Chords OD, and OB
of the smaller circle are drawn.
 $AH : AG = 1 : 2$



a) Give a reason why:

1) $\angle AHB = \angle HCB$

alternating $\angle$ s

$\angle 1$
(1)

2) $\angle CAB = \angle HCB$

$\angle$'s in same seg

$\angle 1$
(1)

3) $\angle OBG = 90^\circ$

rad $\perp$ tan

$\angle 1$
(1)

4) $AC = AB$

line $\parallel$ to one side Δ / conv midpt

$\angle 1$
(1)

Downloaded from eduresourceza.com

- b) Prove that:
 $\angle ODB = 90^\circ$

$B_3 + B_4 = 90^\circ$ ✓
 $D_2 + D_3 = 90^\circ$ ✓

cR. $B_2 + B_1 = 90^\circ$ ✓
 $\angle ODB = 90^\circ$
 conv. L in Semi Circle

$AD = DB$ ✓
 $D_2 + D_3 = 90^\circ$ line from centre bisect chord.

c) Determine the ratio of $\frac{\text{Area of } \triangle AHD}{\text{Area of } \triangle AGB} = \frac{\frac{1}{2} AH \cdot AD \cdot \sin A}{\frac{1}{2} AG \cdot AB \cdot \sin A}$
 $= \frac{1 \cdot 1}{2 \cdot 2}$
 $= \frac{1}{4}$

L 1
(2)

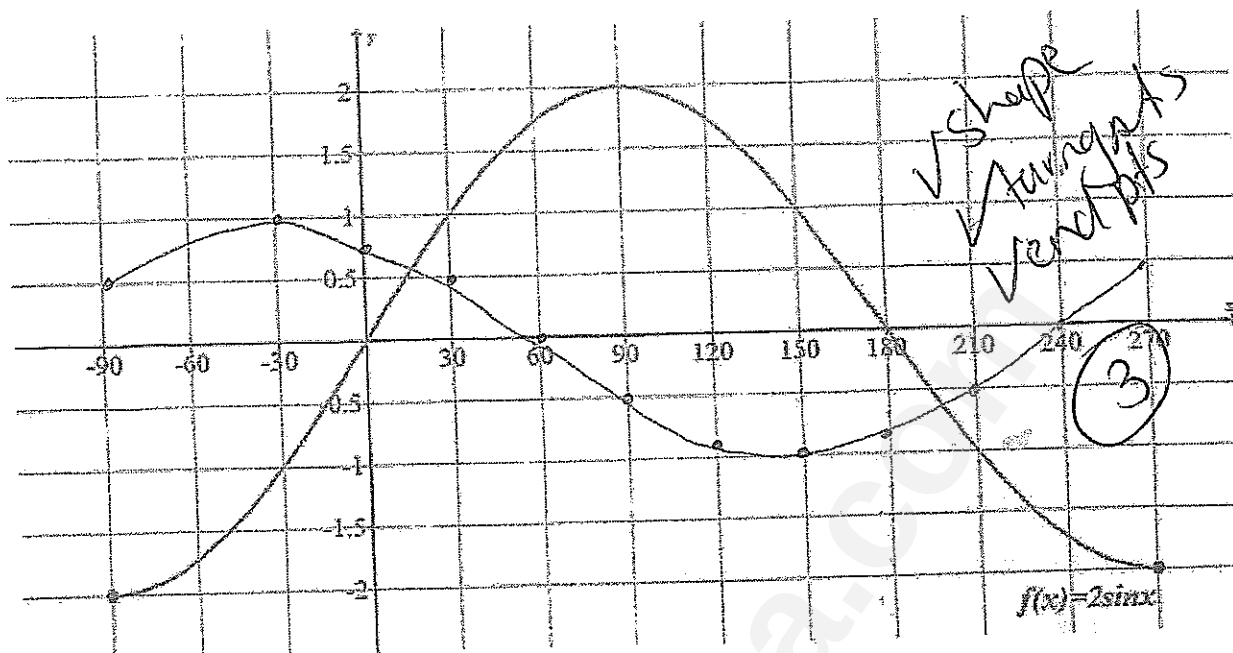
answer only 2/2.

[8]

QUESTION 4

Downloaded from eduresourceza.com

In the diagram the graph of $f(x) = 2\sin x$ is drawn for the interval $x \in [-90^\circ; 270^\circ]$.



- a) Write down the period of f

360° ✓ (1)

- b) Write down the values of x in the interval $x \in [-90^\circ; 270^\circ]$ for which f is decreasing. (1)

$x \in (90^\circ; 270^\circ)$ ✓ (1)

- c) Use the graph of f to determine for which value(s) of x will $f(x) \geq 1$ for $x \in [-90^\circ; 270^\circ]$. (1)

$x \in [30^\circ; 150^\circ]$ ✓ (1)

- d) The function h is obtained by translating the graph of f , 30° to the right and 1 unit down. Write down the equation of h . (2)

$h(x) = 2\sin(x - 30^\circ) - 1$ ✓ (2)

- e) Draw the graph of g if $g(x) = \cos(x + 30^\circ)$, for the interval $x \in [-90^\circ; 270^\circ]$ on the diagram above. Show all the intercepts with the axes, as well as the turning points. (3)

use your calculator.

Downloaded from eduresourceza.com

f) We can determine the point of intersection of $g(x)$ and $f(x)$.

1) Show that $2\sin x = \cos(x + 30^\circ)$ can also be expressed as $\tan x = \frac{\sqrt{3}}{5}$ (4) L2

$$2\sin x = \cos x \cos 30^\circ - \sin x \sin 30^\circ$$

$$2\sin x = \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x$$

$$2\frac{1}{2}\sin x = \frac{\sqrt{3}}{2} \cos x$$

$$\frac{\sin x}{\cos x} = \frac{\sqrt{3}}{5}$$

$$\tan x = \frac{\sqrt{3}}{5}$$

(4)

2) Hence, determine the values of x , where $f(x) = g(x)$, for $x \in [-90^\circ; 270^\circ]$ (3) L2

$$\tan x = \frac{\sqrt{3}}{5}$$

$$\text{ref } x = 19.1^\circ$$

(3)

$$x = 19.1 + k180^\circ \quad k \in \mathbb{Z}$$

$$x \in \{19.1^\circ; 199.1^\circ\}$$

(-1 no k ∈ Z)

g) Determine for which values of x , $x \in [0^\circ; 270^\circ]$ is:

$$f(x) \cdot g(x) \leq 0$$

$$x \neq 0$$

$$[60^\circ; 180^\circ] \cup [240^\circ; 270^\circ]$$

(3)

[18]

$$\cos(x + 30^\circ)$$

$$= \sin[90^\circ - (x + 30^\circ)]$$

$$= \sin(60^\circ - x)$$

NO CALCULATORS ALLOWED IN THIS QUESTION

$$\frac{y}{r} = \frac{y}{\sqrt{x^2 + y^2}}$$

L2
(3)

- a) If $\tan 24^\circ = m$, write $\sin 336^\circ$ in terms of m .

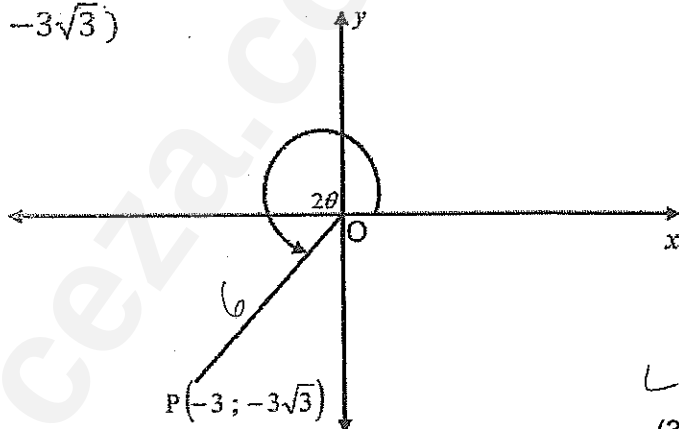
$$r = \sqrt{m^2 + 1} \checkmark$$

$$\begin{aligned} \sin 336^\circ &= -\sin 24^\circ \checkmark \\ &= -\frac{m}{\sqrt{m^2 + 1}} \checkmark \end{aligned}$$

(3)

- b) In the diagram alongside, point $P(-3; -3\sqrt{3})$ and reflex angle 2θ are shown.

$$r = 6 \checkmark$$



Determine the value of:

1) $\cos 2\theta = \frac{-3}{6} = -\frac{1}{2} \checkmark$

(3)

L2
(3)

- 2) $\sin \theta$

$$\cos 2\theta = 1 - 2\sin^2 \theta = -\frac{1}{2} \checkmark$$

$$-2\sin^2 \theta = -\frac{3}{2} \checkmark$$

$$-\sin^2 \theta = -\frac{3}{4} \checkmark$$

$$\sin \theta = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2} \checkmark$$

$$= \pm 0,9$$

L3
(3)

c) Show that $\cos 2x + \cos(-x) \cdot \tan x + \cos(90^\circ + x) = 1 - 3\sin^2 x$

L3
(5)

$$\begin{aligned} &\Rightarrow \cos 2x + \cos x \times \tan x - \sin x \\ &= 1 - 2\sin^2 x + \cos x \times \frac{\sin x}{\cos x} - \sin x \\ &= 1 - 2\sin^2 x - \sin^2 x \\ &= 1 - 3\sin^2 x. \checkmark \quad (5) \end{aligned}$$

d) Given: $\cos(x - y) - \cos(x + y)$

L2
(4)

1) Prove $\cos(x - y) - \cos(x + y) = 2\sin x \sin y$

$$\begin{aligned} &\cos x \cos y + \sin x \sin y - (\cos x \cos y - \sin x \sin y) \\ &= \cos x \cos y + \sin x \sin y - \cos x \cos y + \sin x \sin y \\ &= 2\sin x \sin y \checkmark \quad (4) \end{aligned}$$

2) Hence, calculate the numerical value of $2\sin 195^\circ \sin 45^\circ$

L3
(4)

$$\begin{aligned} &\cos(195 - 45) - \cos(195 + 45) \checkmark \text{ sub} \\ &= \cos 150 - \cos 240 \\ &= -\cos 30 + \cos 60 \checkmark \text{ acute L's} \\ &= -\frac{\sqrt{3}}{2} + \frac{1}{2} \checkmark \text{ special.} \\ &= \frac{1 - \sqrt{3}}{2} \checkmark \text{ No calculator.} \quad (4) \end{aligned}$$

[22]

$$\sin(180 + 15) - \sin(45)$$

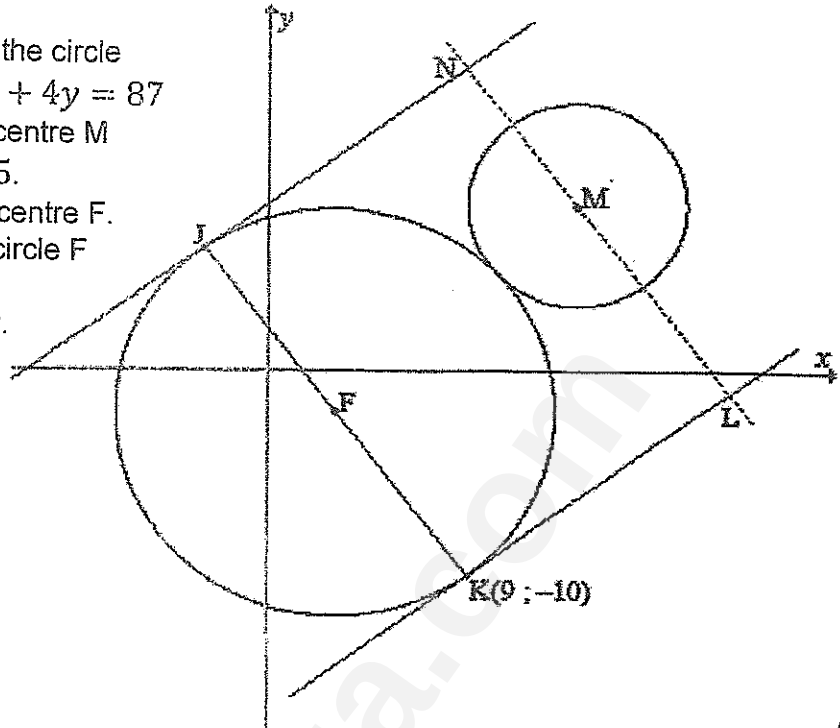
$$-\sin 15.$$

$$-\frac{1}{\sqrt{2}}$$



max $\frac{2}{4}$
for not using
d(1).

In the diagram, the equation of the circle with centre F is $x^2 + y^2 - 6x + 4y = 87$
 The equation of the circle with centre M is $(x - 14)^2 + (y - h)^2 = 25$.
 The point K(9; -10) lies on the circle F.
 JN and KL are tangents to the circle F at the points J and K.
 The two circles touch externally.



- a) Determine the radius and the coordinates of centre of the circle F.

L1
(3)

$$x^2 - 6x + \left(-\frac{6}{2}\right)^2 + y^2 + 4y + \left(\frac{4}{2}\right)^2 - \left(-\frac{6}{2}\right)^2 - (2)^2 = 87$$

$$(x-3)^2 + (y+2)^2 = 100 \quad \checkmark$$

$$F(3; -2) \quad r=10 \quad \checkmark \text{ both } \textcircled{3}$$

- b) Show, with calculations, that $h = -2 + 2\sqrt{26}$

L3
(4)

$$FM = 15 \text{ units } \checkmark$$

$$15 = \sqrt{(14-3)^2 + (h+2)^2} \quad \checkmark$$

$$225 = 121 + h^2 + 4h + 4$$

$$0 = h^2 + 4h - 100 \quad \checkmark$$

$$h = -2 + 2\sqrt{26} \quad \checkmark \quad h = -2 - 2\sqrt{26}$$

$\textcircled{4}$

- c) Determine the equation of the tangent JN.

$$m_{FK} = \frac{-10+2}{9-3} = -\frac{4}{3} \checkmark$$

$$m_{JN} = \frac{3}{4} \tan \perp \text{rad.} \checkmark$$

$$m_1 \times m_2 = -1 \checkmark$$

$$3 = \frac{x+9}{2} - 2 = \frac{y-10}{2}$$

$$J(3;6)$$

$$y-6 = \frac{3}{4}(x+3) \checkmark$$

$$y = \frac{3}{4}x + \frac{33}{4} \checkmark$$

$$y = \frac{3}{4}x + 8,3$$

- d) The line NL passes through the centre M, such that NL $\parallel$ JK. The circle with centre M, is translated along the line NL and between the two tangents of circle F. Determine the x-value of N, such that the circle M will not intersect the tangent.

$$y_{JN} = \frac{3}{4}x + \frac{33}{4}$$

$$m_{MN} = -\frac{4}{3} \checkmark$$

$$m_{FK} = -\frac{4}{3}$$

$$\text{OR } m_{MN} = -\frac{4}{3}$$

FK $\parallel$ MN.

$$M(14; -2 + 2\sqrt{26})$$

$$y - (-2 + 2\sqrt{26}) = -\frac{4}{3}(x - 14)$$

$$y = -\frac{4}{3}x + \frac{56}{3} + (-2 + 2\sqrt{26})$$

$$y = -\frac{4}{3}x + 26,9 \checkmark$$

$$-\frac{4}{3}x + 26,9 = \frac{3}{4}x + \frac{33}{4} \checkmark$$

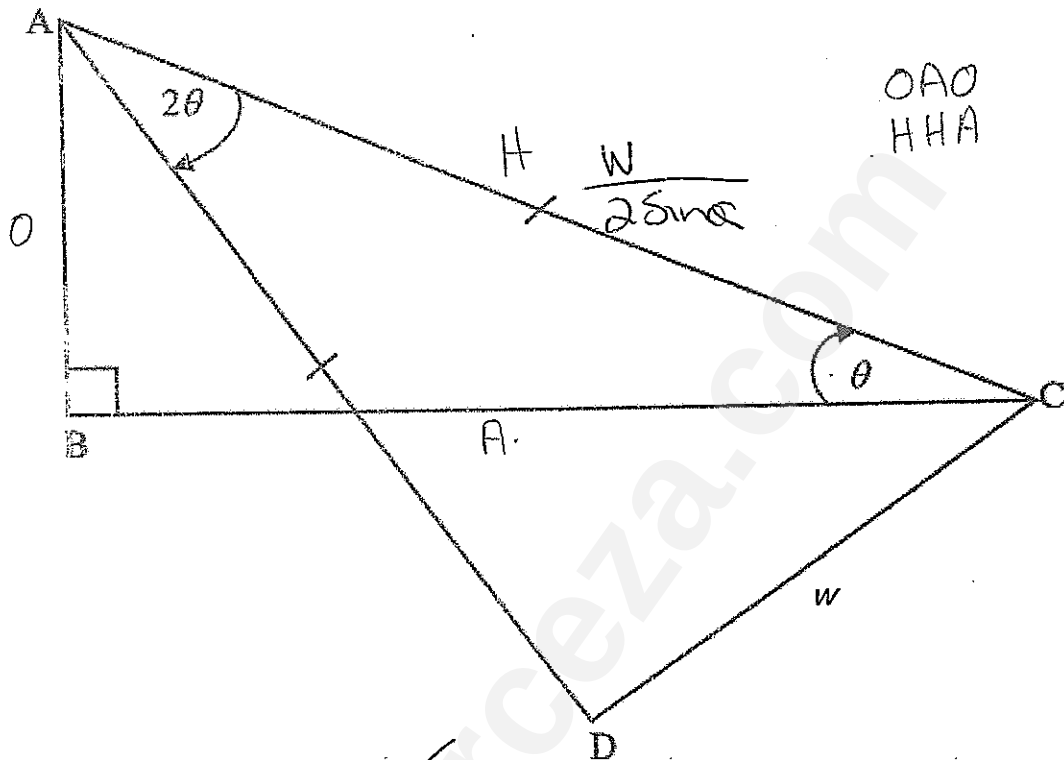
$$-\frac{25}{12}x = -\frac{373}{20}$$

$$x = 8,9 \checkmark$$

QUESTION 7

Downloaded from eduresourceza.com

In the diagram below, AB is a tower, anchored by two cables at C and D. B, C and D are in the same horizontal plane. The angle of elevation from C to the top of the pole, A, is θ . $\angle DAC = 2\theta$ and $AC = AD$ and CD, the distance between the two anchors, is w . Show that the height of the tower (AB) is $\frac{w}{2}$. (6)



$$\hat{D} = \frac{180^\circ - 2\theta}{2} = 90^\circ - \theta \quad \text{sum } \angle \text{ in } \triangle ADC$$

$$\frac{AC}{\sin(90^\circ - \theta)} = \frac{w}{\sin 2\theta} \quad \checkmark$$

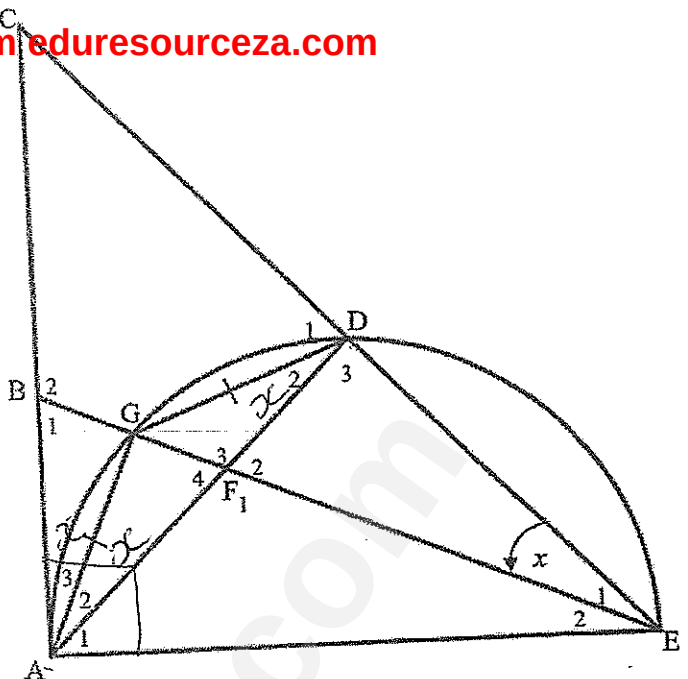
$$AC = \frac{w \cos \theta}{2 \sin \theta \cos \theta} = \frac{w}{2 \sin \theta} \quad \checkmark$$

$$\frac{AB}{AC} = \sin \theta \quad \checkmark$$

$$\begin{aligned} AB &= \sin \theta \cdot AC \\ &= \sin \theta \times \frac{w}{2 \sin \theta} \\ &= \frac{w}{2} \quad \checkmark \end{aligned}$$

(6)

- a) In the figure AGDE is a semicircle.
AC is the tangent to the semicircle at A and EG produced intersects AC at B.
AD intersects BE in F.
AG = GD.
 $\widehat{E_1} = x$



- 1) Write down, with reasons, FOUR other angles each equal to x .

L2
(8)

$\hat{E}_2$ ✓ eq. lines sub = LS

$\hat{A}_3$ ✓ tan chord

$\hat{D}_2$ ✓ tan chord

$\hat{A}_2$ ✓ iso Δ ✓ / $\angle$ in same seg. (8)

L3
(6)

- 2) Prove that $BE \cdot DE = AE \cdot FE$

In ΔBEA and ΔDEF

$\hat{A} = 90^\circ$ rad tan $\hat{D}_3 = 90^\circ$ $\angle$ in semi $\odot$

$\hat{E}_1 = \hat{E}_2 = x$ proven.

$\hat{B}_1 = \hat{F}_2$ sum Δ . (6)

$\therefore \Delta AEB \sim \Delta DEF$ (AAA)

$\frac{AE}{DE} = \frac{EB}{EF} = \frac{AB}{DF}$ ✓ $\therefore BE \cdot DE = AE \cdot EF$

L3
(3)

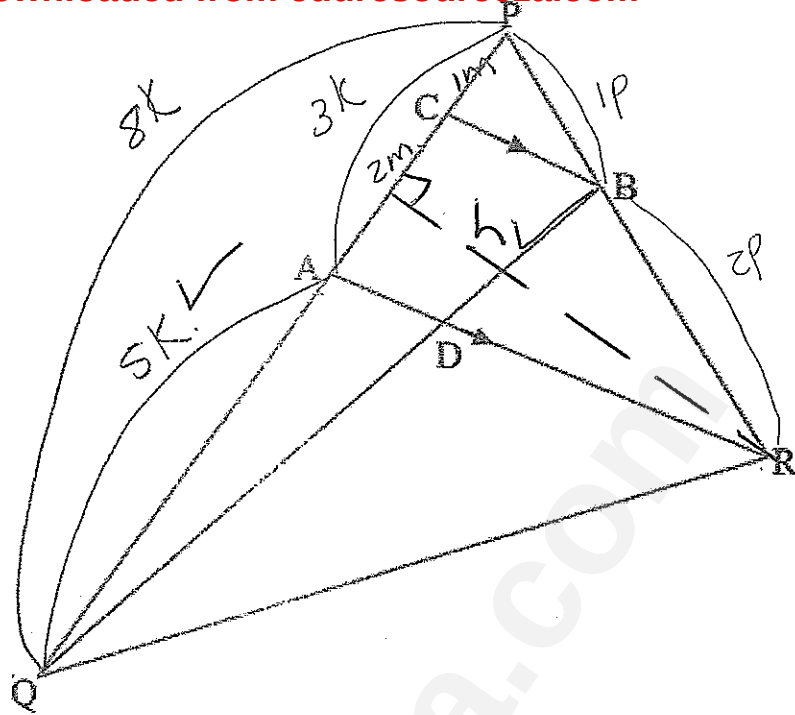
- 3) Prove that $EA^2 = DE \cdot EC$

$\Delta ACE \sim \Delta DAE$ ✓ line from 90° vertex $\perp$ hyp

$\frac{AE}{DE} = \frac{CE}{AE}$ ✓

$AE^2 = DE \cdot EC$ ✓ (3)

- b) In $\triangle PQR$ below, B lies on PR such that $2PB = BR$. A lies on PQ such that $PA : PQ = 3 : 8$. BC is drawn parallel to AR.



1) Determine the ratio of $\frac{\text{Area of } \triangle PRA}{\text{Area of } \triangle QRA} = \frac{\frac{1}{2} \times 3K \times 1h}{\frac{1}{2} \times 5K \times 1h}$

$$\frac{\frac{1}{2} PA \times 1h}{\frac{1}{2} PQ \times 1h} = \frac{3}{5} \checkmark \textcircled{3}$$

L2
t3
(3)

$\frac{1h \checkmark}{5K \checkmark}$

- 2) Calculate the value of the ratio $BQ : BD$. Show all working to support your answer. (3)

$\frac{BQ}{BD} = \frac{PC}{AC} = \frac{PB}{BR} = \frac{1}{2}$ line $\parallel$ (BC || AR) to one side

$PA = 3m = 3K$
 $AC = 2K$
 $AG = 5K$
 $CQ = 7K$

$\frac{BQ}{BD} = \frac{CQ}{CA} = \frac{7K}{2K}$ line $\parallel$ to one side $\checkmark$ reason

$\frac{BQ}{BD} = \frac{7}{2}$

$\textcircled{5}$

[25]

a) Solve for α :

$$\sin(3\alpha - \beta) = \frac{1}{\sqrt{2}} \text{ if } 3\alpha - \beta \in [0^\circ; 90^\circ] \text{ and}$$

$$\tan(2\alpha + \beta) = \frac{1}{\sqrt{3}} \text{ if } 2\alpha + \beta \in [90^\circ; 270^\circ]$$

L4
(5)

$$3\alpha - \beta = 45^\circ \checkmark$$

$$3\alpha - 45 = \beta \quad \text{--- (1)}$$

$$2\alpha + \beta = 30^\circ + 180^\circ \checkmark$$

$$2\alpha + \beta = 210^\circ \checkmark \quad \text{--- (2)}$$

$$2\alpha + 3\alpha - 45^\circ = 210^\circ \checkmark$$

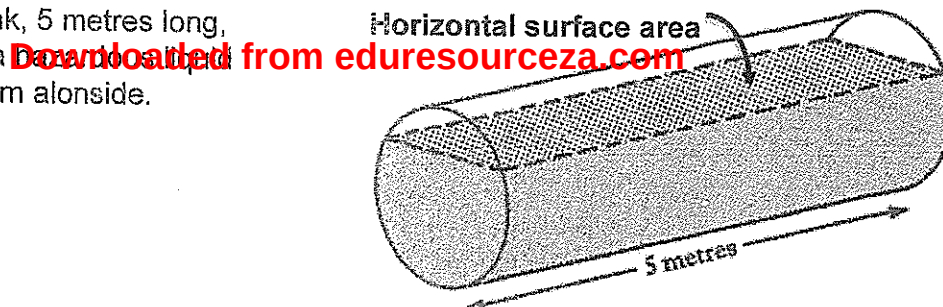
$$5\alpha = 225$$

$$\alpha = 45^\circ \checkmark$$

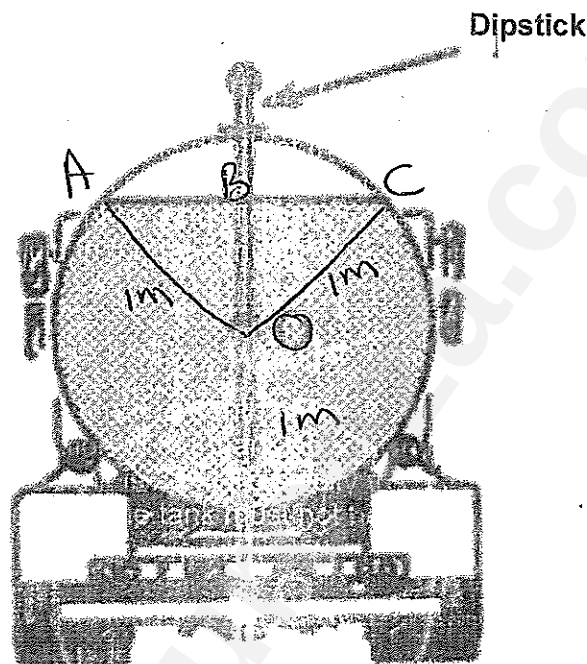
(5)

$$\left(\begin{array}{l} 3\alpha - 45^\circ = -2\alpha + 30^\circ \\ \alpha = 15 \quad 4/5 \end{array} \right)$$

- b) A cylindrical tank, 5 metres long, is used to store a liquid as in the diagram alongside.



A dipstick is used to measure the depth of the liquid. The dipstick passes through the centre, O, of a cross-section of the tank as shown in the diagram below. The diameter of the cross-section of the tank is 2 m. To satisfy the storage conditions, the horizontal surface area of the liquid in the tank must not be less than 2 m^2 as shown in the diagrams.



Calculate the minimum volume of the liquid that is allowed in the tank.

$$\text{Surface area} \Rightarrow 2 = x \times 5$$

$$AC \times = \frac{2}{5} \checkmark = AC$$

$$AB = BC = \frac{0.4}{2} \text{ line from centre to chord.} \checkmark$$

$$OB = \frac{2\sqrt{6}}{5}$$

$$\angle AOC = 23.0739^\circ$$

$$\text{Area } AOC = \frac{1}{2} \times 0.4 \times \frac{2\sqrt{6}}{5}$$

$$= \frac{2\sqrt{6}}{25}$$

$$\text{Volume } AOC = \frac{2\sqrt{6}}{25} \times 5 \checkmark$$

$$= \frac{2\sqrt{6}}{5} \text{ m}^3 = 0.9768$$

$$\text{Volume rest} = \frac{337}{360} \times \pi \times 1^2 \times 5$$

$$= \frac{337}{72} \pi = 14.7$$

$$\text{Total } V = 15.7 \text{ m}^3 \checkmark$$

$$x = \frac{2}{5} \checkmark = AC^{(7)}$$

$$\angle AOC \quad (7)$$

$$\cos \angle AOC = \frac{(\frac{2}{5})^2 - 1^2 - 1^2}{-2(1)(1)} \checkmark$$

$$\angle AOC = 23.0739^\circ$$

$$\text{Volume } AOC \checkmark$$

$$= \frac{1}{2} \times 1 \times 1 \times 5 \sin 23.0739^\circ$$

$$= 0.9768 \checkmark$$

$$\text{Volume rest.}$$

$$\frac{337}{360} \times \pi \times 1^2 \times 5$$

$$= \frac{337}{72} \pi \checkmark$$

$$\text{Total} = 15.7 \text{ m}^3 \checkmark$$