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ST STITHIANS GIRLS' COLLEGE

Department of Mathematics

GRADE 12	
PRELIMINARY EXAMINATION – PAPER 1	
DATE: 5 September 2022	TIME: 3 hours
TOTAL MARKS: 147	
EXAMINER: Boys' College	MODERATOR: Girls' College

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This question paper consists of 12 questions and 24 pages.
2. Questions end on Page 21 then 2 additional pages of lined paper for extra working are provided.
3. A separate information (formula) sheet will be provided to you.
4. Answer ALL the questions and clearly show the calculations you have used to determine your answers.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless specified otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Please note that diagrams are not drawn to scale.
8. It is in your own interest to write legibly and to present your work neatly.
9. PLEASE FILL IN YOUR NAME AND MARK YOUR TEACHER'S NAME ON THE BACK PAGE.

SECTION A:**Question 1:****[16]**Solve for x in the following:

(a) $(2x+3)(3-x) = 4$ (4)

$$\begin{aligned}
 (2x+3)(3-x) &= 4 \\
 6x - 2x^2 + 9 - 3x &= 4 \quad \checkmark \\
 0 &= 2x^2 - 3x - 5 \quad \checkmark \\
 0 &= (2x-5)(x+1) \\
 x &= \frac{5}{2} \quad \checkmark \text{ or } x = -1 \quad \checkmark
 \end{aligned}$$

(b) $3\sqrt{x+1} - x = 3$ (5)

$$\begin{aligned}
 3\sqrt{x+1} - x &= 3 \\
 3\sqrt{x+1} &= 3 + x \quad \checkmark \\
 9(x+1) &= (3+x)^2 \quad \checkmark \\
 9x + 9 &= 9 + 6x + x^2 \\
 0 &= x^2 - 3x \quad \checkmark \\
 0 &= x(x-3) \\
 x &= 0 \quad \checkmark \quad x = 3 \quad \checkmark
 \end{aligned}$$

(c) $x(5-x) < 0$ (2)

$$\begin{aligned}
 x(5-x) &< 0 \\
 x &< 0 \quad \checkmark \text{ or } 5 < x \quad \checkmark
 \end{aligned}$$

(d) $2^x + 2^{6-x} = 16$

(5)

$$2^x + 2^{6-x} = 16$$

$$2^x + \frac{2^6}{2^x} = 16 \quad \text{let } k = 2^x$$

$$k + \frac{64}{k} = 16 \quad \checkmark$$

$$k^2 + 64 = 16k$$

$$k^2 - 16k + 64 = 0 \quad \checkmark$$

$$(k-8)^2 = 0$$

$$k = 8$$

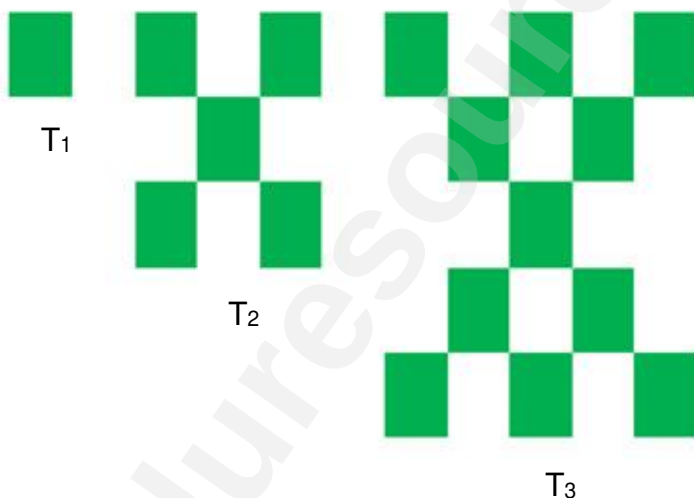
$$2^x = 8 \quad \checkmark$$

$$x = 3 \quad \checkmark$$

Question 2:

[9]

(a) The shaded squares below grow in a quadratic pattern:



Determine the general term of the pattern.

(5)

$$\begin{array}{ccc} 1 & ; & 5 & ; & 11 \\ 4 & \checkmark & 6 & & \\ 2 & & & & \end{array}$$

$$2a = 2$$

$$a = 1 \quad \checkmark$$

$$3a + b = 4$$

$$b = 1 \quad \checkmark$$

$$a + b + c = 1$$

$$c = -1 \quad \checkmark$$

$$T_n = n^2 + n - 1 \quad \checkmark$$

- (b) A pattern has a general term of $T_n = n^2 - 20n + 1$

Determine the position, and value, of the term that is the minimum of this pattern.

(4)

$$T_n = n^2 - 20n + 1$$

$$= n^2 - 20n + \left(\frac{20}{2}\right)^2 - \left(\frac{20}{2}\right)^2 + 1 \quad \text{there for the position is } 10 \text{ and the minimum is } 99$$

$$= (n-10)^2 - 99$$

OR

$$T_n = 2n - 20$$

$$0 = 2n - 20$$

$$n = 10$$

$$\therefore T_{10} = (10)^2 - 20(10) + 1 = -99$$

$$x = -\frac{b}{2a}$$

$$x = -\frac{(-20)}{2}$$

OR

$$x = 10$$

$$\therefore T_{10} = (10)^2 - 20(10) + 1 = -99$$

Question 3:

[9]

- (a) Determine the general term of the following geometric sequence:

3 ; -6 ; 12 ; -24 ; ...

(2)

$$a = 3$$

$$r = -2$$

$$T_n = ar^{n-1}$$

$$= 3(-2)^{n-1}$$

(b) Consider the following pattern:

3 ; 1 ; 3 ; 2 ; 3 ; 4 ; 3 ; 8 ; 3 ; 16 ; ...

(1) Write down the next two terms.

(2)

✓ ✓
3 ; 32

(2) Determine the sum of the first 50 terms of this pattern.

(5)

$$S_{50} = (\text{sum of } 3\text{s}) + (\text{sum of geometric pattern})$$

$$= (3 \times 25) + \left(\frac{1(2^{25} - 1)}{2 - 1} \right)$$

$$= 75 + 33554431$$

$$= 33554506$$

Question 4:

[12]

(a) You are given the word **STRAWBERRIES**.

(1) How many unique twelve-letter words can be formed using the letters in the word above?

(2)

SS T RRR A W B EE I

$$\frac{12!}{2! 3! 2!} = 19958400$$

- (2) How many unique twelve-letter words can be formed such that the words start and end with an S. (2)

$$\frac{10!}{3! 2!} = 302\,400$$

- (3) Determine the probability of twelve-letter words formed such that the letters W and B are next to each other. (4)

$$\frac{11! 2!}{3! 2! 2!} \div \frac{12!}{2! 3! 2!} = \frac{1}{6}$$

- (b) In a Mathematics quiz, two teams work independently on a problem. The probabilities that each team will solve the problem are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. Calculate the probability that the problem will be solved. (4)

$$P(\text{problem solved}) = \frac{1}{2} + \frac{1}{3} - \left(\frac{1}{2} \times \frac{1}{3} \right) \quad \checkmark \text{ independent events}$$

$$= \frac{2}{3} \quad \checkmark$$

$$= 0,67$$

OR

$$P(\text{problem solved}) = 1 - P(\text{not solved})$$

$$= 1 - \left(\frac{1}{3} \times \frac{2}{3} \right)$$

$$= \frac{2}{3} = 0,67$$

Question 5:**[13]**

Determine:

- (a) $f'(x)$ from first principles if $f(x) = 2x^2 - 1$ (5)

$$f(x) = 2x^2 - 1$$

$$f(x+h) = 2(x+h)^2 - 1$$

$$= 2x^2 + 4xh + 2h^2 - 1 \quad \checkmark$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 1 - (2x^2 - 1)}{h} \quad \checkmark$$

$$= \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 1 - 2x^2 + 1}{h} \quad \checkmark$$

$$= \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(4x + 2h)}{h} \quad \checkmark$$

$$= \lim_{h \rightarrow 0} (4x + 2h)$$

$$= 4x \quad \checkmark$$

(b) $\frac{dy}{dx}$ if $y = (x+1)(4x^2 - 2)$ (4)

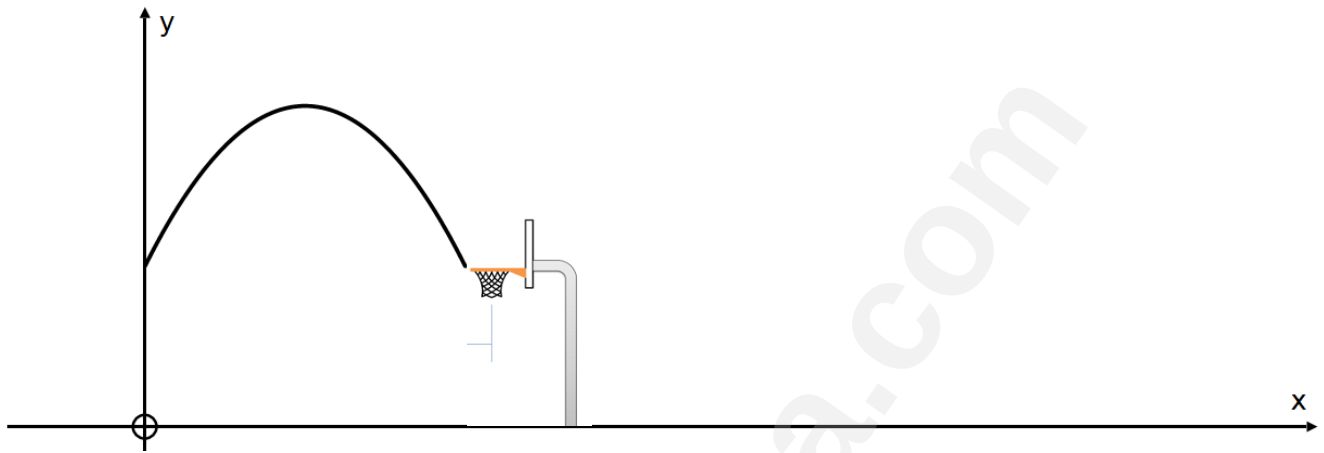
$$\begin{aligned}
 y &= (x+1)(4x^2 - 2) \\
 &= 4x^3 + 4x^2 - 2x - 2 \quad \checkmark \\
 \frac{dy}{dx} &= 12x^2 + 8x - 2 \quad \checkmark \quad \checkmark \quad \checkmark
 \end{aligned}$$

(c) $D_t \left[\frac{7t^2 - 2}{\sqrt{t}} \right]$ (4)

$$\begin{aligned}
 &D_t \left[\frac{7t^2 - 2}{\sqrt{t}} \right] \\
 &= D_t \left[\frac{7t^2}{\sqrt{t}} - \frac{2}{\sqrt{t}} \right] \quad \checkmark \\
 &= D_t \left[7t^{\frac{3}{2}} - 2t^{-\frac{1}{2}} \right] \quad \checkmark \\
 &= \frac{21}{2} t^{\frac{1}{2}} + t^{-\frac{3}{2}} \quad \checkmark \quad \checkmark
 \end{aligned}$$

Question 6:**[15]**

- (a) A basketball is thrown, and it follows the path of a parabola. If a basketball is thrown from a starting point $(0 ; 2)$ and reaches its maximum height at a point $(2 ; 4)$.



Determine the equation of the parabola

(4)

$$y = a(x - p)^2 + q$$

$$\text{sub } TP(2 ; 4) \checkmark$$

$$y = a(x - 2)^2 + 4$$

$$\text{sub } (0 ; 2) \checkmark$$

$$2 = a(0 - 2)^2 + 4$$

$$-2 = 4a$$

$$a = -\frac{1}{2} \checkmark$$

$$y = -\frac{1}{2}(x - 2)^2 + 4 \checkmark$$

(b) Given: $f(x) = \frac{2}{x+1} - 3$

- (1) Give the equations of the asymptotes of $f(x)$. (2)

$$x = -1 \quad \checkmark$$

$$y = -3 \quad \checkmark$$

- (2) Find the x and y intercepts of $f(x)$. (3)

$$X\text{-int: } y = 0$$

$$0 = \frac{2}{x+1} - 3$$

$$3 = \frac{2}{x+1} \quad \checkmark$$

$$3x + 3 = 2$$

$$3x = -1$$

$$x = -\frac{1}{3} \quad \checkmark$$

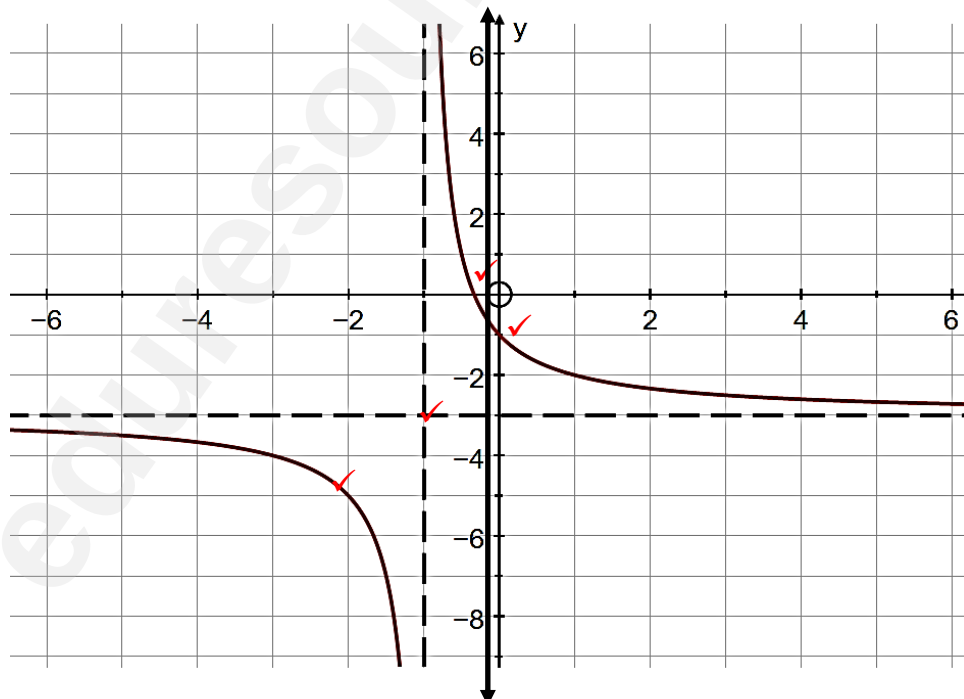
$$Y\text{-int: } x = 0$$

$$f(x) = \frac{2}{x+1} - 3$$

$$f(0) = \frac{2}{0+1} - 3$$

$$y = -1 \quad \checkmark$$

- (3) Sketch the graph of $f(x)$, clearly showing the asymptotes and intercepts with the axes. (4)



- (4) One of the axes of symmetry of $f(x)$ is a decreasing function. Write down the equation of this function. (2)

$$y = -x - 4 \quad \checkmark \quad \checkmark$$

END OF SECTION A [73 MARK]

SECTION B:**Question 7:****[9]**

- (a) Given any function $y = f(x)$, describe the transformation occurring in the following images:

(1) $y = f(x) + 1$ (1)

Vertical Shift of 1 unit upwards ✓

(2) $y = f(x - 2)$ (1)

Horizontal Shift of 2 units to the right ✓

(3) $y = f(-x)$ (1)

Reflection on the y-axis ✓

- (b) Consider the function $h(x) = 2^x - 6$

(1) Find $h^{-1}(x)$. (3)

$$h(x): y = 2^x - 6$$

$$h^{-1}(x): x = 2^y - 6 \quad \checkmark$$

$$2^y = x + 6 \quad \checkmark$$

$$y = \log_2(x + 6) \quad \checkmark$$

(2) Determine the domain of $h^{-1}(x)$. (2)

$$x > -6 \quad \checkmark$$

(3) Does the point $(-4;1)$ lie on $h^{-1}(x)$? (1)

$$y = \log_2(x + 6)$$

$$y = \log_2(-4 + 6)$$

$$= 1 \quad \checkmark$$

$\therefore$ Point $(-4;1)$ lies on $h^{-1}(x)$

Question 8:**[8]**

Consider the following sequence: $\log x$; $\log x^2$; $\log x^3$; $\log x^4$; $\log x^5$;...

- (a) Is this pattern arithmetic, geometric, or neither?
Provide a mathematical explanation for your answer. (2)

$$\begin{aligned} & \log x ; \log x^2 ; \log x^3 ; \log x^4 ; \log x^5 ; \dots \\ & = 1 \log x ; 2 \log x ; 3 \log x ; 4 \log x ; 5 \log x ; \dots \\ d &= 5 \log x - 4 \log x && \text{therefore, an arithmetic pattern} \\ &= (5 - 4) \log x \quad \checkmark \\ &= \log x \end{aligned}$$

- (b) Determine the general term of the pattern in terms of x . (2)

$$\begin{aligned} T_n &= a + (n - 1)d \\ &= \log x + (n - 1) \log x \quad \checkmark \\ &= \log x + n \log x - \log x \\ &= n \log x \\ &= \log x^n \quad \checkmark \end{aligned}$$

- (c) Calculate the sum of the first 50 terms of the pattern in terms of x . (4)

$$\begin{aligned} S_n &= \frac{n}{2} [2a + (n - 1)d] \quad \checkmark \\ S_{50} &= \frac{50}{2} [2 \log x + (50 - 1) \log x] \quad \checkmark \\ &= 25 [51 \log x] \quad \checkmark \\ &= 1275 \log x \quad \checkmark \end{aligned}$$

Question 9:**[13]**

- (a) The cost of a bus is R1,2 million. It is expected that the value of the bus will depreciate on a reducing balance per annum to R491 520 in 4 years' time. The price of a new bus is expected to increase by 15% per annum.

- (1) Calculate the percentage annual rate of depreciation of the bus. (2)

$$F_v = P_v (1-i)^n$$

$$491520 = 1200000 (1-i)^4 \quad \checkmark$$

$$\frac{491520}{1200000} = (1-i)^4$$

$$(1-i) = \sqrt[4]{\frac{491520}{1200000}}$$

$$i = 1 - \sqrt[4]{\frac{491520}{1200000}} \quad \checkmark$$

$$i = 0,2 \quad \checkmark$$

$$r = 20\%$$

- (2) If the bus needs to be replaced in 4 years' time, calculate the value of the sinking fund that needs to be set up to pay for the new bus. Assume that the old bus will be traded in at its depreciation value of R491 520. (4)

$$F_{\text{sinking}} = P_v (1+i)^n - \text{trade-in value}$$

$$= 1200000 (1+0,15)^4 - 491523 \quad \checkmark \quad \checkmark \quad \checkmark$$

$$= 1607287,50 \quad \checkmark$$

- (b) Tshepiso bought a car for R500 000 on an agreement in which he will repay it in monthly instalments at the end of each month for 5 years. Interest is charged at 18% p.a. compounded monthly.

(1) Calculate Tshepiso's monthly instalments.

(3)

$$500\,000 = x \left[\frac{1 - \left(1 + \frac{0,18}{12}\right)^{-5 \times 12}}{\frac{0,18}{12}} \right]$$

$$x = \frac{500\,000}{\frac{1 - \left(1 + \frac{0,18}{12}\right)^{-5 \times 12}}{\frac{0,18}{12}}} = R12\,696,71$$

- (2) Tshepiso decided to pay R12 700 each month as his repayment. Calculate the outstanding balance on the car after 2 years.

(4)

$$\text{Balance outstanding} = 500\,000 \left(1 + \frac{0,18}{12}\right)^{2 \times 12} - 12\,700 \left[\frac{\left(1 + \frac{0,18}{12}\right)^{2 \times 12} - 1}{\frac{0,18}{12}} \right] = R351\,105,69$$

OR

$$\text{Balance outstanding} = 12\,700 \left[\frac{1 - \left(1 + \frac{0,18}{12}\right)^{-36}}{\frac{0,18}{12}} \right] = R351\,290,69$$

Question 10:**[17]**

- (a) Determine the equation of the tangent to the curve, $y = \frac{8}{x}$, $x > 0$ that is perpendicular to the line $y = \frac{1}{2}x + 4$ (5)

$$m_{\text{original}} = \frac{1}{2}$$

$$m_{\text{tangent}} = -2 \checkmark$$

$$y = \frac{8}{x}$$

$$y = 8x^{-1}$$

$$\frac{dy}{dx} = -8x^{-2}$$

$$= -\frac{8}{x^2} \checkmark$$

$$m_{\text{tangent}} = \frac{dy}{dx}$$

$$-2 = -\frac{8}{x^2}$$

$$x^2 = 4$$

$$x = \pm 2 \text{ however restriction}$$

$$x = 2 \checkmark$$

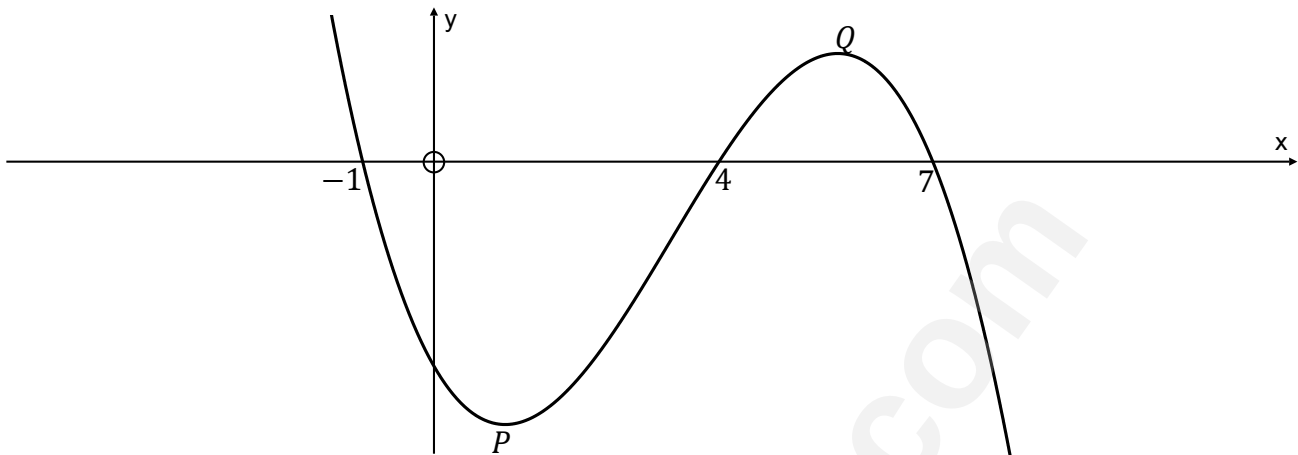
$$\therefore y = 4 \checkmark$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -2(x - 2)$$

$$y = -2x + 8 \checkmark$$

- (b) The diagram represents the graph of $y = -x^3 + ax^2 + bx + c$. Points P and Q are the stationary points of the graph.



- (1) Show that $a = 10$, $b = -17$ and $c = -28$. (4)

$$y = -(x+1)(x-4)(x-7)$$

$$y = -(x+1)(x^2 - 11x + 28)$$

$$y = -(x^3 - 10x^2 + 17x + 28)$$

$$y = -x^3 + 10x^2 - 17x - 28$$

$$a = 10$$

$$b = -17$$

$$c = -28$$

OR

$$-x^3 + ax^2 + bx + c = w(x+1)(x-4)(x-7)$$

$$-x^3 + ax^2 + bx + c = w(x+1)(x^2 - 11x + 28)$$

$$-(x^3 - ax^2 - bx - c) = w(x^3 - 10x^2 + 17x + 28)$$

$$a = 10$$

$$b = -17$$

$$c = -28$$

$$y = -x^3 + 10x^2 - 17x - 28$$

- (2) Determine the length of line PQ . (6)

$$\frac{dy}{dx} = -3x^2 + 20x - 17 \quad \checkmark$$

$$0 = (-3x + 17)(x - 1)$$

$$x = \frac{17}{3} \quad \checkmark \text{ or } x = 1 \quad \checkmark$$

$$y = \frac{400}{27} \quad \checkmark \quad y = -36 \quad \checkmark$$

$$PQ = \sqrt{\left(\frac{17}{3} - 1\right)^2 + \left(\frac{400}{27} - (-36)\right)^2}$$

$$= 51,0286..$$

$$\approx 51 \text{ units } \checkmark$$

- (3) For which value(s) of x is the graph concave up? (2)

$$\frac{d^2y}{dx^2} = -6x + 20$$

$$0 = -6x + 20 \quad \checkmark$$

$$6x = 20 \quad \text{or}$$

$$x = \frac{10}{3}$$

$$x < \frac{10}{3} \quad \checkmark$$

$$x = \frac{-b}{3a}$$

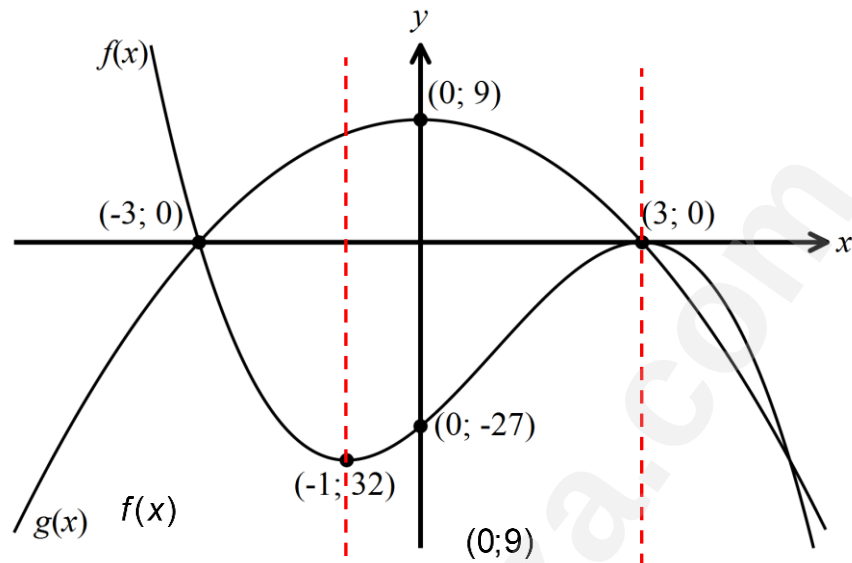
$$= \frac{-10}{3(-1)} \quad \checkmark$$

$$= \frac{10}{3}$$

$$x < \frac{10}{3} \quad \checkmark$$

Question 11:**[14]**

- (a) The figure below represents the graphs of $f(x)$ and $g(x)$.



- (1) Calculate the value of $g(3) - f(0)$ (2)
- $(-3; 0) \quad (3; 0)$

$$\begin{aligned} g(3) - f(0) \\ &= 0 - (-27) \quad \checkmark \\ &= 27 \quad \checkmark \end{aligned}$$

- (2) State the common factor(s) of $f(x)$ and $g(x)$. (1)

$$(x-3) \quad \text{or} \quad (x+3)$$

- (3) State the values of x for which $\frac{f'(x)}{x} > 0$? (3)

$$x < -1 \quad \text{or} \quad 0 < x < 3$$

- (4) State a possible restriction so that $g(x)$ has an inverse that is a function. (1)

$$x > 0 \text{ or } x < 0 \quad \text{or} \quad x \geq 0 \text{ or } x \leq 0 \quad \checkmark$$

- (5) Determine the equation of $g^{-1}(x)$ in the form $y = \dots$ based on your given restriction in part (4) of this question. (4)

$$g(x) = -x^2 + 9 \quad \checkmark$$

$$y = -x^2 + 9$$

$$x = -y^2 + 9 \quad \checkmark$$

$$y^2 = 9 - x$$

$$y = \pm \sqrt{9 - x} \quad \checkmark$$

- (6) For which values of k will $f(x) = k$ have 2 roots? (3)

$$k = 0 \quad \checkmark \text{ or } k = -32 \quad \checkmark \checkmark$$

Question 12:**[16]**

- (a) Solve simultaneously for
- x
- and
- y
- :

$$3y = 2 - 3x + x^2$$

$$\sum_{k=3}^4 x(k-2)^2 = 5y + 10 \quad (5)$$

$$\begin{aligned} \sum_{k=3}^4 x(k-2)^2 &= 5y + 10 \\ x(3-2)^2 + x(4-2)^2 &= 5y + 10 \quad \checkmark \\ x + 4x &= 5y + 10 \\ 5x &= 5y + 10 \\ x &= y + 2 \\ y &= x - 2 \quad \checkmark \end{aligned}$$

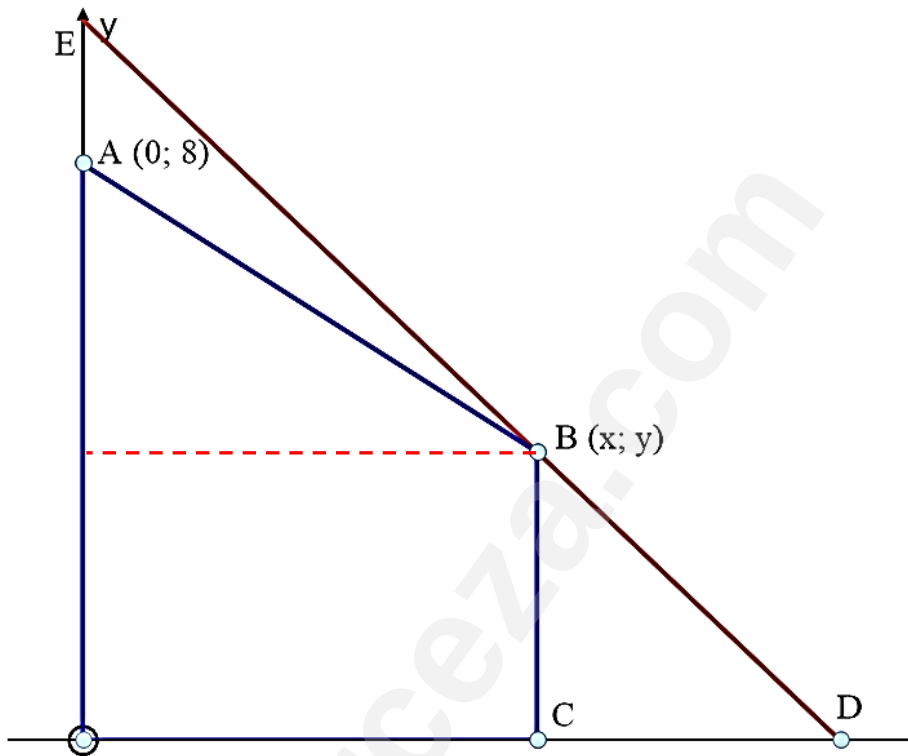
$$\begin{aligned} 3y &= 1 + x + x^2 \\ \text{sub } y &= x - 2 \\ 3(x-2) &= 2 - 3x + x^2 \quad \checkmark \\ 3x - 6 &= 2 - 3x + x^2 \\ 0 &= x^2 - 6x + 8 \\ 0 &= (x-4)(x-2) \quad \checkmark \\ x &= 4 \quad \text{or} \quad x = 2 \\ y &= 2 \quad \checkmark \quad y = 0 \end{aligned}$$

- (b) For which values of
- k
- does the following equation have real roots?

$$k = \frac{1-x^2}{5-4x} \quad (5)$$

$$\begin{aligned} k &= \frac{1-x^2}{5-4x} \\ k(5-4x) &= 1-x^2 \\ x^2 - 4kx + 5k - 1 &= 0 \quad \checkmark \\ \Delta &\geq 0 \quad (\text{real roots}) \\ b^2 - 4ac &\geq 0 \quad \checkmark \\ (-4k)^2 - 4(1)(5k-1) &\geq 0 \\ 16k^2 - 20k + 4 &\geq 0 \quad \checkmark \\ 4k^2 - 5k + 1 &\geq 0 \\ (4k-1)(k-1) &\geq 0 \\ k &\leq \frac{1}{4} \quad \checkmark \quad \text{or} \quad k \geq 1 \quad \checkmark \end{aligned}$$

- (c) In the diagram, the equation of ED is $y = -2x + 10$, $x > 0$, $y > 0$ and B ($x; y$) is any point on ED. Furthermore A (0;8) and $BC \perp OD$. Determine the coordinates of B for the area of AOCB to be a maximum. (6)



Area = Square + Triangle ✓

$$= x \cdot y + \frac{1}{2} x \cdot (8 - y)$$

$$= xy + 4x - \frac{xy}{2}$$

$$= \frac{xy}{2} + 4x$$

$$y = -2x + 10$$

$$\therefore A(x) = \frac{x(-2x + 10)}{2} + 4x$$

$$A(x) = -x^2 + 5x + 4x$$

$$= -x^2 + 9x \quad \checkmark$$

$$A(x) = -x^2 + 9x$$

$$A'(x) = -2x + 9 \quad \checkmark$$

$$0 = -2x + 9 \quad \checkmark$$

$$2x = 9$$

$$x = \frac{9}{2} \quad \checkmark$$

$$y = 1 \quad \checkmark$$

$$B\left(\frac{9}{2}; 1\right)$$

END OF SECTION B [77 MARKS]

FOR OFFICIAL USE ONLY

MARK RECORD SHEET**Full Name:** _____**Teacher (mark with a x):**☐ Mrs Frankiskos ☐ Mrs Robertson☐ Mr Lombard ☐ Mrs McLachlan ☐ Mr Stolk

Question	Algebra	Patterns	Finance	Functions	Calculus	Probability
1	/ 16					
2		/ 9				
3		/ 9				
4						/ 8
5					/ 13	
6				/ 15		
7				/ 9		
8		/ 8				
9			/ 13			
10					/ 17	
11				/ 14		
12	/ 10				/ 6	
TOTAL	/ 26	/ 26	/ 13	/ 38	/ 36	/ 8
TOTAL	/ 147					%