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ST STITHIANS GIRLS' COLLEGE

Department of Mathematics

MEMO

GRADE 12	
PRELIMINARY EXAMINATION – PAPER 1	
DATE: 6 September 2021	TIME: 3 hours
TOTAL MARKS: 150	

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This question paper consists of 14 questions and 24 pages.
2. A separate information (formula) sheet will be provided to you.
3. Answer all the questions and clearly show **ALL** calculations, diagrams, graphs etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
4. You may use an approved scientific calculator (non-programmable and non-graphical) unless specified otherwise. Ensure that your calculator is in **DEGREE** mode.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Please note that diagrams are not drawn to scale.
7. It is in your own interest to write legibly and to present your work neatly.
8. PLEASE FILL IN YOUR NAME AND SURNAME AND CIRCLE YOUR TEACHER'S NAME ON THE BACK PAGE.

SECTION A**QUESTION 1**

Solve for x in each of the following, rounding off to 2 decimal places where necessary:

(a) $x + 4 = \sqrt{3(x + 4)}$

$$(x + 4)^2 = (\sqrt{3(x + 4)})^2$$

$$x^2 + 8x + 16 = 3x + 12$$

$$x^2 + 5x + 4 = 0$$

$$(x + 4)(x + 1) = 0$$

$$\begin{array}{cc} x = -4 & \text{OR} & x = -1 \\ \longrightarrow & & \longrightarrow \end{array}$$

(4)

(b) $(3^x - 1)(3^x - 12) = 0$

$$3^x = 1 \quad \text{OR} \quad 3^x = 12$$

$$3^x = 3^0 \qquad \log_3 12 = x$$

$$\begin{array}{cc} x = 0 & x = 2,26 \\ \longrightarrow & \longrightarrow \end{array}$$

(4)

(c) $-x^2 + 14x \geq -15$

$$0 \geq x^2 - 14x - 15$$

$$0 \geq (x - 15)(x + 1)$$

$$-1 \leq x \leq 15$$



(4)

[12]

QUESTION 2

(a) The first four terms of a quadratic number pattern are -1 ; 2 ; 9 ; 20 .

(1) Determine the general term of the quadratic number pattern.

$$\begin{array}{cccc}
 -1 & ; & 2 & ; & 9 & ; & 20 \\
 & \swarrow & / & & \swarrow & / & \swarrow & / \\
 & & 3 & & 7 & & 11 \\
 & & & \swarrow & / & & \swarrow & / \\
 & & & & 4 & & 4
 \end{array}$$

$$2a = 4$$

$$a = 2$$

$$3a + b = 3$$

$$3(2) + b = 3$$

$$b = -3$$

$$a + b + c = -1$$

$$2 + (-3) + c = -1$$

$$c = 0$$

$$T_n = 2n^2 - 3n$$

(4)

(2) Calculate the value of the 48th term of the quadratic number pattern.

$$T_{48} = 2(48)^2 - 3(48)$$

$$= 4464$$

(2)

(3) Show that the sum of the FIRST DIFFERENCES of this quadratic number pattern can be given by $S_n = 2n^2 + n$.

$$\begin{array}{ccc}
 3 & ; & 7 & ; & 11 \\
 & \swarrow & / & & \swarrow & / \\
 & & 4 & & 4
 \end{array}$$

$$a = 3$$

$$d = 4$$

$$S_n = \frac{n}{2} [2(3) + (n-1)(4)]$$

$$S_n = \frac{n}{2} [6 + 4n - 4]$$

$$S_n = \frac{n}{2} [4n + 2]$$

(3)

$$S_n = 2n^2 + n$$

- (4) Determine the largest number of terms if the sum of the first differences calculated in (3) is less than 10 000.

$$10\ 000 = 2n^2 + n$$

$$0 = 2n^2 + n - 10\ 000$$

$$\underline{n = 70,46 \quad \text{OR} \quad n = -70,46}$$

$\therefore$ largest number of terms is 70

(3)

- (b) Calculate the following, showing all working.

$$\sum_{n=0}^{15} 81 \left(\frac{2}{3} \right)^n$$

$$\begin{aligned} \text{Number of terms} &= (15+0) + 1 \\ &= 16 \text{ terms} \end{aligned}$$

$$a = 81$$

$$r = \frac{2}{3}$$

$$S_{16} = \frac{81 \left[\left(\frac{2}{3} \right)^{16} - 1 \right]}{\frac{2}{3} - 1}$$

$$\underline{S_{16} = 242,63} \rightarrow$$

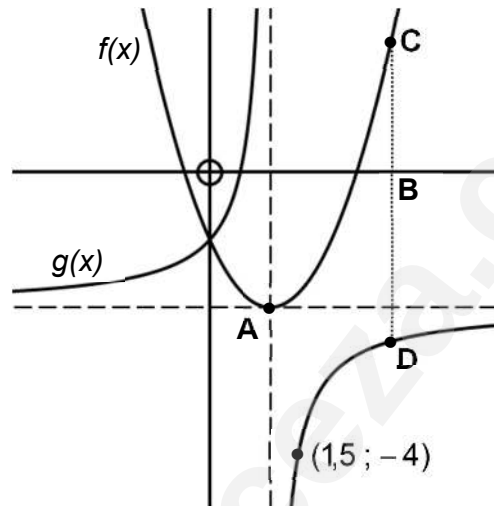
(4)

[16]

QUESTION 3

Sketched below are the graphs of $g(x) = \frac{a}{x-p} + q$ and $f(x) = x^2 - 2x - 1$.

The turning point of $f(x)$ lies on the point of intersection of the asymptotes of $g(x)$ (at A).



- (a) Calculate the values of a , p and q .

coordinate of A (TP of $f(x)$)

$$\begin{aligned} TP_x &= \frac{-b}{2a} \\ &= \frac{-(-2)}{2(1)} \\ &= 1 \end{aligned}$$

$$\begin{aligned} TP_y &= (1)^2 - 2(1) - 1 \\ &= -2 \end{aligned}$$

$$A(1; -2) \therefore p=1 \text{ and } q=-2$$

$$\begin{aligned} g(x) &= \frac{a}{x-1} - 2 \\ -4 &= \frac{a}{1.5-1} - 2 \\ a &= -1 \end{aligned}$$

(7)

- (b) Write down the domain and range of $g(x)$.

Domain: $x \in \mathbb{R} ; x \neq 1$

Range: $y \in \mathbb{R} ; y \neq -2$

(2)

- (c) Determine the equation of $h(x)$, the translation of $g(x)$ if $g(x)$ is moved 3 units up and 2 units to the left.

$$h(x) = \frac{-1}{x-1+2} - 2 + 3$$

$$h(x) = \frac{-1}{x+1} + 1$$

(2)

- (d) Given that $OB = 3$ units and $CD \parallel y$ -axis, determine the distance of CD .

$$B(3;0)$$

$$C(3; y_c)$$

$$D(3; y_D)$$

$$\begin{aligned} f(3) &= (3)^2 - 2(3) - 1 \\ &= 2 \end{aligned}$$

$$\therefore C(3; 2)$$

$$\begin{aligned} g(3) &= \frac{-1}{3-1} - 2 \\ &= -5/2 \end{aligned}$$

$$\therefore D(3; -5/2)$$

$$d_{CD} = 2 - (-5/2)$$

$$= 4,5 \text{ units}$$



(4)

[15]

QUESTION 4

The graph of $g(x) = a^x + q$ passes through the points $(2; 3)$ and $(0; 0)$.

- (a) Find the values of a and q .

$$0 = a^0 + q$$

$$\therefore q = -1$$

$$\longrightarrow$$

$$3 = a^2 - 1$$

$$4 = a^2$$

$$\therefore a = 2$$

$$\longrightarrow$$

(4)

- (b) Determine $g^{-1}(x)$ in the form $g^{-1}(x) = \dots$

$$y = 2^x - 1$$

$$x = 2^y - 1$$

$$\therefore x + 1 = 2^y$$

$$\therefore \log_2(x+1) = y$$

$$\therefore g^{-1}(x) = \log_2(x+1)$$

$$\longrightarrow$$

(3)

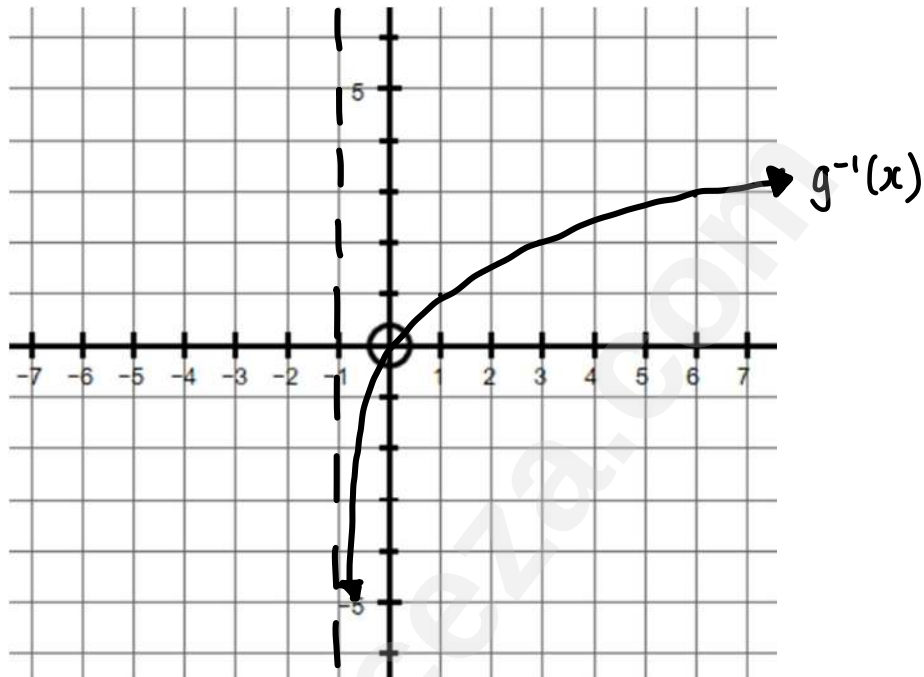
- (c) Determine the domain of g^{-1} .

$$x > -1$$

$$\longrightarrow$$

(1)

- (d) Sketch the graph of g^{-1} on the set of axes below. Clearly show all intercepts with the axes and any asymptotes.



(3)

[11]

QUESTION 5

If the letters of the word EXAMINATION are arranged randomly in a row, find:

- (a) The number of different arrangements that can be made with all the letters

$$\frac{11!}{2! \times 2! \times 2!}$$

(2)

- (b) The number of words that can be made if the word starts with an E and ends with an M.

$$\frac{9!}{2! \times 2! \times 2!}$$

(2)

- (c) The probability of a word starting and ending with the same letter.

$$\frac{3 \times \left(\frac{9!}{2! \times 2!} \right)}{\frac{11!}{2! \times 2! \times 2!}}$$

$$= 5,5\%$$

(4)

[8]

QUESTION 6

Given: $(x-2)^2 + (x-2)^3 + (x-2)^4 + \dots$

- (a) For which values of x will the series converge?

$$r = x - 2$$

$$-1 < x - 2 < 1$$

$$\therefore 1 < x < 3$$

(3)

- (b) If $S_{\infty} = 4$, find the value of x .

$$4 = \frac{(x-2)^2}{1 - (x-2)}$$

$$4 = \frac{(x-2)^2}{3-x}$$

$$4(3-x) = (x-2)^2$$

$$12 - 4x = x^2 - 4x + 4$$

$$0 = x^2 - 8$$

$$\therefore x = \pm \sqrt{8}$$

(4)

[7]

SECTION B

QUESTION 7

- (a) Determine $f'(x)$ from first principles if $f(x) = 3x^2 - x$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned} f(x+h) &= 3(x+h)^2 - (x+h) \\ &= 3x^2 - x + 6xh - 3h^2 - h \end{aligned}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x^2 - x + 6xh - 3h^2 - h - (3x^2 - x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{6xh + 3h^2 - h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(6x + 3h - 1)}{h}$$

$$f'(x) = 6x + 3(0) - 1$$

$$f'(x) = 6x - 1$$

—————>

(5)

- (b) Determine $D_x \left[\frac{3x^3 - 7x^2 - 6x}{x-3} \right]$

$$= D_x \left[\frac{x(3x^2 - 7x - 6)}{(x-3)} \right]$$

$$= D_x \left[\frac{x(3x+2)\cancel{(x-3)}}{\cancel{(x-3)}} \right]$$

$$= D_x [3x^2 + 2x]$$

(4)

$$= 6x + 2$$

—————>

- (c) Evaluate $f'(1)$ if $f(x) = \frac{2}{\sqrt{x}} + \sqrt{12}x - 6$

$$f(x) = 2 \cdot x^{-1/2} + \sqrt{12}x - 6$$

$$f'(x) = -x^{-3/2} + \sqrt{12}$$

$$f'(1) = -(1)^{-3/2} + \sqrt{12}$$

$$= -1 + 2\sqrt{3}$$

$$= 2,46$$



(4)

[13]

QUESTION 8

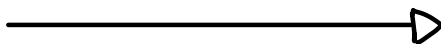
Ms Judd has R180 000 as a deposit to buy her dream apartment. The sale price of the apartment is R812 000. The financial institution offers her a mortgage rate of 18% per annum compounded monthly over a period of 20 years.

- (a) Calculate Ms Judd's monthly repayment.

$$\begin{aligned} \text{Loan amount} &= 812\,000 - 180\,000 \\ &= 632\,000 \end{aligned}$$

$$\therefore 632\,000 = x \left[\frac{1 - \left(1 + \frac{0,18}{12}\right)^{-20 \times 12}}{\frac{0,18}{12}} \right]$$

$$\therefore x = R\,9753,73$$



(4)

- (b) Calculate the outstanding balance on the loan immediately after her 84th payment.

$$B_0 = 9753,73 \left[\frac{1 - \left(1 + \frac{0,18}{12}\right)^{-13 \times 12}}{\frac{0,18}{12}} \right]$$

$$= \underline{R586\,513,83} \rightarrow$$

(4)

- (c) How much interest will Ms Judd pay during the first 7 years?

$$\text{Amount paid after 7 years} = 9753,73 \times 84$$

$$= 819\,313,32$$

$$\text{Amount paid off of loan} = 632\,000 - 586\,513,83$$

$$= 45\,486,17$$

$$\therefore \text{Interest} = 819\,313,32 - 45\,486,17$$

$$= \underline{R773\,827,15} \rightarrow$$

(4)

- (d) Determine the effective interest rate Ms Judd was charged on her loan.

$$1 + i_{\text{eff}} = \left(1 + \frac{0,18}{12}\right)^{12}$$

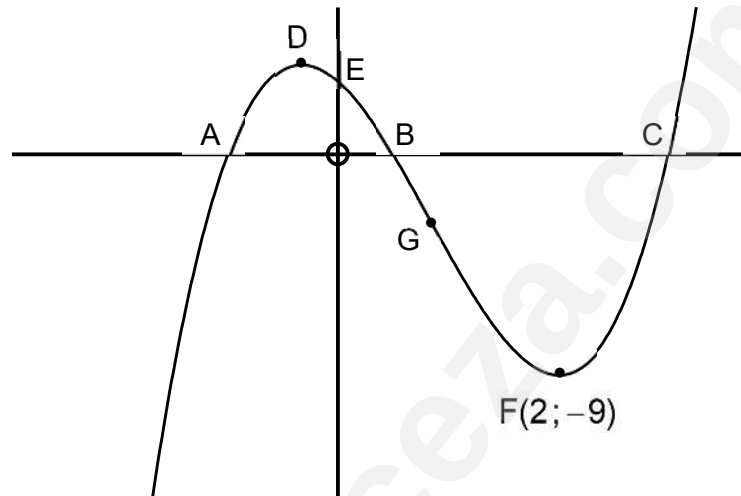
$$i_{\text{eff}} = \underline{19,56\%} \rightarrow$$

(3)

[15]

QUESTION 9

The figure below represents the graph of $f(x) = 2x^3 - 5x^2 - 4x + 3$, which has a local minimum at $F(2; -9)$. The x-intercepts are respectively given by $A(-1; 0)$, $B(\frac{1}{2}; 0)$ and $C(3; 0)$.



- (a) Determine the x-coordinate of D, the local maximum turning point.

$$f'(x) = 0$$

$$f'(x) = 6x^2 - 10x - 4$$

$$0 = 6x^2 - 10x - 4$$

$$\therefore x_0 = -1/3 \text{ or } x = 2$$

→

(3)

- (b) For which value(s) of x is the graph concave up?

$$\text{POI : } f''(x) = 0$$

$$\therefore 12x - 10 = 0$$

$$\therefore x = 5/6$$

concave up for $x > 5/6$

(3)

(c) For which value(s) of x :

(1) is the function increasing?

$$(-\infty; -\frac{1}{3}) \cup (2; \infty)$$

(2)

(2) is $f(x) \cdot f'(x) < 0$?

$$(-\infty; -1) \cup (-\frac{1}{3}; \frac{1}{2}) \cup (2; 3)$$

(3)

(d) Given $h(x) = f(x + k)$, determine the value(s) of k for which $h(x)$ will have three positive roots.

$$k < -1$$


(2)

[13]

QUESTION 10

- (a) It is given that two events, A and B, are independent.

If $P(A) = \frac{2}{5}$ and $P(B) = 0,35$, then determine the value of $P(A \cup B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\begin{aligned} P(A \cap B) &= P(A) \times P(B) \\ &= \frac{2}{5} \times 0,35 \\ &= 0,14 \end{aligned}$$

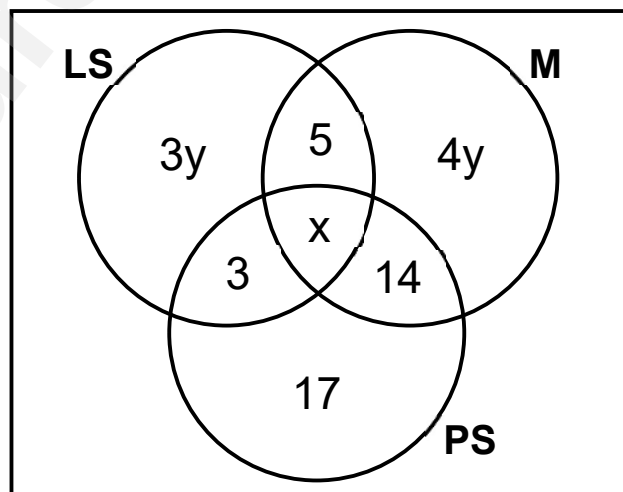
$$\begin{aligned} P(A \cup B) &= \frac{2}{5} + 0,35 - 0,14 \\ &= 0,61 \end{aligned}$$

(4)

- (b) At a certain high school there are 120 Grade 12 students. A survey is conducted to see how many take Life Sciences, Mathematics and Physical Sciences.

44 students take Life Sciences (LS) and 65 students take Mathematics (M).

The Venn-diagram below represents the data:



- (1) Determine the values of x and y in the Venn diagram.

Life sciences : $3y + 5 + 3 + x = 44$
 $\therefore x = 36 - 3y \dots \textcircled{1}$

Mathematics : $4y + 14 + 5 + x = 65$
 $\therefore x = 46 - 4y \dots \textcircled{2}$

$$36 - 3y = 46 - 4y$$

$$\therefore y = 10$$

$$x = 36 - 3(10)$$

$$\therefore x = 6$$

(5)

- (2) Hence, determine the probability that a student chosen at random will take none of the 3 subjects.

$$P(\bar{A}) = 120 - (30 + 5 + 40 + 6 + 3 + 14 + 17)$$

$$P(\bar{A}) = 5$$

$$P(\bar{A}) = 5/120$$

$$= 1/24$$

(2)

[11]

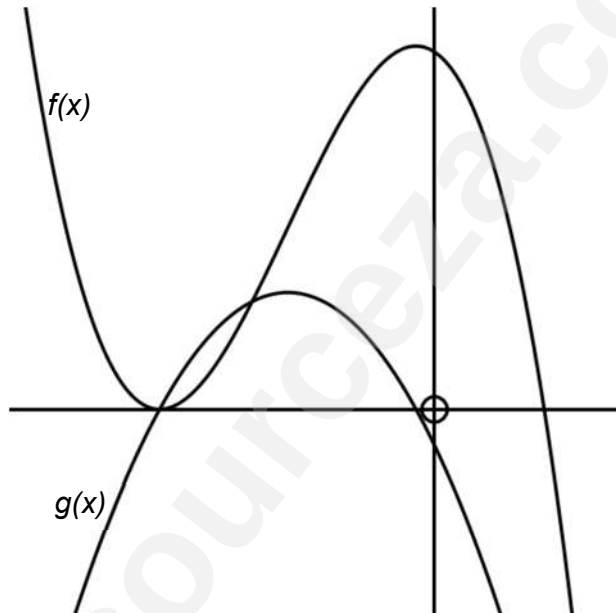
QUESTION 11

The sketch below shows the graphs of the two functions:

$$f(x) = ax^3 + bx^2 - 5x + 50$$

$$g(x) = f'(x) = px^2 + qx + t$$

The x-intercepts of $f(x)$ are $(-5; 0)$ and $(2; 0)$ while the x-intercepts of $g(x)$ are $(-5; 0)$ and $(-\frac{1}{3}; 0)$.



Use the given information and the sketch to answer the following questions:

- (a) Determine the value of $f[g(-5)]$.

$$g(-5) = 0$$

$$f(0) = 50$$

→

(2)

- (b) Calculate the average gradient of $f(x)$ over the interval $x \in [-5; 0]$.

$$M_{AB} = \frac{0 - 50}{-5 - 0}$$

$$= 10$$

→

(2)

- (c) Write down the x-values of the stationary points of $f(x)$.

$$\underline{x = -5} \rightarrow$$

$$\underline{x = -1/3} \rightarrow$$

(2)

- (d) Determine the value(s) of x for which $f''(x) < 0$.

$$f''(x) < 0 \quad \text{concave down}$$

$$\text{POI}_x = -8/3$$

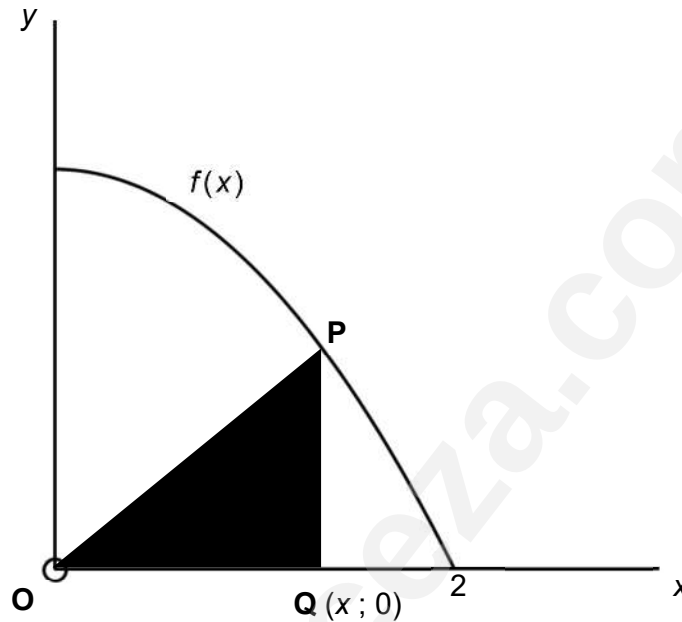
$$\therefore x > -8/3$$

(2)

[8]

QUESTION 12

Given $f(x) = \frac{-x^2}{2} + 2$ and $PQ \parallel y$ -axis as shown in the sketch below.



- (a) Show that the area (A) of $\triangle POQ$ is given by $A = \frac{-x^3}{4} + x$.

$$P(x; -\frac{x^2}{2} + 2)$$

$$\text{Area}_{POQ} = \frac{1}{2} (x) \left(-\frac{x^2}{2} + 2 \right)$$

$$= \frac{-x^3}{4} + x \rightarrow$$

- (b) Determine how far Q should be from O for the area of $\triangle POQ$ to be a maximum. Leave your answer in simplest surd form.

$$\text{Max Area : } A' = 0$$

$$A' = -\frac{3}{4}x^2 + 1$$

$$0 = -\frac{3}{4}x^2 + 1$$

$$x^2 = \frac{4}{3}$$

$$\therefore x = \frac{2}{\sqrt{3}} \text{ units}$$

(3)

- (c) Calculate the area of $\triangle POQ$ when P is at the point in question (b) above. Give your answer correct to 2 decimal places.

$$A = \frac{-x^3}{4} + x$$

$$A_{\max} = \frac{-\left(\frac{2}{\sqrt{3}}\right)^3}{4} + \left(\frac{2}{\sqrt{3}}\right)$$

$$= \frac{4\sqrt{3}}{9}$$

$$= 0,77 \text{ units}^2 \rightarrow$$

(2)

[9]

QUESTION 13

When the polynomial $f(x) = x^3 - ax^2 + 12x + b$ is divided by $g(x) = x + 5$ the quotient is $x^2 + 10x + c$ and the remainder is 150.

Find the values of a , b and c .

$$(x+5)(x^2 + 10x + c) + 150 = x^3 - ax^2 + 12x + b$$

$$x^3 + 10x^2 + cx + 5x^2 + 50x + 5c + 150 = x^3 - ax^2 + 12x + b$$

$$\underbrace{x^3}_{\text{}} + \underbrace{15x^2}_{\text{}} + \underbrace{(50+c)x}_{\text{}} + \underbrace{5c+150}_{\text{}} = \underbrace{x^3}_{\text{}} - \underbrace{ax^2}_{\text{}} + \underbrace{12x}_{\text{}} + \underbrace{b}_{\text{}}$$

$$\therefore a = -15$$

—————>

$$50 + c = 12$$

$$\therefore c = -38$$

—————>

$$5c + 150 = b$$

$$5(-38) + 150 = b$$

$$\therefore b = -40$$

—————>

(5)

[5]

QUESTION 14

Show that if $3\log_x y + 3\log_y x = 10$, then $y = x^3$ or $x = y^3$.

$$3\log_x y + \frac{3\log_x x}{\log_x y} = 10$$

$$3\log_x y + \frac{3}{\log_x y} = 10$$

$$\text{let } k = \log_x y \quad \therefore \quad 3k + \frac{3}{k} = 10$$

$$3k^2 + 3 = 10k$$

$$3k^2 - 10k + 3 = 0$$

$$(3k - 1)(k - 3) = 0$$

$$k = \frac{1}{3} \quad k = 3$$

$$\log_x y = \frac{1}{3} \quad \text{or} \quad \log_x y = 3$$

$$x^{1/3} = y$$

$$x^3 = y$$

$$x = y^3$$

(7)

[7]

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Full Name: _____**Teacher:** KAUR JUDD SMITH STATHAM

Question	Algebra	Patterns	Finance	Functions	Calculus	Probability
1	/ 12					
2		/ 16				
3				/ 15		
4				/ 11		
5						/ 8
6		/ 7				
7					/ 13	
8			/ 15			
9					/ 13	
10						/ 11
11					/ 8	
12				/ 9		
13					/ 5	
14	/ 7					
SUB-TOTAL	/ 19	/ 23	/ 15	/ 35	/ 39	/ 19
TOTAL	/ 150			%		