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ST STITHIANS GIRLS' COLLEGE

Departments of Mathematics

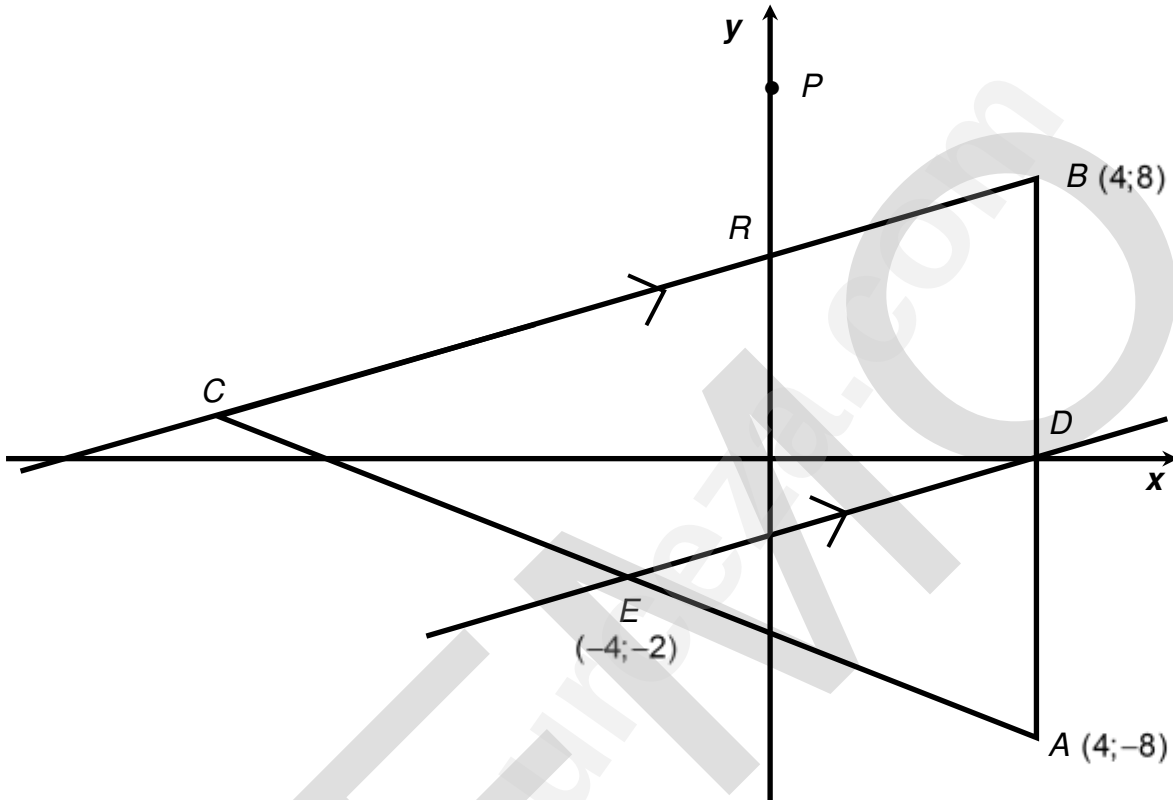
GRADE 12	
PRELIMINARY EXAMINATION – PAPER 2	
DATE: September 2020	TIME: 3 hours
TOTAL MARKS: 150	
EXAMINER: Girls' College	MODERATOR: Boys' College

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This question paper consists of 13 questions and 24 pages.
2. A separate information (formula) sheet will be provided to you.
3. Answer ALL the questions and clearly show the calculations you have used to determine your answers.
4. You may use an approved scientific calculator (non-programmable and non-graphical) unless specified otherwise.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Please note that diagrams are not drawn to scale.
7. It is in your own interest to write legibly and to present your work neatly.
8. PLEASE FILL IN YOUR NAME AND CIRCLE YOUR TEACHER'S NAME ON THE BACK PAGE.

Question 1:

In the diagram below, $\triangle ABC$ is shown with coordinates of $A(4; -8)$ and $B(4; 8)$ given. AB intersects the x -axis at D and AC intersects the line DE at $E(-4; -2)$. BC intersects the y -axis at R . P is a point on the y -axis. $DE \parallel BC$.



- (a) Write down the coordinates of D. (1)

(4;0) ✓	Answer
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- (b) Determine the gradient of line DE. (2)

$m = \frac{0 - (-2)}{4 - (-4)} = \frac{1}{4} \quad \checkmark$	Substitute points
	Answer

(c) Determine the equation of line BC.

(3)

$y = \frac{1}{4}x + c$ $(8) = \frac{1}{4}(4) + c \quad c = 7$ $y = \frac{1}{4}x + 7$	Gradient Substitution Equation
--	---

(d) Calculate $\hat{P}RB$.

(3)

$\tan \theta = \frac{1}{4}$ $\theta = 14,04^\circ$ $PRB = 90^\circ - 14,04^\circ = 75,96^\circ$	$\tan \theta = \text{Gradient}$ Answer θ $(90 - \theta)$ Answer
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(e) Determine the equation of the circle with centre at B and passing through point R. (3)

$(x - 4)^2 + (y - 8)^2 = r^2$ $((0) - 4)^2 + ((7) - 8)^2 = r^2$ $r^2 = 17$ $(x - 4)^2 + (y - 8)^2 = 17$	Substitution centre Substitution Point Equation
---	--

(f) Determine the equation of the tangent at R.

(2)

$y = -4x + 7$	Gradient $y\text{-int} = 7$
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(g) Determine the midpoint of DE.

(2)

$\left(\frac{4 + (-4)}{2}, \frac{0 + (-2)}{2} \right)$ $(0; -1)$	Substitution Answer
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(h) Determine the point Q such that BDQE is a parallelogram.

(3)

$\frac{x+4}{2} = 0 \quad \frac{y+8}{2} = -1$ $x = -4 \quad y = -10$ $(-4; -10)$	Method (Can use inspection) x value y value
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(i) Calculate the length of DE in simplest surd form.

(2)

$DE = \sqrt{(4 - (-4))^2 + (0 - (-2))^2}$ $= \sqrt{68} = 2\sqrt{17}$	Substitution Answer
--	----------------------------

(j) (1) Write down the ratio AD: AB in its simplest form.

(2)

$1:2$	Answer
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(2) Hence, or otherwise, calculate the length of BC with reasons.

(3)

$\frac{DE}{BC} = \frac{AD}{AB} \quad (\text{Prop Int Thm})$ $\frac{2\sqrt{17}}{BC} = \frac{1}{2}$ $BC = 4\sqrt{17}$	Reason Ratio Answer
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[26]

Question 2:

In the current COVID-19 pandemic, a significant proportion of cases shed SARS-Coronavirus-2 (SARS-CoV-2) with their faeces. To determine if SARS-CoV-2 RNA was present in sewage during the emergence of COVID-19 in the Netherlands, sewage samples were tested for the nucleocapsid gene (N3). Below is a table of the data recorded of the number of cases per 100 000 people and the presence of the Gene (N3) per ml in the sewerage system. (This data was taken from the early days of the outbreak.)

Number of cases per 100 000 people (x)	Gene (N3) per ml of Sewerage (N)
2	9
3	23
4	90
8	132
6	486
7	486
10	775
22	357
38	711
57	1643
64	3250
67	2929
67	2286
93	839

- (a) Determine the correlation coefficient, r , for the set of data, and describe the relationship between N and x that this value represents. (Round off to 3 decimal places) (3)

0,2426 ✓✓✓	Answer
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- (b) Determine the equation of the least squares regression line for N in terms of x in the form $N = A + Bx$. (Round off to 3 decimal places). (2)

$N = 126,314 + 25,776x$ ✓✓	Answers
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- (c) Predict the expected concentration of the N3 Gene when the area has 150 cases per 100 000 people. Comment on the reliability of your answer. (2)

4042,7 ✓ Extrapolation – not reliable ✓	Answer comment
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[7]

Question 3:

The table below indicates the COVID-19 deaths by age in South Africa up until the 24th June 2020. Answer the questions below.

Age of deceased	Number of deaths	
0-9	3	4,5
10-19	5	14,5
20-29	30	24,5
30-39	127	34,5
40-49	280	44,5
50-59	529	54,5
60-69	581	64,5
70-79	392	74,5
80-89	190	84,5
90-99	55	94,5
100- 109	1	104,5
Total Deaths	2193	

- (a) Determine the estimated mean age of the deceased population. (3)

61,34 ✓✓	Midpoints Answer 2 marks
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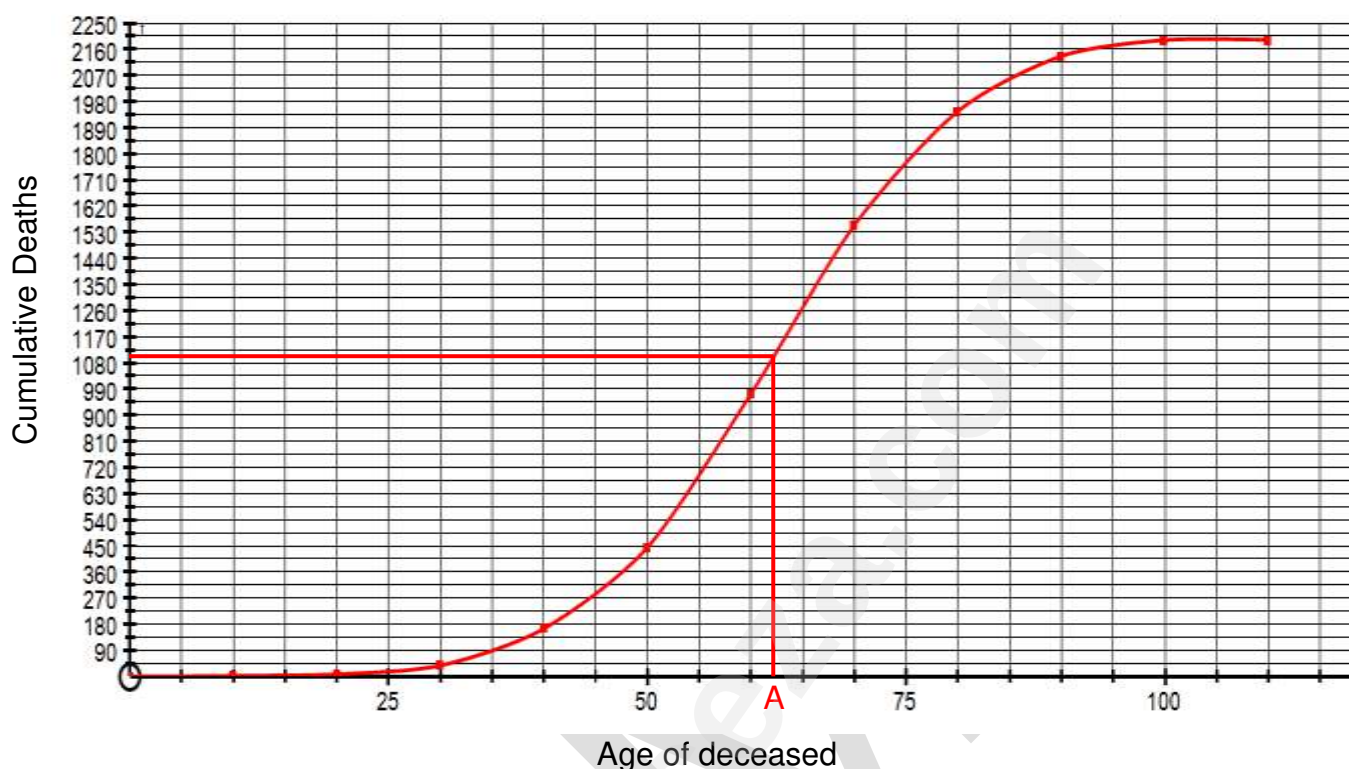
- (b) Determine the standard deviation of the data. (2)

15,02 ✓✓	Answer
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- (c) Give the full interval of possible ages within one standard deviation. (3)

$61,34 \pm 15,02$ ✓ $(46,32 ; 76,36)$	Method 1 mark Answer 2 marks
--	---------------------------------

- (d) Given the Ogive below of the data in the table above. Answer the following questions:



- (1) Determine the median value of the age on your Ogive. Indicate with the letter A where you read it on the graph. (2)

62 – 63 years ✓✓

Answer in range

Indication of A

- (2) Determine the interquartile range from the Ogive. (3)

72 / 73 – 52 / 53 ✓ ✓ ✓
 ≈ 20 years

Q1

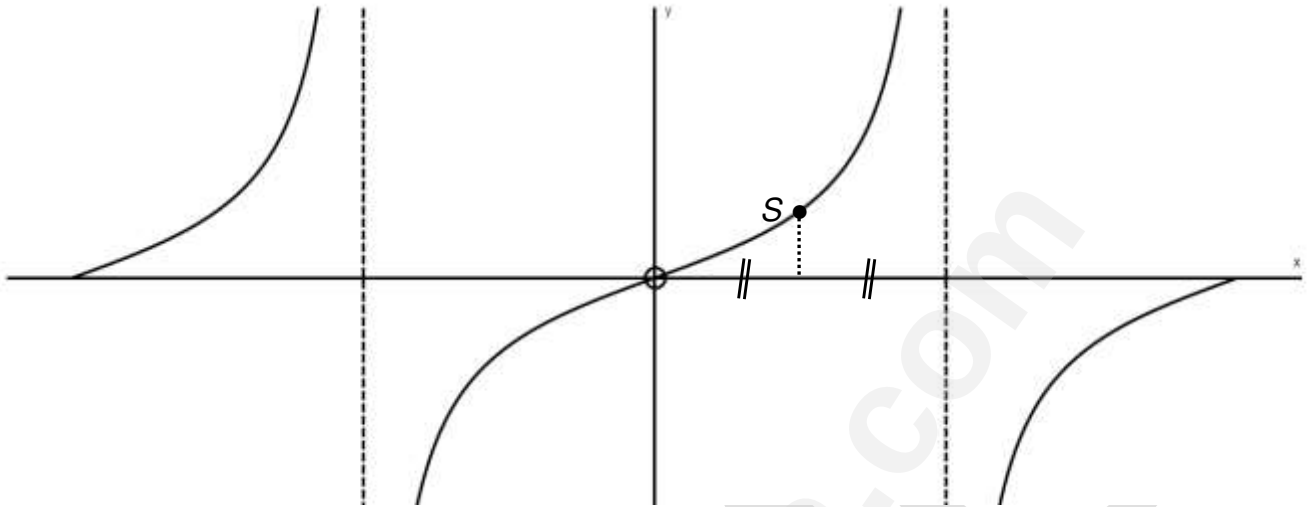
Q2

Answer

[13]

Question 4:

The function $f(x) = \tan 2x$ is sketched for $x \in [-90^\circ; 90^\circ]$.



- (a) Write down the equation of the asymptote to the right of the y-axis. (1)

$x = 45^\circ$ ✓	Answer
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- (b) Give the co-ordinates of the point S. (2)

$(22.5^\circ; 1)$ ✓	x value y value
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- (c) If $g = \frac{\sin x - 2\sin^3 x}{2\sin^2 x \cdot \cos x}$, show that $g = \frac{1}{f(x)}$. (5)

$ \begin{aligned} g &= \frac{\sin x(1 - 2\sin^2 x)}{2\sin^2 x \cdot \cos x} \quad \checkmark \\ &= \frac{\sin x \cdot \cos 2x}{2\sin^2 x \cdot \cos x} \quad \checkmark \\ &= \frac{\cos 2x}{2\sin x \cos x} \quad \checkmark \\ &= \frac{\cos 2x}{\sin 2x} = \frac{1}{\tan 2x} \quad \checkmark \end{aligned} $	Factorise cosine double angle cancel sine sine double angle tan2x
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- (d) Hence, state the values of $x \in [-90^\circ; 90^\circ]$ where g is undefined. (2)

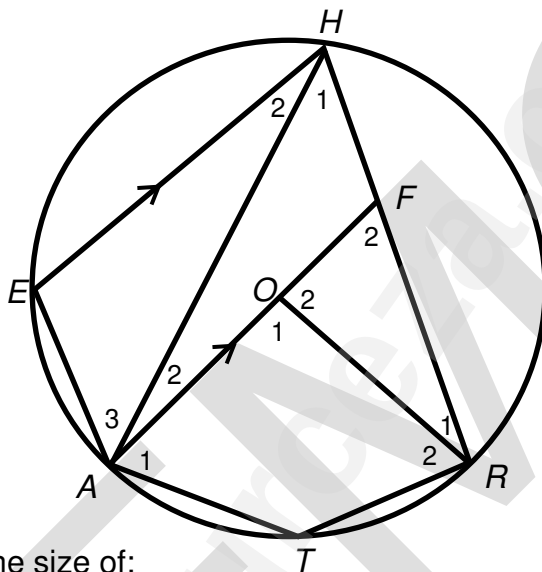
$x \in \{-90^\circ; -45^\circ; 0^\circ; 45^\circ; 90^\circ\}$	Asymptotes 1 mark 0° & $\pm 90^\circ$ 1 mark
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[10]

Question 5:

In the diagram O is the centre of the circle $HEATR$. AOF is parallel to EH .

$$\hat{F}_2 = 78^\circ \text{ and } \hat{R}_1 = 22^\circ.$$



Calculate, with reasons, the size of:

- (a) $\hat{O}_1$ (2)

100° (Ext $\angle \Delta$)	Answer Reason
------------------------------------	------------------

- (b) $\hat{H}_1$ (2)

50° ($\angle$ at centre = $2 \times \angle$ at circum)	Answer Reason
--	------------------

- (c) $\hat{T}$ (2)

130° (Opp $\angle$'s cyclic quad)	Answer Reason
---	------------------

- (d) $\hat{H}_2$ (2)

$E\hat{H}F = 78^\circ$ (Corr $\angle$'s $EH \parallel AF$) $\hat{H}_2 = 78^\circ - 50^\circ = 28^\circ$	Answer Answer
--	------------------

[8]

Question 6:

- (a) Complete the theorem that states: A line drawn parallel to one side of a triangle cuts the other 2 sides ... (1)

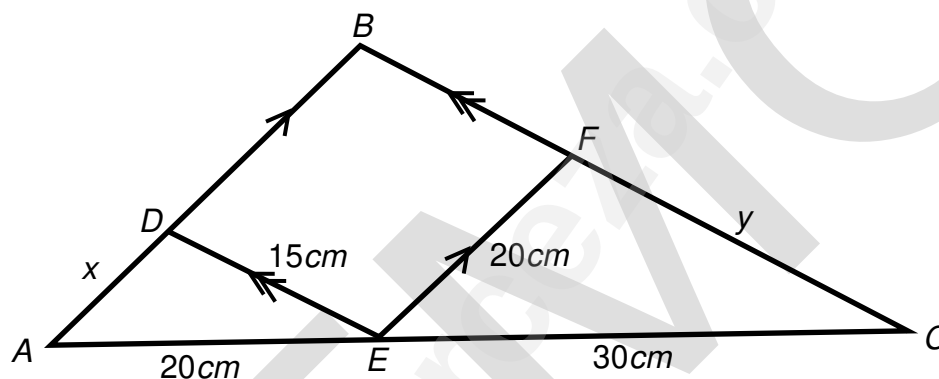
In the same proportion or ratio ✓	Answer
-----------------------------------	--------

- (b) Given $\triangle ABC$ with D a point on AB such that $DE \parallel BC$ and F is a point such that $AB \parallel EF$.

Given:

$DE = 15\text{cm}$, $EF = 20\text{cm}$, $EC = 30\text{cm}$ and $AE = 20\text{cm}$.

$AD = x$ and $FC = y$.



Determine, giving reasons the values of x and y . (5)

$BF = 15\text{cm}$ (Oppsides parm) $\frac{y}{15} = \frac{30}{25}$ ✓ (Propint Thm) ✓ $y = 18$ ✓	Ratio
	Reason
	Answer
$BD = 20$ (Oppsides parm) $\frac{x}{20} = \frac{20}{30}$ ✓ (Propint Thm) $x = \frac{40}{3}$ ✓	Ratio
	Answer

[6]

Question 7:

- (a) Simplify to one trigonometric ratio:

$$\frac{2\cos(90^\circ + \theta)\sin 216^\circ \cos 396^\circ}{\sin 72^\circ} \quad (5)$$

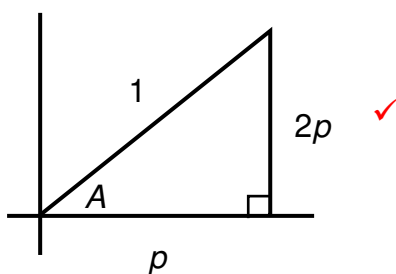
$= \frac{2(-\sin \theta)(-\sin 36^\circ)(\cos 36^\circ)}{2\sin 36^\circ \cos 36^\circ}$ $= \sin \theta$	1 mark each for reduction 1 mark double angle 1 mark answer
---	---

- (b) Prove the following identity:

$$\frac{\sin \theta + \sin 2\theta}{1 + \cos \theta + \cos 2\theta} = \tan \theta \quad (4)$$

$LHS = \frac{\sin \theta + 2\sin \theta \cos \theta}{1 + \cos \theta + 2\cos^2 \theta - 1}$ $= \frac{\sin \theta(1 + 2\cos \theta)}{\cos \theta(1 + 2\cos \theta)}$ $= \tan \theta = RHS$	Double angle Double angle Factorise Tan identity
---	---

(b) Given $\sin A = 2p$ and $\cos A = p$ and $0^\circ < A < 90^\circ$



(1) Determine the value of $\tan A$

(2)

$$\tan A = \frac{2p}{p} = 2 \quad \checkmark$$

Diagram

Answer

(2) Without a calculator, determine the value of p .

(3)

$$(2p)^2 + p^2 = 1 \quad \checkmark \quad (\text{Pythag})$$

$$5p^2 = 1$$

$$p^2 = \frac{1}{5} \quad \checkmark$$

$$p = \frac{\sqrt{5}}{5} \quad \checkmark$$

Method Pythag

Working

Answer

[14]

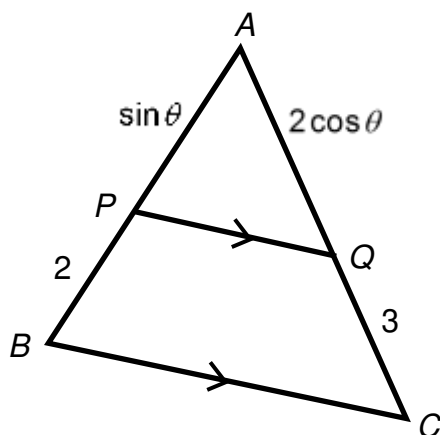
Question 8:

In the triangle below, $PQ \parallel BC$.

$$AP = \sin \theta, PB = 2, AQ = 2 \cos \theta, QC = 3$$

Determine the values of θ for which the diagram is valid, where $\theta \in [0^\circ; 360^\circ]$

(6)



$$\frac{\sin \theta}{2} = \frac{2 \cos \theta}{3} \quad \checkmark \quad (\text{Propint Thm})$$

$$\tan \theta = \frac{1}{3} \quad \checkmark$$

$$\theta = 53,1^\circ + k180^\circ \quad \checkmark \quad k \in \mathbb{Z}$$

$$\theta \in \{53,1^\circ; 133,1^\circ; 313,1^\circ\} \quad \checkmark \checkmark$$

Ratio

Reason

 $\tan \theta$

General solution

Interval values

[6]

Question 9:

In the sketch below,

DA and DB are tangents to the circle ABF at A and B .

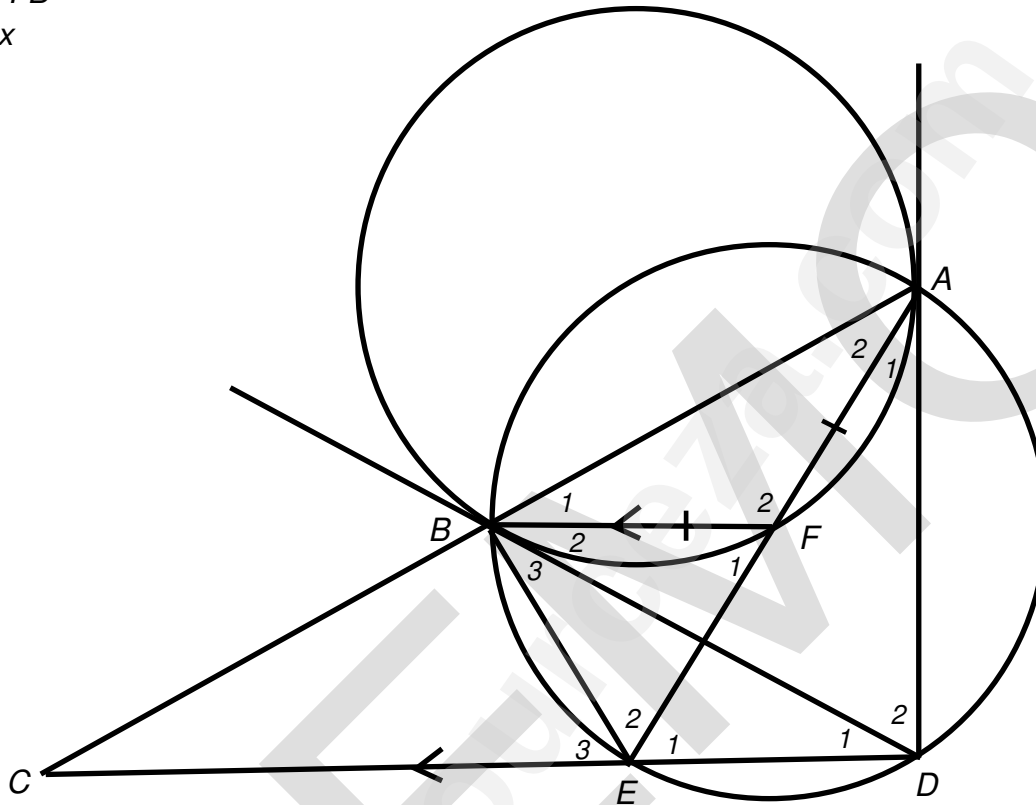
AB produced meets the line DC at C .

$DC \parallel BF$

AF produced meets DC at E .

$AF = FB$

$\hat{A}_1 = x$



(a) Determine, giving reasons, 4 other angles which are equal to x .

(4)

$\hat{B}_1$ <i>tan chord</i> ✓ $\hat{A}_2$ <i>Isos Δ</i> ✓ $\hat{C}$ <i>Corr $\angle$'s $BF \parallel CD$</i> ✓ $\hat{B}_2$ <i>tan chord</i> ✓ $\hat{B}_3$ <i>$\angle$'s same seg</i>	1 mark per statement and reason
--	---------------------------------

(b) Prove $\hat{A}BE = 3\hat{A}_1$ (3)

$\hat{B}_3 = x$ $\hat{B}_2 = x$ ✓✓ $\hat{B}_1 = x$ $\therefore \hat{A}BE = 3x$ ✓ $\therefore \hat{A}BE = 3\hat{A}_1$	Statements 2 marks Conclusion 1 mark
--	--

(c) Prove $AD = BC$ (3)

$AD = BD$ (Tan from same point) ✓ $\hat{B}_2 = \hat{D}_1 = x$ (Alt $\angle$'s $BF \parallel CD$) $\therefore \hat{C} = \hat{D}_1 = x$ ✓ $\therefore BC = BD$ (Isos Δ) ✓ $\therefore BC = AD$	Statement and reason Showing isos Conclusion
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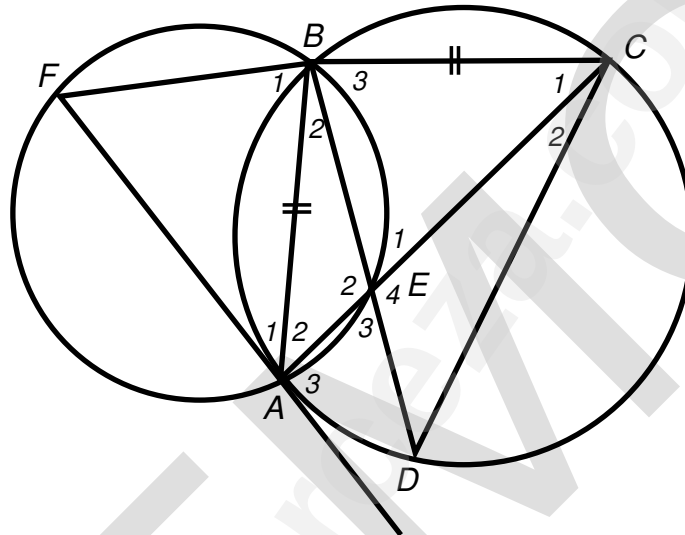
[10]

Question 10:

- (a) Complete the statement: "The exterior angle of a cyclic quadrilateral..." (1)

Is equal to the opposite interior angle ✓	Answer
---	--------

- (b) A, B, C, D are 4 points on the circumference of a circle such that $AB = BC$.
 AC and BD intersect at E .
 FA is a tangent to the circle $ABCD$.



Prove:

- (1) $\triangle BCD \parallel \triangle BEC$ (4)

In $\triangle BCD$ & $\triangle BEC$ (1) $\hat{C}_1 = \hat{D}$ ($\angle$'s opp = chords) ✓ (2) $\hat{B}_3 = \hat{B}_3$ (Common) ✓ (3) $\hat{BCD} = \hat{E}_1$ (3rd $\angle \Delta$) ✓ $\therefore \triangle BCD \parallel \triangle BEC$ (AAA) ✓	1 mark each angle 1 mark final statement
---	--

- (2) $\hat{C}_1 + \hat{C}_2 = \hat{F}$ (3)

$\hat{F} = \hat{E}_1$ (Ext $\angle$ cyclic quad) ✓ $\hat{E}_1 = \hat{BCD}$ ($\parallel \Delta$'s) ✓ $\therefore \hat{F} = \hat{BCD}$ ✓	1 mark each angle 1 mark final statement
--	--

[8]

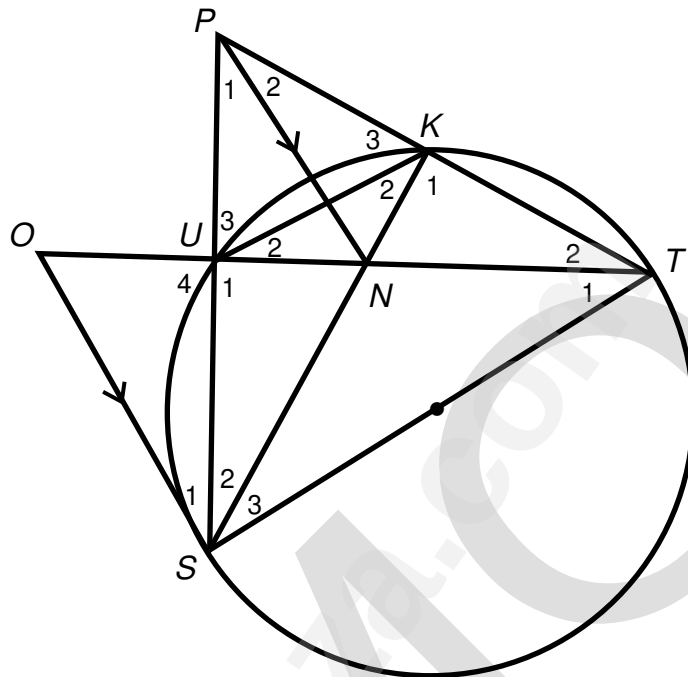
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Question 11:

Refer to the diagram:

ST is a diameter of the circle.

OS // PN,

TO bisects $\hat{S}TP$.

Prove that:

(a) $PUNK$ is a cyclic quadrilateral.

(5)

$\hat{U}_1 = 90^\circ$ ✓ $\hat{K}_1 = 90^\circ$ ✓ $P\hat{K}N = 90^\circ$ ✓ $\therefore PUNK$ cyclic ✓	$(\angle \text{semi circle})$ ✓ (Common) ✓ $(\angle \text{st line})$ ✓ $(\text{Ext } \angle = \text{oppint } \angle \text{'s})$ ✓	1 mark each 1 st angle and reason 1 mark angle & reason 1 mark final reason
--	---	--

(b) SO is a tangent to circle $KUST$.

(5)

$\hat{P}_1 = \hat{K}_2$ ✓ $\hat{P}_1 = \hat{S}_1$ ✓ $\therefore \hat{K}_2 = \hat{S}_1$ ✓ $\therefore SO$ is a tangent ✓	$(\angle \text{same seg})$ ✓ $(\text{Alt } \angle \text{'s } PN // OS)$ $(\text{Conv tan chord thm})$ ✓	1 mark each 1 st angle and reason 1 mark angle & reason 1 mark conclusion 1 mark final reason
--	---	---

(c) *POST* is a cyclic quadrilateral

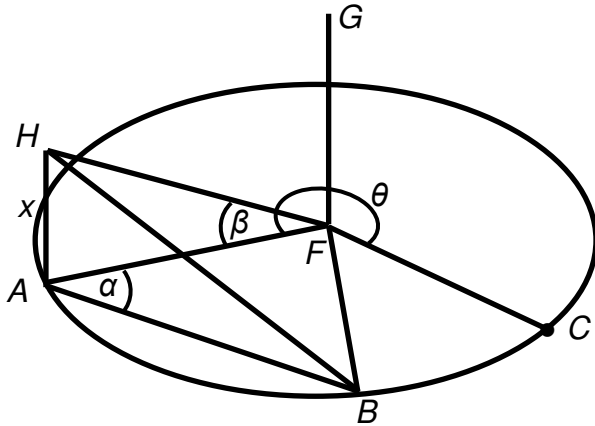
(4)

$\hat{S}_1 = \hat{T}_1$ ✓ $\therefore \hat{T}_1 = \hat{T}_2$ $\therefore \hat{S}_1 = \hat{T}_2$ ✓ <i>POST</i> is cyclic	(tan chord thm) ✓ (Given) ($\angle$'s same seg =) ✓	1 mark each 1 st angle and reason 1 mark conclusion 1 mark final reason
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[14]

Question 12:

Anele and Bina are playing on a merry-go-round. F is the centre of the merry-go-round and FG is the pole in the centre. Anele is leaning against pole AH which is x metres tall and Bina is standing at B . $\hat{AFH} = \beta$, $\hat{FAB} = \alpha$ and reflex angle $\hat{AFC} = \theta$.



- (a) Determine the radius of the merry-go-round in terms of x and β . (2)

$$\tan \beta = \frac{x}{AF} \checkmark$$

$$AF = \frac{x}{\tan \beta} \checkmark$$

1 mark each

- (b) Prove $AB = \frac{2x \cos \alpha}{\tan \beta}$ (4)

$$\frac{AB}{\sin(180^\circ - 2\alpha)} = \frac{AF}{\sin \alpha} \checkmark \checkmark$$

$$AB = \frac{x \cdot \sin 2\alpha}{\tan \beta \cdot \sin \alpha} \checkmark$$

$$= \frac{x \cdot 2 \sin \alpha \cos \alpha}{\tan \beta \sin \alpha} \checkmark$$

$$= \frac{2x \cos \alpha}{\tan \beta}$$

2 mark sine rule

1 mark reduction

1 mark double angle

- (c) If $x = 0,7$ metres, $\alpha = 50^\circ$ and $\beta = 25^\circ$. Determine the length AB to 2 decimal places. (2)

$AB = \frac{2(0,7)\cos 50^\circ}{\tan 25^\circ} \checkmark$ $= 1,93m \quad \checkmark$	Substitution Answer
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- (d) A third friend Carmen gets on at point C. Bina and Carmen are 0,85 metres apart and $\theta = 247^\circ$. How far apart are Anele and Carmen? (5)

$ABC = 123,5^\circ \quad \checkmark \quad (\angle \text{ at centre } = 2 \times \angle \text{ at circum}) \quad \checkmark$ $AC^2 = 1,92^2 + 0,85^2 - 2(1,92)(0,85)\cos(123,5) \quad \checkmark$ $AC = 2,50m \quad \checkmark$	Angle and reason Substitution cos rule 2 marks Answer
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[13]

Question 13:

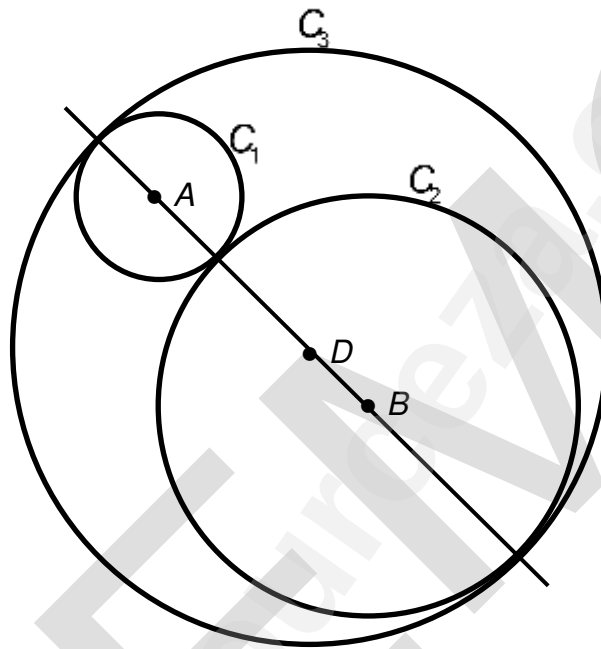
Circle C_1 has the equation $x^2 + y^2 + 6x - 10y + 9 = 0$ and its centre at A .

The centre of C_2 is $B(9; -11)$.

Circle C_1 and C_2 touch externally.

Circle C_3 , with centre D , is drawn so that C_1 , C_2 and C_3 touch internally.

The centres, A , B , and D are collinear.



(a) Determine the centre of circle C_1 .

(3)

$x^2 + 6x + 9 + y^2 - 10y + 25 = -9 + 9 + 25 \quad \checkmark$ $(x+3)^2 + (y-5)^2 = 25 \quad \checkmark$ $(-3;5) \quad \checkmark$	Method Equation Centre
--	---

- (b) Determine the radius of C_2 . (4)

$AB = \sqrt{(9 - (-3))^2 + (-11 - 5)^2} \quad \checkmark$ $= 20 \quad \checkmark$ $R_{C_2} = AB - 5 = 15 \quad \checkmark \quad \checkmark$	Distance AB 2 marks Method Subtraction Answer
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- (c) Determine the equation of the straight line ADB . (3)

$m = \frac{-11 - 5}{9 - (-3)} = -\frac{4}{3} \quad \checkmark$ $y = -\frac{4}{3}x + c$ $(5) = -\frac{4}{3}(-3) + c \quad \checkmark \quad c = 1$ $y = -\frac{4}{3}x + 1 \quad \checkmark$	Gradient Substitution Answer
---	---

- (d) Determine the equation of C_3 . (5)

$R_{C_3} = \frac{30 + 10}{2} = 20 \quad \checkmark$ $\therefore DA = 15$ $15^2 = (x - 3)^2 + (y + 5)^2 \quad \checkmark$ $225 = (x - (-3))^2 + \left(-\frac{4}{3}x + 1 - 5\right)^2$ $225 = x^2 + 6x + 9 + \frac{16}{9}x^2 + \frac{32}{3}x + 16$ $0 = 25x^2 + 150x - 1800 \quad \checkmark$ $0 = x^2 + 6x - 72$ $0 = (x - 12)(x + 6) \quad \checkmark$ $x = 12 \quad x = -6 \quad \checkmark$ $y = -\frac{4}{3}(-6) + 1 = 9$ $(x + 6)^2 + (x - 9)^2 = 400 \quad \checkmark$	Radius Distance DA Substitution of St line method x & y value
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[15]

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MARK RECORD SHEET**Name:** _____**Teacher:**Mr Nonyane ☐Mrs Marais ☐Mr Schaerer ☐Mrs Smith ☐

Question	Analytical Geometry	Euclidean Geometry & Measurement	Statistics	Trigonometry
1	/26			
2			/7	
3			/13	
4				/10
5		/8		
6		/6		
7				/14
8				/6
9		/10		
10		/8		
11		/14		
12				/13
13	/15			
TOTAL	/ 41	/ 46	/ 20	/ 43
TOTAL	/ 150		%	