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ST STITHIANS GIRLS' COLLEGE

Departments of Mathematics

GRADE 12	
PRELIMINARY EXAMINATION – PAPER 1	
DATE: September 2020	TIME: 3 hours
TOTAL MARKS: 150	
EXAMINER: Boys' College	MODERATOR: Girls' College

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This question paper consists of 12 questions and 24 pages.
2. Questions end on Page 19 then 4 additional pages of lined paper for extra working are provided.
3. A separate information (formula) sheet will be provided to you.
4. Answer ALL the questions and clearly show the calculations you have used to determine your answers.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless specified otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Please note that diagrams are not drawn to scale.
8. It is in your own interest to write legibly and to present your work neatly.
9. PLEASE FILL IN YOUR NAME AND CIRCLE YOUR TEACHER'S NAME ON THE BACK PAGE.

Question 1:(a) Solve for x , leaving answers in surd form where appropriate

(i) $3^x = 10$

(2)

$3^x = 10$ $x = \log_3 10$ ✓ $x = 2,09590... \approx 2,1$ ✓	log form answer if directly to answer full marks
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(ii) $5x^2 + 1 = 5x$

(4)

$5x^2 + 1 = 5x$ $5x^2 - 5x + 1 = 0$ ✓ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(5)(1)}}{2(5)}$ $= \frac{5 \pm \sqrt{5}}{10}$ ✓	set to zero sub into quadratic formula two surd solutions if directly to answer full marks $x=0,72$ or $x=0,28$ if not surd
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(b) Given $f(x) = \sqrt{2x+1}$ (i) Give the domain of $f(x)$

(1)

$2x+1 \geq 0$ $x \geq -\frac{1}{2}$ ✓	correct domain
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(ii) If $g(x) = 2x - 1$, Solve for $f(x) = g(x)$,

(5)

$\sqrt{2x+1} = 2x - 1$ $2x+1 = 4x^2 - 4x + 1$ ✓ $0 = 4x^2 - 6x$ ✓ $0 = 2x(2x - 3)$ critical values: $x = 0$ or $x = \frac{3}{2}$ ✓ check: $x \neq 0$ only one solution $x = \frac{3}{2}$ ✓	square both sides equate to zero critical values validity
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[12]

Question 2:

- (a) Solve for x , in the following inequality $2x^2 - 7x + 3 \leq 0$ (4)

$2x^2 - 7x + 3 \leq 0$ $(2x - 1)(x - 3) \leq 0$ ✓ critical values: $x = \frac{1}{2}$ or $x = 3$ ✓ $\frac{1}{2} \leq x \leq 3$ ✓	factorise critical values included solution set
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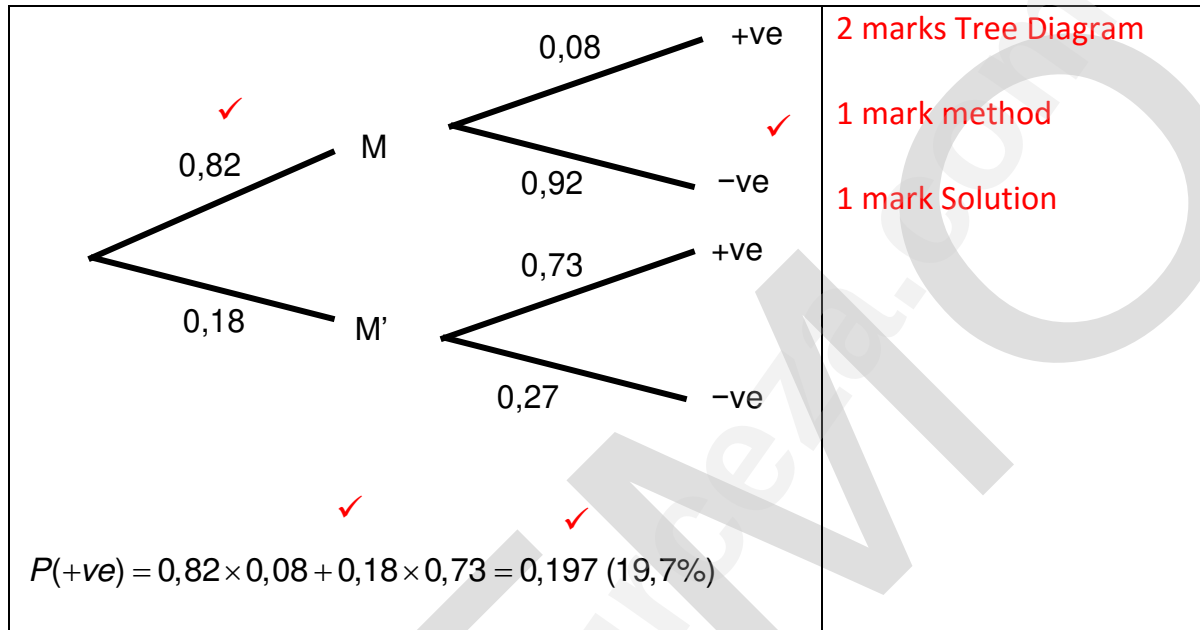
- (b) For which value(s) of q , will the equation $3x^2 - 6qx + 3q^2 = 0$ have real roots? (3)

$3x^2 - 6qx + 3q^2 = 0$ $\Delta = b^2 - 4ac$ $= (-6q)^2 - 4(3)(3q^2)$ ✓ $= 36q^2 - 36q^2$ $= 0$ ✓ $\therefore$ one real repeated root $\therefore q \in \mathbb{R}$ ✓	set discriminant result values of q as per their discriminant
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[7]

Question 3:

- (a) If Thato encountered someone who was positive for the corona virus, the probability they were wearing a mask is 82%. If they were wearing a mask, the probability that she would contract the virus is 8%. If they were not wearing a mask, the probability that she would not contract the virus is 27%. What is the probability that Thato will test positive for the virus? (Disclaimer: These probabilities have not been verified) (4)



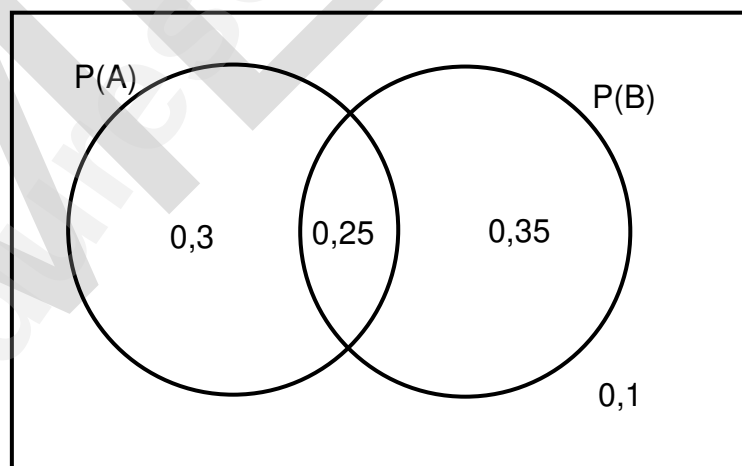
- (b) For two events, A and B, in the Sample space S, it is given that:

$$P(A) = 0,55$$

$$P(B) = 0,6$$

$$P(A \text{ and } B) = 0,25$$

- (i) Draw a Venn Diagram to represent the information. (4)



Determine:

- (ii) $P(A \text{ and } B')$ (1)

$P(A \text{ and } B') = 0,3$ ✓	Answer
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(iii) $P(A \text{ and } B)'$

(1)

$P(A \text{ and } B)' = 1 - 0,25 = 0,75$ ✓	Answer
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- (c) The table below summarises the results of all the driving tests taken in a certain town during the first week of January.

	Took test in the Morning	Took test in the Afternoon	Totals
Pass	43	32	75
Fail	15	8	23
Totals	58	40	98

A person is chosen at random from those who took their test during the first week of January.

- (i) Find the probability that the person took the test in the afternoon and failed. (2)

$P(\text{afternoon} \cap \text{failed}) = \frac{8}{98}$ $= 0,0816...$	numerator denominator
---	--------------------------

- (ii) The person chosen took the test in the morning.

Find the probability that they passed the test

(2)

$P(\text{passed} / \text{morning}) = \frac{43}{58}$ $= 0,74137...$	numerator denominator
--	--------------------------

- (iii) Was the time of day the test was taken independent from outcome? (4)

$P(\text{afternoon}) = \frac{40}{98}$ $P(\text{fail}) = \frac{23}{98}$ $P(\text{afternoon}) \times P(\text{fail}) = \frac{230}{4801} = 0,0957...$ $\neq P(\text{afternoon} \cap \text{failed})$ not independent	one line one column product check conclusion
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[18]

Question 4:

- (a) The first three terms of a geometric sequence are 6 ;
- x
- ; 54 .

Determine the value(s) of x .

(3)

for geometric : constant r : $\frac{x}{6} = \frac{54}{x}$ $x^2 = 324$ $x = \pm 18$	set up r solve for x
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- (b) Evaluate, by showing all your working:
- $\sum_{k=4}^{179} (3k - 1)$
- .

(4)

$\sum_{k=4}^{179} (3k - 1) = (3 \cdot 4 - 1) + (3 \cdot 5 - 1) + (3 \cdot 6 - 1) + \dots + (3 \cdot 179 - 1)$ $= 11 + 14 + 17 + \dots + 536$ $= \frac{n}{2}(a + l)$ $= \frac{179 - 4 + 1}{2}(11 + 536)$ $= 48136$	first term last term number of terms answer
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- (c) The sum of the first
- n
- terms of a series is given by the formula
- $S_n = 3n^2 - 2n$
- .

Determine T_{10} the tenth term of this series.

(3)

$T_{10} = S_{10} - S_9$ $= (3(10)^2 - 2(10)) - (3(9)^2 - 2(9))$ $= 280 - 225$ $= 55$	Sum of 10 Sum of 9 10 th term
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[10]

Question 5:(a) Given the series $(m-3) + (m-3)^2 + (m-3)^3 + \dots$ (i) For which value(s) of m will the series converge? (3)

for converge: $-1 < r < 1$ ✓ $-1 < m-3 < 1$ ✓ $2 < m < 4$ ✓	converging r inequality min max
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(ii) If $m = \frac{5}{2}$ determine S_{∞} of the series. (2)

$S_{\infty} = \frac{a}{1-r}$ $= \frac{\left(\frac{5}{2}-3\right)}{1-\left(\frac{5}{2}-3\right)} \checkmark$ $= \frac{-\frac{1}{2}}{\frac{3}{2}}$ $= -\frac{1}{3} \checkmark$	a and r into infinite sum formula answer
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41; 43; 47; 53; 61; 71; 83; ...

(b) Consider the sequence of numbers: $\begin{matrix} 2 & 4 & 6 & 8 & 10 & 12 \\ 2 & 2 & 2 & 2 & 2 \end{matrix}$ (i) Determine a formula for T_n (4)

$2a = 2$ $3a + b = 2$ $a + b + c = 41$ $a = 1 \checkmark$ $b = -1 \checkmark$ $c = 41 \checkmark$ $T_n = n^2 - n + 41 \checkmark$	find the a,b,c values expression for Tn
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(ii) An interesting property of this sequence is that the first 40 terms are all prime numbers.

Calculate the 41st term and show that it is **not** a prime number (3)

$T_n = n^2 - n + 41$ $T_{41} = 41^2 - 41 + 41 \checkmark$ $= 41^2 \checkmark$ $\therefore$ not prime ✓	sub 41 answer conclusion
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[12]

Question 6:

(a) Given $f(x) = 1 - 3x^2$.

(i) Determine $f'(x)$ from first principles.

(5)

$f(x) = 1 - 3x^2$ $f(x+h) = 1 - 3x^2 - 6xh - 3h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{1 - 3x^2 - 6xh - 3h^2 - 1 + 3x^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-6x - 3h)}{h}$ $= \lim_{h \rightarrow 0} -6x - 3h$ $= -6x - 0$ $= -6x$	$x+h$ distribute negative factorise h set limit to 0 derivative
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(ii) Determine the equation of the tangent for $f(-2)$.

(4)

$f(-2) = -11$ $f'(-2) = 12$ $y = 12x + 13$	y m equation
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(b) Evaluate the following derivatives:

(i) $D_x \left[\left(1 - x^{\frac{1}{2}}\right) \left(1 - \frac{2}{x^{\frac{1}{2}}}\right) \right]$

(4)

$D_x \left[\left(1 - x^{\frac{1}{2}}\right) \left(1 - \frac{2}{x^{\frac{1}{2}}}\right) \right]$ $= D_x \left[1 - \frac{2}{x^{\frac{1}{2}}} - x^{\frac{1}{2}} + 2 \right]$ $= D_x \left[3 - 2x^{-\frac{1}{2}} - x^{\frac{1}{2}} \right]$ $= +x^{-\frac{3}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$	expand sort out powers derive
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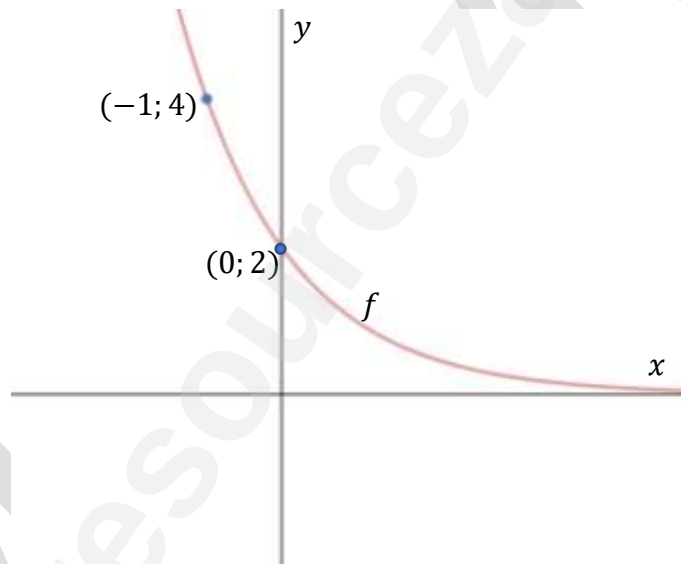
(ii) $\frac{dy}{dx}$ if $y = \frac{x^2 - 1}{x + 1}$ (3)

$y = \frac{x^2 - 1}{x + 1}$ $= \frac{(x + 1)(x - 1)}{x + 1} \checkmark$ $= x - 1 \checkmark$ $\frac{dy}{dx} = 1 \checkmark$	expand simplify derive
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[16]

Question 7:

Shown below the graph of $f(x) = a.b^x$



(a) Show that $a = 2$ and $b = \frac{1}{2}$ (2)

$f(x) = a.b^x \checkmark$ sub(0;2) $2 = a.b^0 \checkmark$ $\therefore a = 2$	$f(x) = 2.b^x$ sub(-1;4) $\checkmark$ $4 = 2.b^{-1}$ $2 = \frac{1}{b} \checkmark$ $\therefore b = \frac{1}{2}$	substitutions simplifications
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- (b) What is the equation of $g(x)$, the reflection of $f(x)$ in the y-axis? (2)

$f(x) = 2 \cdot \left(\frac{1}{2}\right)^x$ reflection on y-axis $(x; y) \rightarrow (-x; y)$ $g(x) = 2 \cdot \left(\frac{1}{2}\right)^{-x}$ $= 2 \cdot 2^x$ $= 2^{x+1}$	reflect new formula
--	--------------------------------

- (c) What is the equation of $f^{-1}(x)$? (3)

$f(x) = 2 \cdot \left(\frac{1}{2}\right)^x$ $y = 2 \cdot \left(\frac{1}{2}\right)^x$ $x = 2 \cdot \left(\frac{1}{2}\right)^y$ $\frac{x}{2} = \left(\frac{1}{2}\right)^y$ $\log\left(\frac{x}{2}\right) = y \cdot \log\left(\frac{1}{2}\right)$ $y = \log_{\left(\frac{1}{2}\right)}\left(\frac{x}{2}\right)$ $f^{-1}(x) = \log_{\left(\frac{1}{2}\right)}\left(\frac{x}{2}\right)$	map x onto y take log simplify log
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- (d) What is the domain of $f^{-1}(x)$? (2)

Domain: $x > 0$	correct inequality correct reference
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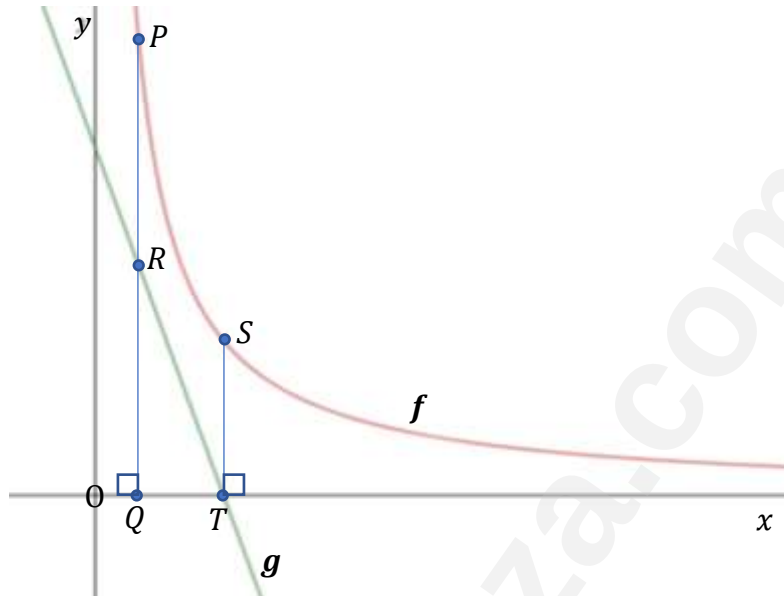
- (e) If $f(x)$ and $f^{-1}(x)$ intersect at point $(1; 1)$,
Determine the value(s) of x for which $f(x) \leq f^{-1}(x)$? (2)

$f(x) \leq f^{-1}(x)$ $0 < x \leq 1$	correct inequality correct reference
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[11]

Question 8:

Shown below the graphs of $f(x) = \frac{12}{x}$ for all $x \geq 0$ and $g(x) = -3x + 9$



- (a) Determine the co-ordinates of S. (2)

$g(x) = -3x + 9$ $0 = -3x + 9$ $x = 3$ $f(3) = 4$ $S(3; 4)$	x-value correct corresponding y-value based on their x
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- (b) Write down the axis of symmetry for $f(x)$. (1)

Axis of symmetry: $y = x$	answer
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- (c) The following three scenarios require you to visualise specific situations in order to answer the questions. So read the unique scenarios carefully before answering.

- (i) If $OQ = 1,5$ units determine the length of PR . (3)

$OQ = 1,5$ units $PR = f\left(\frac{3}{2}\right) - g\left(\frac{3}{2}\right)$ $= 8 - \frac{9}{2}$ $= \frac{7}{2} = 3,5$ units	determine $f(1,5)$ determine $g(1,5)$ PR answer
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- (ii) If
- $PR = 6$
- units determine possible length(s) of
- OQ
- .

(4)

$PR = 6$ units $6 = \frac{12}{x} - (-3x + 9)$ ✓ $6 = \frac{12}{x} + 3x - 9$ $6x = 12 + 3x^2 - 9x$ $0 = 3x^2 - 15x + 12$ ✓ $0 = x^2 - 5x + 4$ $0 = (x - 4)(x - 1)$ $OQ = 4$ or ✓ $OQ = 1$ ✓	set difference equal to 6 setup a quadratic Solutions
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- (iii) If
- Q
- is a variable point on the x-axis for all
- $x \geq 0$
- ,
-
- determine the minimum distance of
- PR
- .

(5)

let $m(x) = f(x) - g(x)$ $m(x) = \frac{12}{x} - (-3x + 9)$ $= \frac{12}{x} + 3x - 9$ ✓ $= 12x^{-1} + 3x - 9$ $m'(x) = -12x^{-2} + 3$ ✓ $0 = \frac{-12}{x^2} + 3$ $0 = -12 + 3x^2$ $0 = x^2 - 4$ $x = \pm 2$ min $PR = m(2)$ ✓ $= 3$ units apart ✓	set difference of equations derive set to zero solve for turning points choose correct TP and find min PR
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[15]

Question 9:

You take out a home loan of R1,2 million in order to purchase an apartment. You have to pay off the loan over a period of 20 years at 8% p.a compounded monthly.

- (a) Determine the size of the monthly instalments if the first payment is made one month after you take out the loan. (4)

$1200000 \left(1 + \frac{0.08}{12}\right)^{20 \times 12} = \frac{x \left[\left(1 + \frac{0.08}{12}\right)^{20 \times 12} - 1 \right]}{\frac{0.08}{12}}$ $5912163,33 = x(589,02\dots)$ $x = 10037,28$	grow loan set power set interest rate solve
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- (b) Determine the outstanding balance on the loan after 12 years. (4)

$OB_{12} = 10037,28 \left[\frac{1 - \left(1 + \frac{0.08}{12}\right)^{-8 \times 12}}{\frac{0.08}{12}} \right] = R710016,82$ or $OB_{12} = 1200000 \left(1 + \frac{0.08}{12}\right)^{12 \times 12} - \frac{10037,28 \left[\left(1 + \frac{0.08}{12}\right)^{12 \times 12} - 1 \right]}{\frac{0.08}{12}}$ $= 3124067,09 - 2414050,02$ $= 710017,07$	grow loan use monthly from above grow payments solve
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- (c) After a period of 12 years you inherit R50000 and decide to pay it into your bond. Determine the size of your new monthly instalment assuming that you will continue to pay off your loan for a further 5 years at the same rate of interest. (5)

new loan = $710017,07 - 50000$ $= 660017,07$ ✓ $660017,07 \left(1 + \frac{0,08}{12}\right)^{5 \times 12} = \frac{x_{new} \left[\left(1 + \frac{0,08}{12}\right)^{5 \times 12} - 1 \right]}{\frac{0,08}{12}}$ $983323,60 = x_{new} (73,476...)$ $x_{new} = 13382,77$ ✓	determine new loan from the above OB grow loan set power set interest rate solve
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- (d) How much money will you have saved by making the deposit of R50000 after 12 years and restructuring the loan? (4)

original cost = $10037,28 \times 12 \times 20$ $= 2408947,20$ ✓ new cost = $(10037,28 \times 12 \times 12) + (13382,77 \times 12 \times 5) + 50000$ $= 2298334,30$ savings = $2408947,20 - 2298334,30$ $= 110612,90$ ✓	calculate original cost calculate the split cost taking into account deposit find saving
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[17]

Question 10:Solve for x :

(a) $3^{4x} - 27 \cdot 3^x = 0$

(5)

$3^{4x} - 27 \cdot 3^x = 0$ $\text{let } k = 3^x$ $k^4 - 27k = 0$ $k(k^3 - 27) = 0$ $k(k-3)(k^2 - 3k + 9) = 0$ $k = 0 \text{ or } k = 3$ $3^x = 0 \text{ or } 3^x = 3$ $n.s. \quad x = 1$	$3^{4x} - 27 \cdot 3^x = 0$ $3^{4x} - 3^3 \cdot 3^x = 0$ $3^{4x} - 3^{x+3} = 0$ $3^{4x} = 3^{x+3}$ $4x = x + 3$ $3x = 3$ $x = 1$	suitable substitution factorise solve for roots check answer
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(b) By completing the square and leaving your answers in terms of b

$2x^2 + 5x - 2b = 3$

(4)

$2x^2 + 5x - 2b = 3$ $2x^2 + 5x = 2b + 3$ $x^2 + \frac{5}{2}x = b + \frac{3}{2}$ $x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = b + \frac{3}{2} + \left(\frac{5}{4}\right)^2$ $\left(x + \frac{5}{4}\right)^2 = b + \frac{3}{2} + \frac{25}{16}$ $\left(x + \frac{5}{4}\right)^2 = \frac{16b + 49}{16}$ $x + \frac{5}{4} = \pm \sqrt{\frac{16b + 49}{16}}$ $x = -\frac{5}{4} \pm \frac{\sqrt{16b + 49}}{4}$ $x = \frac{-5 \pm \sqrt{16b + 49}}{4}$	divide by 2 complete the square solve for x in terms of b
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[9]

Question 11:

Airlines have various restrictions on the size of bags that can be taken into the cabin of a plane. Some require that the largest rectangular box shape has sum of the length, breath and height equal to 115 cm with length 55 cm.

Determine the maximum volume of the bag you can take on the plane that satisfies these criteria.

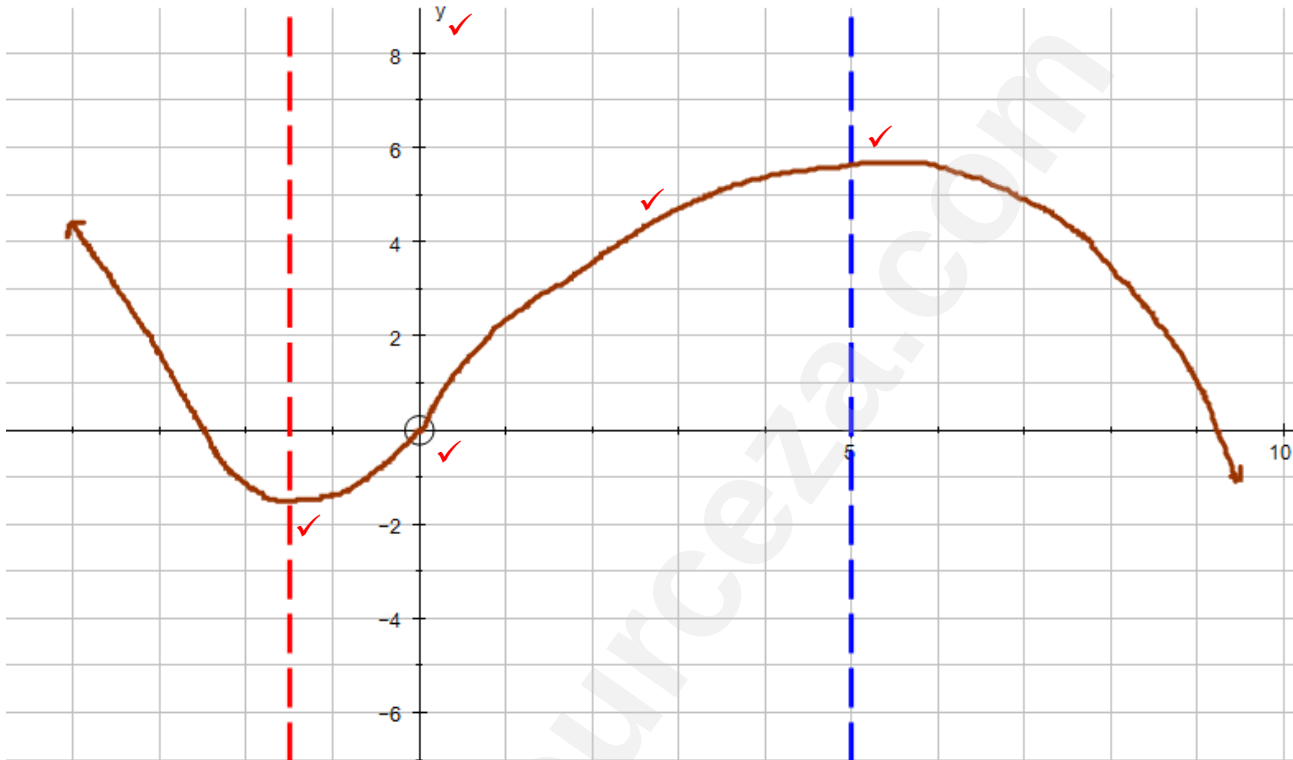
Sum of dimensions = $l + b + h$ $115 = 55 + b + h$ $60 = b + h$ $b = 60 - h \quad \checkmark$ Volume = $l.b.h$ $V(h) = 55.(60 - h).h$ $= 3300h - 55h^2 \quad \checkmark$ $V(h) = 3300h - 55h^2$ $V'(h) = 3300 - 110h \quad \checkmark$ $\checkmark 0 = 3300 - 110h$ $110h = 3300$ $h = 30 \quad \checkmark$ $b = 60 - h$ $= 60 - 30$ $= 30$ Max Volume = $l.b.h$ $= 55.30.60$ $= 49500\text{cm}^3 \quad \checkmark$	Will need to follow student logic. This method solves sets up a volume expression that needs to be maximized using calculus. Set variables Create volume expression derive set to zero solve for a root find other dimensions determine max volume
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[6]

Question 12:

Given $f(x) = ax^3 + bx^2 + cx$ with $f'\left(\frac{-3}{2}\right) = f'(5) = 0$ and $f(x)$ increasing for $\frac{-3}{2} < x < 5$

- (i) Use this information to draw a rough sketch of $y = f(x)$ (5)



- (ii) Determine the value(s) of x for which:

- (1) $f'(x) < 0$ (2)

$f'(x) < 0$ $x < \frac{-3}{2}$ or $5 < x$	two arms of the parabola derivative
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- (2) $f(x)$ is concave upwards (3)

$5 + \left(\frac{3}{2}\right) = \frac{13}{2} = 6,5$ $-\frac{3}{2} + \frac{13}{4} = \frac{7}{4}$ $x < \frac{7}{4}$	find the point of inflection between the two turning points then recognize that concave up is the the left
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[17]

Question 13:

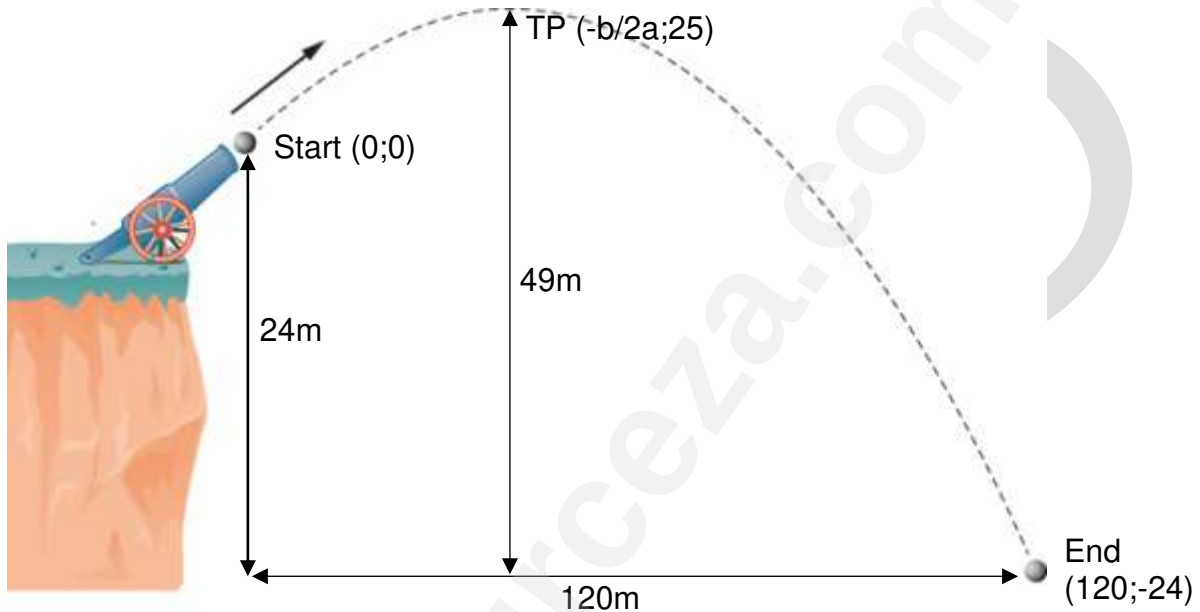
Projectiles like cannon balls go through the air along the path of a parabola.
As shown below in the rough sketch.

Let the point where the cannon ball leaves the cannon be point $(0; 0)$, and note that the cannon ball turns 40m above its end point.

Find an equation that will match the path of the cannon ball in the form:

$$y = ax^2 + bx + c$$

(7)



$y = ax^2 + bx + c$ sub $(0;0)$ $0 = c$ ✓	$y = ax^2 + bx$ sub $\left(\frac{-b}{2a}; 25\right)$ ✓ $25 = \frac{b^2}{4a} - \frac{b^2}{2a}$ $100a = -b^2$ $a = \frac{-b^2}{100}$ ✓	$y = ax^2 + bx$ ✓ sub $(120; -24)$ $-24 = 14400a + 120b$	Will need to follow student logic. This method does substitution of points and uses the TP x-value then solves simultaneously.
from above $a = \frac{-b^2}{100}$ $-24 = -144b^2 + 120b$ $0 = -144b^2 + 120b + 24$	$b = 1$ ✓ $b = -\frac{1}{6}$ N/A	$(-ve b \rightarrow TP - ve x \text{ value})$	
$a = -\frac{1}{100}$			
$y = \frac{-x^2}{100} + x$ ✓			

END OF PAPER 1

MEMORANDUM