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FORM V MATHEMATICS: PAPER II  
PRELIMINARY EXAMINATION 2021  
ANALYSIS SHEET

EXAMINATION NUMBER: \_\_\_\_\_ TEACHER: \_\_\_\_\_

| <u>Question</u>        | <u>Topic</u>                | <u>Mark obtained</u> |
|------------------------|-----------------------------|----------------------|
| 1                      | Analytical Geometry<br>[20] |                      |
| 2                      | Data Handling<br>[7]        |                      |
| 3                      | Trigonometry<br>[23]        |                      |
| 4                      | Measurement<br>[11]         |                      |
| 5                      | Euclidian Geometry<br>[11]  |                      |
| <b>TOTAL SECTION A</b> | <b>[72]</b>                 |                      |
| 6                      | Data Handling<br>[15]       |                      |
| 7                      | Trigonometry<br>[11]        |                      |
| 8                      | Euclidean Geometry<br>[11]  |                      |
| 9                      | Euclidian Geometry<br>[6]   |                      |
| 10                     | Analytical Geometry<br>[19] |                      |
| 11                     | Trigonometry<br>[16]        |                      |
| <b>TOTAL SECTION B</b> | <b>[78]</b>                 |                      |

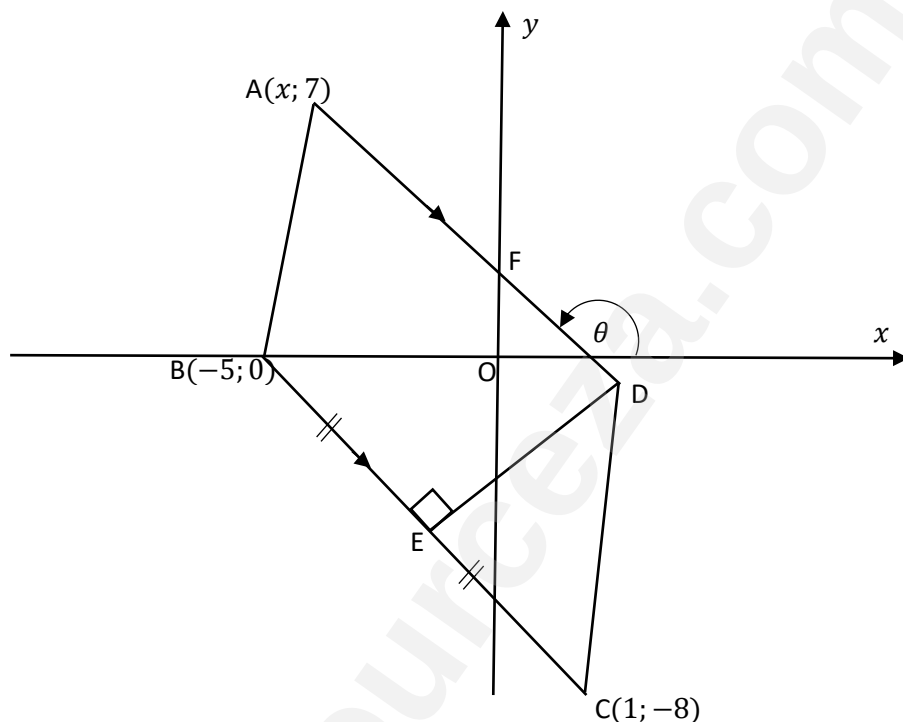
**TOTAL:**  $\frac{\quad}{150} = \quad \%$

Comment:

## SECTION A

### QUESTION 1

In the diagram, ABCD is a trapezium with  $AD \parallel BC$  and vertices  $A(x; 7)$ ,  $B(-5; 0)$ ,  $C(1; -8)$  and D.  $DE \perp BC$  with E on BC such that  $BE = EC$ . The inclination of AD with the positive  $x$ -axis is  $\theta$  and AD cuts the  $y$ -axis at F.



- Calculate the gradient of BC. (2)
- Calculate the coordinates of E. (2)
- Determine the equation of DE in the form  $y = mx + c$ . (3)

(d) Calculate, rounded to two decimal digits:

(1) the size of  $\theta$  (3)

(2) the size of  $\widehat{OFD}$  (2)

(e) Determine the value of  $x$  if the length of  $AB = 5\sqrt{2}$  (5)

(f) Determine the equation of the circle with diameter BC in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3)$$

[20]

## QUESTION 2

The data in the table below represent the percentage scores for 12 Mathematics students in their Grade 12 Preliminary examination and the corresponding Final examination.

|             |    |    |    |    |    |    |    |    |    |    |    |    |
|-------------|----|----|----|----|----|----|----|----|----|----|----|----|
| Prelim Exam | 76 | 64 | 90 | 68 | 70 | 79 | 52 | 64 | 61 | 71 | 84 | 70 |
| Final Exam  | 82 | 69 | 94 | 75 | 80 | 88 | 56 | 81 | 76 | 78 | 90 | 76 |

- (a) Determine the equation of the least squares regression line for the data set. Give your answers rounded to two decimal digits. (3)
- (b) Calculate the correlation coefficient, correct to 3 decimal places, of the above data and comment on the correlation between the Preliminary and Final examination marks. (2)
- (c) Hence, predict the final percentage for a student obtaining 73% in the Preliminary examination, giving your answer to the nearest percentage. (2)

[7]

### QUESTION 3

(a) Given  $\sin 31^\circ = p$ , determine the following in terms of  $p$ , without a calculator.

(1)  $\sin 149^\circ$

(2)

(2)  $\cos(-59^\circ)$

(2)

(3)  $\cos 62^\circ$

(2)

(b) Simplify to a single trigonometric ratio:

$\cos 65^\circ \cdot \sin 40^\circ + \cos 25^\circ \cdot \sin 50^\circ$

(4)

(c) Simplify without using a calculator:

$$\frac{\sin 150^\circ \cdot \tan 225^\circ}{\sin(-30^\circ) \cdot \cos 420^\circ} \quad (5)$$

(d) Consider:  $\frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} = \tan x$

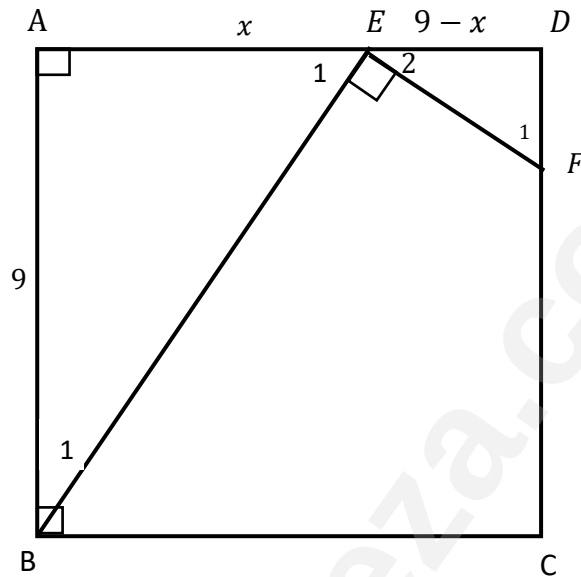
(1) Prove the identity. (5)

(2) Determine the values of  $x$  for which the identity is invalid for  $x \in [0^\circ; 360^\circ]$ . (3)

**[23]**

#### QUESTION 4

In the diagram below, ABCD is a square with sides 9 cm in length and  $BE \perp EF$



(a) Show that  $\hat{F}_1 = a$ , if  $\hat{E}_1 = a$  (2)

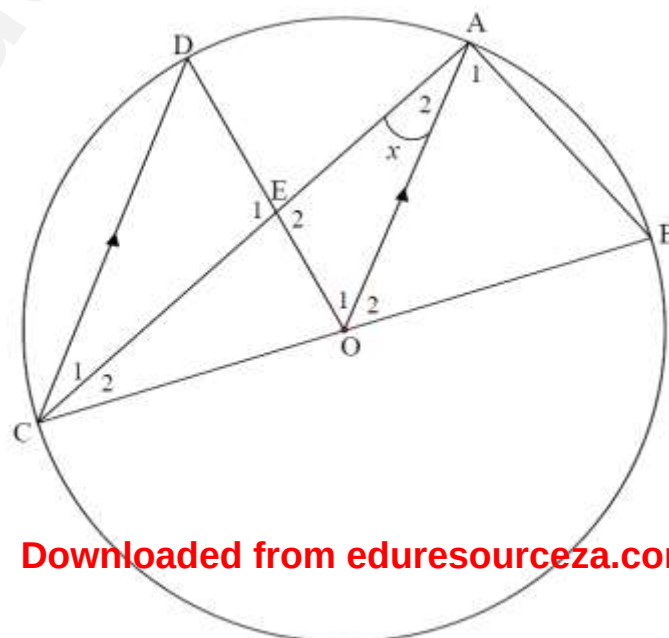
(b) Prove that  $\triangle ABE$  is similar to  $\triangle DEF$  (4)

(c) Show that area of  $\triangle EDF = \frac{x(9-x)^2}{18}$  (5)

[11]

### QUESTION 5

In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle and CB is the diameter of the circle. Chord CA intersects radius OD at E. AB is drawn.  $CD \parallel OA$  and  $\hat{A}_2 = x$ .



(a) Give reasons why

(1)  $\hat{C}_1 = x$  (1)

(2)  $\hat{C}_2 = x$  (1)

(b) Determine the following angles in terms of  $x$ , giving reasons.

(1)  $\hat{A}_1$  (2)

(2)  $\hat{O}_1$  (2)

(3)  $\hat{O}_2$  (2)

- (c) For what values of  $x$  will ABOE be a cyclic quadrilateral? (3)

[11]

**Total Section A: 72 marks**

## SECTION B

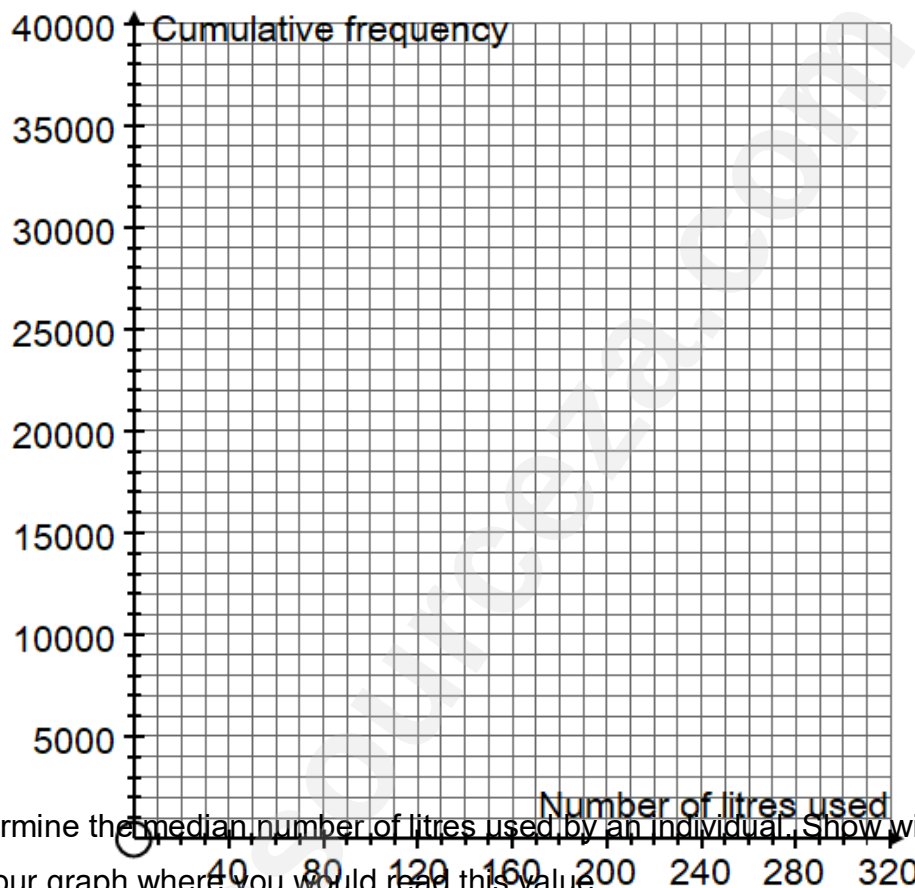
### QUESTION 6

The table below summarises the amount of water consumed by individuals.

| Number of litres used | Frequency | Midpoint | Cumulative Frequency |
|-----------------------|-----------|----------|----------------------|
| $0 < x \leq 40$       | 2 000     | 20       | 2 000                |
| $40 < x \leq 80$      | 3 000     | 60       | 5 000                |
| $80 < x \leq 120$     | 7 000     | 100      | 12 000               |
| $120 < x \leq 160$    | 13 000    | 140      | 25 000               |
| $160 < x \leq 200$    | 6 500     | 180      | 31 500               |
| $200 < x \leq 240$    | 2 500     | 220      | 34 000               |
| $240 < x \leq 280$    | 1 000     | 260      | 35 000               |

- (a) Determine the estimated mean number of litres used by an individual. Give your answer correct to two decimal places. (2)

- (b) Draw a cumulative frequency graph that represents the information in the table above. (3)



- (c) Determine the median number of litres used by an individual. Show with an "M" on your graph where you would read this value. (2)

- (d) How many individuals use more than 220 litres of water? Show with an "A" on your graph where you would read this value. (2)

- (e) At the time of the survey the maximum number of litres permitted per individual was 280 litres. If the restriction was reduced to 240 litres per individual, what would happen to: (You may assume that all the individuals who used between 240 and 280 litres will now use between 200 and 240 litres.)

(1) the median? (Give a reason for your answer.) (2)

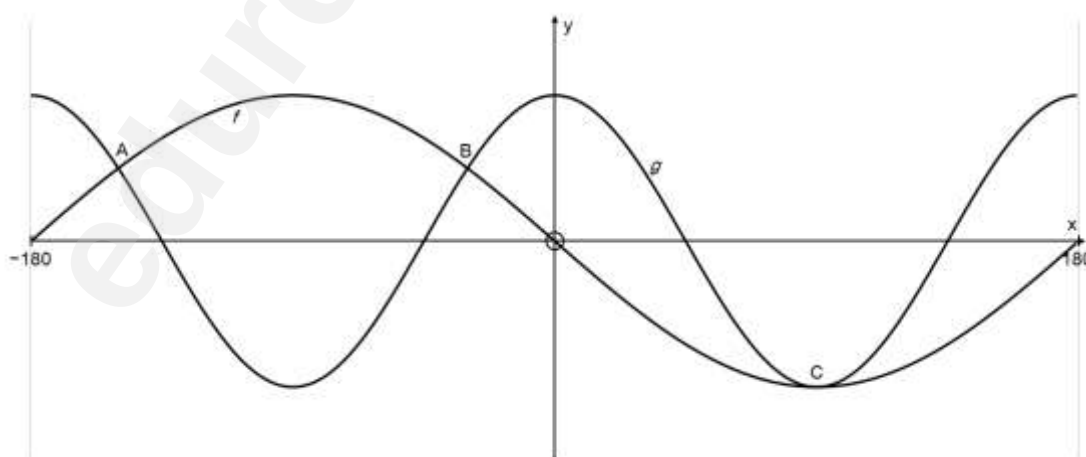
(2) the standard deviation? (Give a reason for your answer.) (2)

(3) the skewness of the data? (Give a reason for your answer.) (2)

[15]

### QUESTION 7

In the diagram below, the graphs of  $f(x) = \cos 2x$  and  $g(x) = -\sin x$  are drawn in the interval  $x \in [-180^\circ; 180^\circ]$ . A, B and C are the points of intersection of  $f$  and  $g$ .



(a) Without using a calculator, determine the  $x$  –coordinates of points A, B and C. (7)

(b) Determine the values of  $x$  for which  $f'(x) \cdot g'(x) > 0$ . (2)

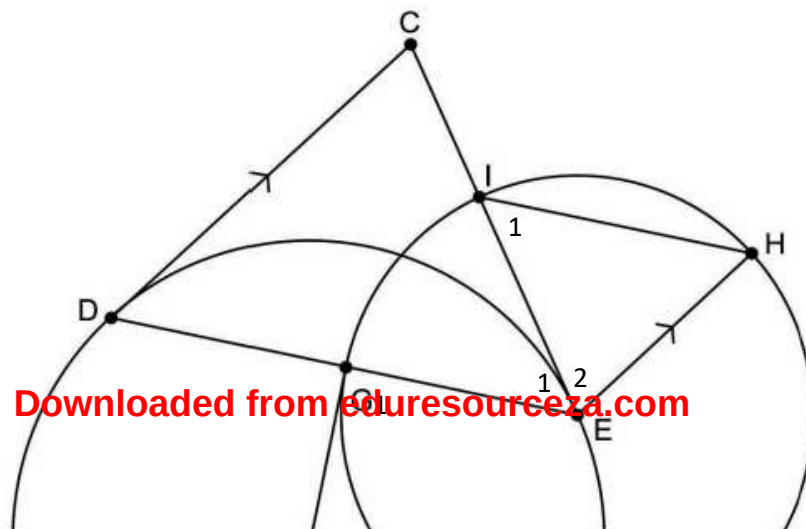
(c) Determine the values of  $k$  for which  $\cos 2x + 3 = k$  will have no solutions. (2)

[11]

### QUESTION 8

In the diagram below:

- I, H and G are points on the circle with centre E
- AB is a diameter of the semi-circle with centre F, and D and E lie on the semi-circle
- CD and CE are tangents to the semi-circle at D and E respectively
- $CD \parallel HE$
- FG is a tangent to the circle with centre E



(a) Prove that  $\triangle DCE \parallel \triangle HEI$  (4)

(b) Prove  $DG = GE$  (2)

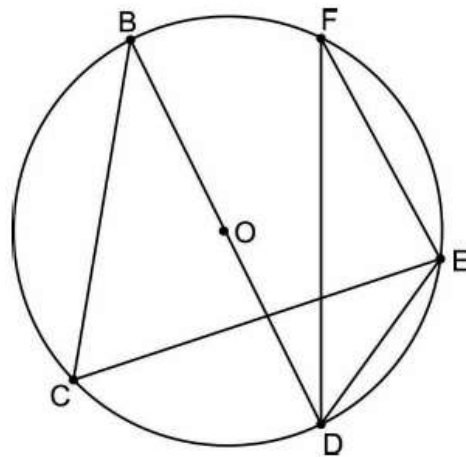
(c) Show that  $2HE^2 = DC \times HI$  (5)

[11]

**QUESTION 9**

In the diagram below:

- O is the centre of the circle passing through B, C, D, E and F
- BD is a diameter
- $D\hat{F}E = x$  and  $B\hat{C}E = 2x + 6^\circ$

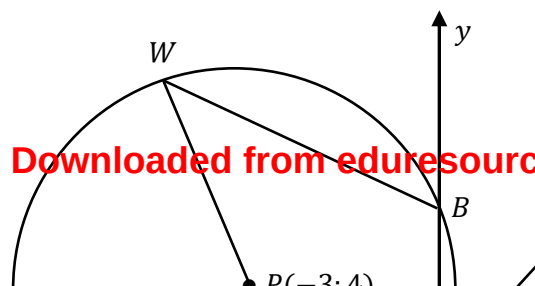


Determine the value of  $x$ .

[6]

### QUESTION 10

In the diagram,  $P(-3; 4)$  is the centre of the circle.  $V(k; 1)$  and  $W$  are the endpoints of a diameter. The circle intersects the  $y$ -axis at  $B$  and  $C$ .  $BCVW$  is a cyclic quadrilateral.  $CV$  is produced to intersect the  $x$ -axis at  $T$ .  $\widehat{OTC} = \alpha$ .



- (a) The radius of the circle with centre  $P$  is  $\sqrt{10}$ . Determine the value of  $k$  if point  $V$  is to the right of point  $P$ . (4)

- (b) Determine the length of  $BC$ . (5)

(c) If  $k = -2$ , calculate the size of: (Round your answers off to one decimal place.)

(1)  $\alpha$  (2)

(2)  $\widehat{VWB}$  (2)

(d) A new circle is obtained by reflecting the given circle about the line  $y = 1$ .

Determine:

(1) The coordinates of  $Q$ , the centre of the new circle. (2)

(2) The equation of the new circle in the form  $(x - a)^2 + (y - b)^2 = r^2$ . (2)

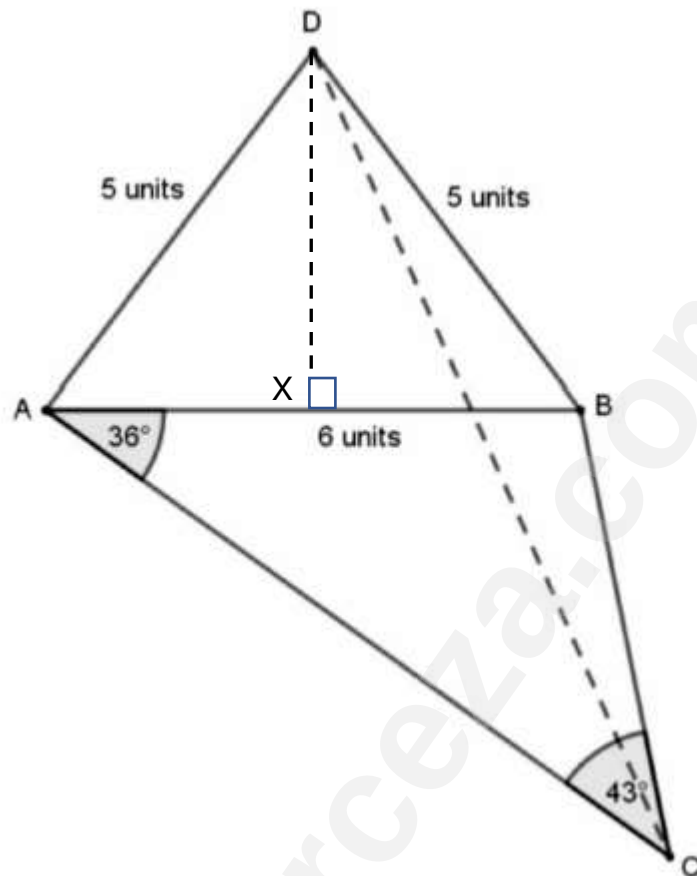
(3) The equations of the lines parallel to the  $y$  –axis and passing through the points of intersection of the two circles. (2)

[19]

### QUESTION 11

In the diagram below,  $\triangle ABC$  lies on the horizontal plane and  $\triangle ABD$  lies on the vertical plane.

- $DB = DA = 5$  units
- $AB = 6$  units
- $\angle CAB = 36^\circ$  and  $\angle BCA = 43^\circ$



(a) Determine the length of DX. (2)

(b) Determine the length of AC, correct to two decimal digits. (3)

(c) Calculate the length of straight-line DC, correct to two decimal digits. (5)

(d) If  $\triangle ADB$  is folded back along line AB away from C so that D is on the same horizontal plane as A, B and C, then calculate the new straight-line distance from D to C. Give your answer correct to two decimal places. (6)

[16]

Total Section B: 78 marks

GRAND TOTAL: 150 MARKS