

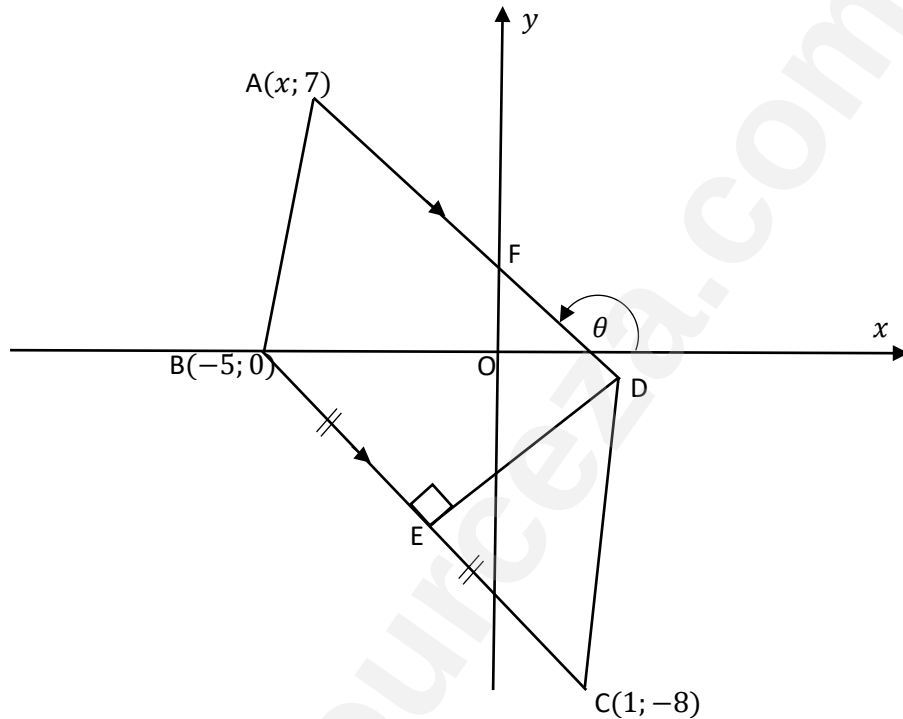


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SECTION A

QUESTION 1

In the diagram, ABCD is a trapezium with $AD \parallel BC$ and vertices $A(x; 7)$, $B(-5; 0)$, $C(1; -8)$ and D. $DE \perp BC$ with E on BC such that $BE = EC$. The inclination of AD with the positive x -axis is θ and AD cuts the y -axis at F.



- (a) Calculate the gradient of BC. (2)

$$m_{BC} = \frac{0+8}{-5-1} = \frac{8}{-6} = -\frac{4}{3} \checkmark \checkmark$$

- (b) Calculate the coordinates of E. (2)

$$E\left(\frac{-5+1}{2}; \frac{0-8}{2}\right) = E(-2; -4) \checkmark \checkmark$$

- (c) Determine the equation of DE in the form $y = mx + c$. (3)

$$m_{DE} = \frac{3}{4} \checkmark$$

$$y - (-4) = \frac{3}{4}(x - (-2)) \checkmark$$

$$y = \frac{3}{4}x - \frac{5}{2} \checkmark$$

(d) Calculate, rounded to two decimal digits:

(1) the size of θ (3)

$$\tan \theta = -\frac{4}{3} \checkmark$$

$$\theta = 180^\circ - 53,13^\circ \checkmark$$

$$= 126,87^\circ \checkmark$$

(2) the size of $\widehat{OFD}$ (2)

$$\widehat{OFD} = 126,87^\circ - 90^\circ \text{ (ext. } \angle \text{ of triangle)} \checkmark$$

$$= 36,87^\circ \checkmark$$

(e) Determine the value of x if the length of $AB = 5\sqrt{2}$ (5)

$$(5\sqrt{2})^2 = (-5 - x)^2 + (0 - 7)^2 \checkmark$$

$$50 = (-5 - x)^2 + (0 - 7)^2 \checkmark$$

$$x^2 + 10x + 24 = 0 \checkmark$$

$$x = -6 \text{ or } x = -4 \checkmark$$

$$n/a \checkmark$$

(f) Determine the equation of the circle with diameter BC in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3)$$

$$(x + 2)^2 + (y + 4)^2 = r^2 \checkmark$$

$$(-5 + 2)^2 + (0 + 4)^2 = r^2 \checkmark$$

$$(x + 2)^2 + (y + 4)^2 = 25 \checkmark$$

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QUESTION 2

The data in the table below represent the percentage scores for 12 Mathematics students in their Grade 12 Preliminary examination and the corresponding Final examination.

Prelim Exam	76	64	90	68	70	79	52	64	61	71	84	70
Final Exam	82	69	94	75	80	88	56	81	76	78	90	76

- (a) Determine the equation of the least squares regression line for the data set. Give your answers rounded to two decimal digits. (3)

$$a = 16,16 \quad \checkmark$$

[full marks: ans. Only]

$$b = 0,88 \checkmark$$

$$\hat{y} = 16,16 + 0,88x \checkmark$$

- (b) Calculate the correlation coefficient, correct to 3 decimal places, of the above data and comment on the correlation between the Preliminary and Final examination marks. (2)

$$r = 0,9177 \checkmark$$

The correlation between the two examinations is very strong and positive. $\checkmark$

- (c) Hence, predict the final percentage for a student obtaining 73% in the Preliminary examination, giving your answer to the nearest percentage. (2)

$$\hat{y} = 16,167 \dots + 0,884 \dots (73) \checkmark$$

[full marks: ans. Only]

$$= 81\% \checkmark$$

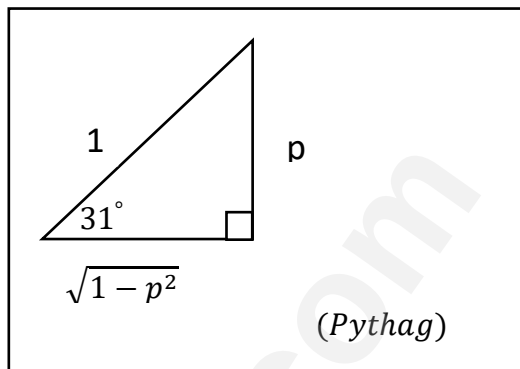
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QUESTION 3

(a) Given $\sin 31^\circ = p$, determine the following in terms of p , without a calculator.

$$(1) \sin 149^\circ = \sin 31^\circ \checkmark$$

$$= p \checkmark$$



(2)

$$(2) \cos(-59^\circ)$$

$$= \cos 59^\circ$$

$$= \sin 31^\circ \checkmark$$

$$= p \checkmark$$

(2)

$$(3) \cos 62^\circ$$

$$= 1 - 2\sin^2 31^\circ \checkmark$$

$$= 1 - 2p^2 \checkmark$$

(2)

(b) Simplify to a single trigonometric ratio:

$$\cos 65^\circ \cdot \sin 40^\circ + \cos 25^\circ \cdot \sin 50^\circ$$

(4)

$$= \sin 25^\circ \cdot \cos 50^\circ + \cos 25^\circ \cdot \sin 50^\circ \checkmark \checkmark$$

$$= \sin (25^\circ + 50^\circ) \checkmark$$

$$= \sin 75^\circ \checkmark$$

(c) Simplify without using a calculator:

$$\begin{aligned} & \frac{\sin 150^\circ \cdot \tan 225^\circ}{\sin(-30^\circ) \cdot \cos 420^\circ} \\ &= \frac{\sin 30^\circ \cdot \tan 45^\circ}{-\sin 30^\circ \cdot \cos 60^\circ} \\ &= \frac{-1}{\frac{1}{2}} \\ &= -2 \end{aligned} \quad (5)$$

(d) Consider: $\frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} = \tan x$

(1) Prove the identity. (5)

$$\begin{aligned} \text{LHS: } & \frac{\sin 2x + \sin x}{\cos 2x + \cos x + 1} \\ &= \frac{2 \sin x \cos x + \sin x}{2 \cos^2 x - 1 + \cos x + 1} \\ &= \frac{\sin x (2 \cos x + 1)}{\cos x (2 \cos x + 1)} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \\ &= \text{RHS} \end{aligned}$$

(2) Determine the values of x for which the identity is invalid for $x \in [0^\circ; 360^\circ]$. (3)

$$\cos x (2 \cos x + 1) = 0 \text{ or at asymptotes}$$

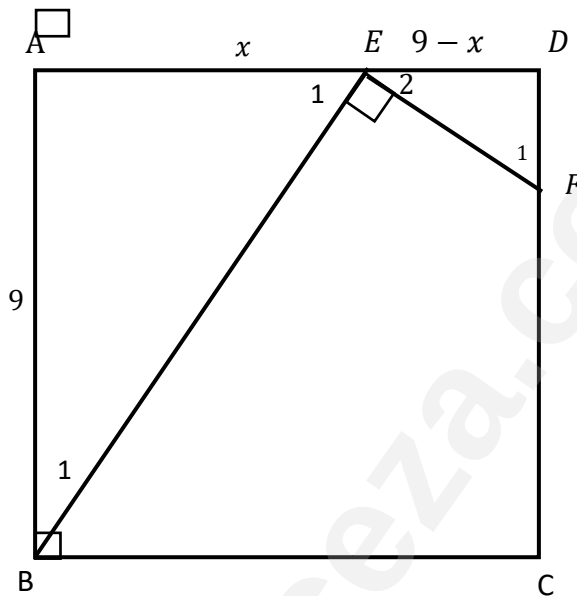
$$x = \{90^\circ; 120^\circ; 240^\circ; 270^\circ\}$$

(2) if only general solutions

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QUESTION 4

In the diagram below, ABCD is a square with sides 9 cm in length and $BE \perp EF$



(a) Show that $\hat{F}_1 = a$, if $\hat{E}_1 = a$ (2)

$$\hat{E}_1 + \hat{E}_2 = 90^\circ \text{ (angles on a str. line) } \checkmark$$

$$\hat{F}_1 + \hat{E}_2 = 180^\circ \text{ (angles in a triangle) }$$

$$\therefore \hat{E}_1 = \hat{F}_1 = a \checkmark$$

(b) Prove that $\triangle ABE$ is similar to $\triangle DEF$ (4)

In $\triangle ABE$ and $\triangle DEF$

$$\hat{A} = \hat{D} \text{ (given) } \checkmark$$

$$\hat{E}_1 = \hat{F}_1 \text{ (proven) } \checkmark$$

$$\hat{B} = \hat{E} \text{ (int. angles of } \Delta \text{) }$$

$$\therefore \triangle ABE \sim \triangle DEF \text{ (AAA) } \checkmark$$

(c) (1) Determine DF in terms of x .

(5)

$$\frac{AE}{DF} = \frac{AB}{DE} = \frac{BE}{EF} \text{ (III } \Delta s) \checkmark$$

$$\frac{x}{DF} = \frac{9}{9-x} \checkmark$$

$$x(9-x) = 9DF$$

$$DF = \frac{x(9-x)}{9} \checkmark$$

$$\text{Area of } \triangle EDF = \frac{1}{2}(ED)(DF)$$

$$= \frac{1}{2}(9-x) \cdot \frac{x(9-x)}{9} \checkmark$$

$$= \frac{x(9-x)^2}{18} \checkmark$$

$$= \frac{x(x-9)^2}{18}$$

or

$$LSF = \frac{9-x}{9} \checkmark$$

$$ASF = \left(\frac{9-x}{9}\right)^2 \checkmark$$

$$\text{Area of } \triangle EDF = \text{Area of } \triangle ABE \times \left(\frac{9-x}{9}\right)^2 \checkmark$$

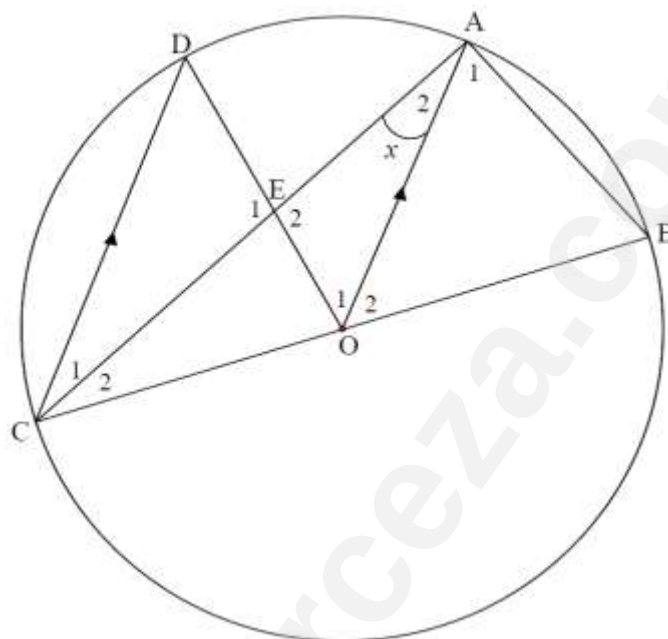
$$= \frac{9x}{2} \times \left(\frac{9-x}{9}\right)^2 \checkmark$$

$$= \frac{x(x-9)^2}{18} \checkmark$$

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QUESTION 5

In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle and CB is the diameter of the circle. Chord CA intersects radius OD at E. AB is drawn. $CD \parallel OA$ and $\hat{A}_2 = x$.



(a) Give reasons for the following

(1) $\hat{C}_1 = x$ (1)

Alt angles, $CD \parallel OA$ ✓

(2) $\hat{C}_2 = x$ (1)

$CO = OA$, radii ✓

(b) Determine the following angles in terms of x , giving reasons.

(1) $\hat{A}_1$ (2)

$\angle CAB = 90^\circ$ (angle in a semi circle) ✓

$\hat{A}_1 = 90^\circ - x$ ✓

(2) $\hat{O}_1 = 2x$ ✓ (angle at centre = 2x angle at circle) ✓ (2)

(3) $\hat{O}_2 = 2x$ ✓ (ext. angle of $\triangle ACO$) ✓ (2)

(c) For what values of x will ABOE be a cyclic quadrilateral? (3)

$$E\hat{O}B = 4x \quad [\hat{O}_1 = 2x \text{ and } \hat{O}_2 = 2x]$$

$$E\hat{O}B + E\hat{A}B = 180^\circ \text{ (Opp. angles of a cyclic quad.) } \checkmark$$

$$4x + 90^\circ = 180^\circ \checkmark$$

$$4x = 90^\circ$$

$$x = 22,5^\circ \quad \checkmark$$

(ca according to question (b))

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Total Section A: 72 marks

SECTION B

QUESTION 6

The table below summarises the amount of water consumed by individuals.

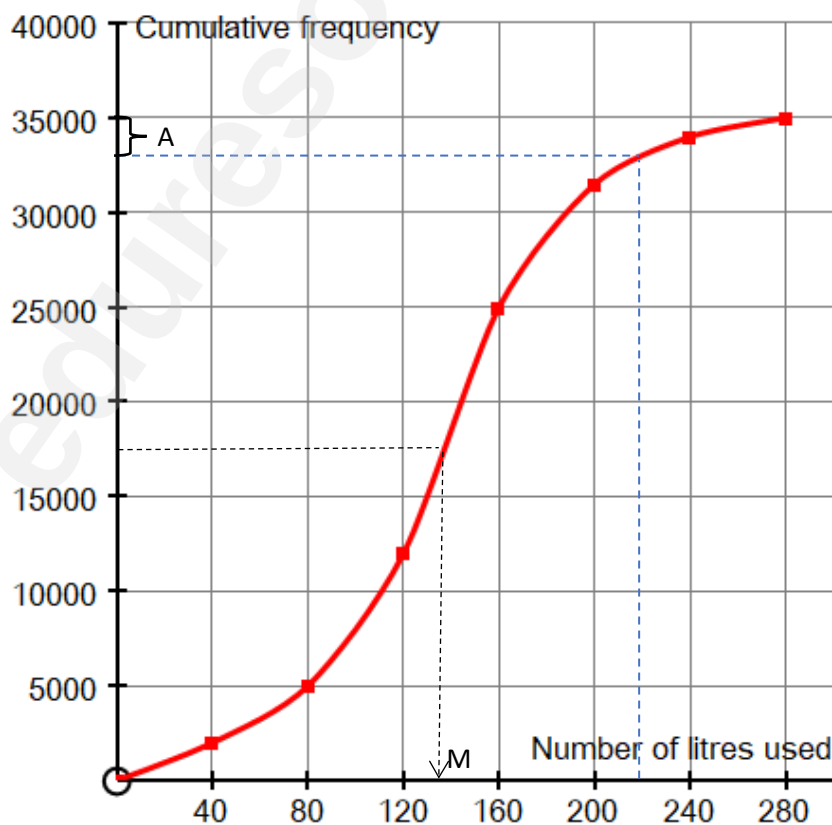
Number of litres used	Frequency	Midpoint	Cumulative Frequency
$0 < x \leq 40$	2 000	20	2 000
$40 < x \leq 80$	3 000	60	5 000
$80 < x \leq 120$	7 000	100	12 000
$120 < x \leq 160$	13 000	140	25 000
$160 < x \leq 200$	6 500	180	31 500
$200 < x \leq 240$	2 500	220	34 000
$240 < x \leq 280$	1 000	260	35 000

- (a) Determine the estimated mean number of litres used by an individual. (2)

$$\bar{X} = \frac{20 \times 2000 + 60 \times 3000 + 100 \times 7000 + 140 \times 13000 + 180 \times 6500 + 220 \times 2500 + 260 \times 1000}{35000} \checkmark$$

$$= 134,86 \text{ l} \checkmark \quad -1 \text{ if no unit}$$

- (b) Draw a cumulative frequency graph that represents the information in the table above. (3)



- (c) Determine the median number of litres used by an individual. Show with an “M” on your graph where you would read this value. (2)

$\approx 135 \text{ l}$

- (d) How many individuals use more than 220 litres of water? Show with an “A” on your graph where you would read this value. (2)

$\approx 35\,000 - 33\,000 = 2000$

- (e) At the time of the survey the maximum number of litres permitted per individual was 280 litres. If the restriction was reduced to 240 litres per individual, what would happen to:

- (1) the median? (Give a reason for your answer.) (2)

It will remain unchanged because the reduction only affects the top half of the data set.

- (2) the standard deviation? (Give a reason for your answer.) (2)

Decrease because the spread of the data will decrease

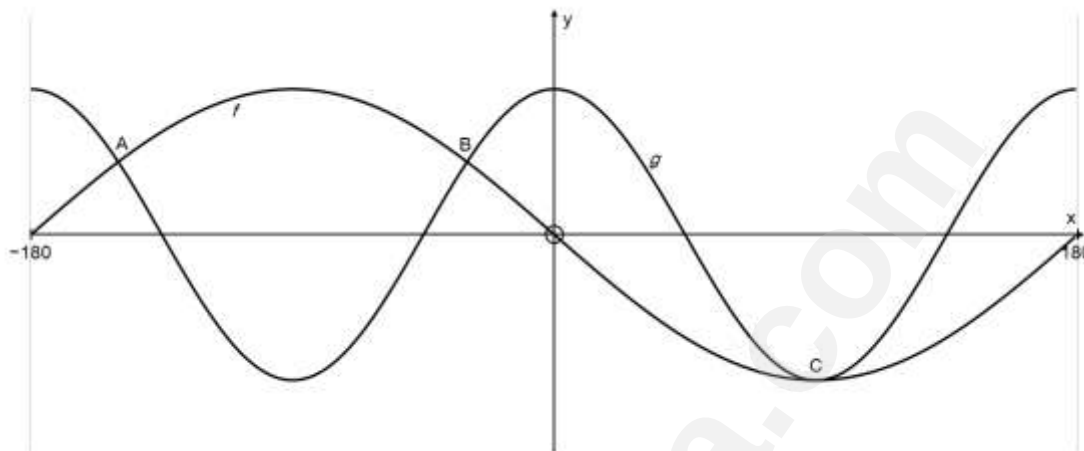
- (3) the skewness of the data? (Give a reason for your answer.) (2)

It will become skewed to the left because the range of the top half of the data will decrease.

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QUESTION 7

In the diagram below, the graphs of $f(x) = \cos 2x$ and $g(x) = -\sin x$ are drawn in the interval $x \in [-180^\circ; 180^\circ]$. A, B and C are the points of intersection of f and g .



(a) Without using a calculator, determine the x -coordinates of points A, B and C. (7)

$$\cos 2x = -\sin x \checkmark$$

or

$$\cos 2x = -\sin x \checkmark$$

$$\cos 2x = \sin(-x)$$

$$1 - 2\sin^2 x + \sin x = 0$$

$$\cos 2x = \cos(90^\circ + x) \checkmark$$

$$2\sin^2 x - \sin x - 1 = 0 \checkmark$$

$$2x = 90^\circ + x + k \cdot 360^\circ \text{ or } 2x = -90^\circ - x + k \cdot 360^\circ$$

$$\sin x = -\frac{1}{2} \text{ or } \sin x = 1$$

$$x = 90^\circ + k \cdot 360^\circ \checkmark \text{ or } 3x = -90^\circ + k \cdot 360^\circ$$

$$x = -30^\circ + k \cdot 360^\circ$$

$$x = -30^\circ + k \cdot 120^\circ \checkmark$$

$$x = 210^\circ + k \cdot 360^\circ \checkmark$$

$$A: x = -150^\circ \checkmark; B: x = -30^\circ \checkmark; C: x = 90^\circ \checkmark$$

$$x = 90^\circ + k \cdot 360^\circ \checkmark$$

(b) Determine the values of x for which $f'(x) \cdot g'(x) > 0$. (2)

$$x \in (0^\circ; 90^\circ) \checkmark \cup (90^\circ; 180^\circ) \checkmark$$

(c) Determine the values of k for which $\cos 2x + 3 = k$ will have no solutions. (2)

$$k - 3 > 1 \text{ or } k - 3 < -1$$

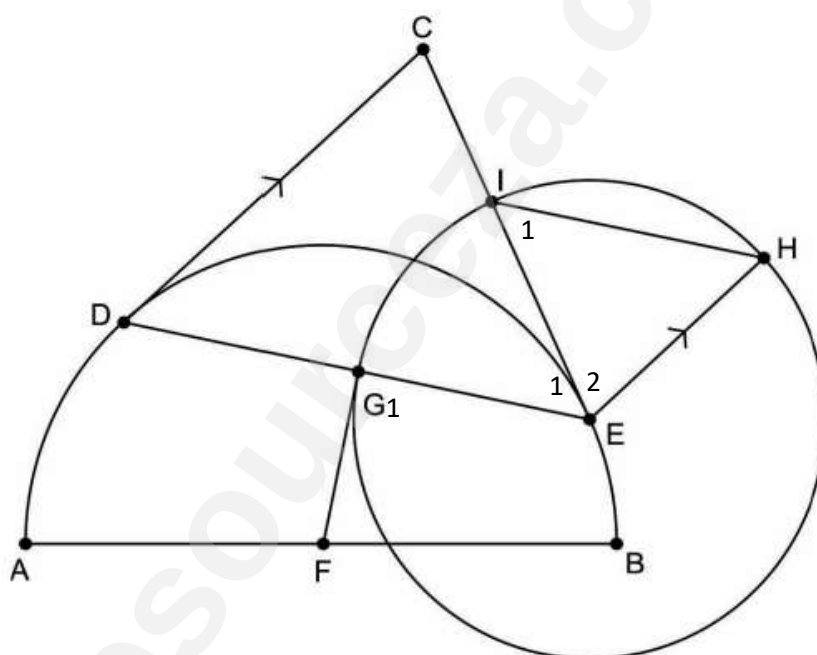
$$k > 4 \checkmark \text{ or } k < 2 \checkmark$$

[11]

QUESTION 8

In the diagram below:

- I, H and G are points on the circle with centre E
- AB is a diameter of the semi-circle with centre F, and D and E lie on the semi-circle
- CD and CE are tangents to the semi-circle at D and E respectively
- $CD \parallel HE$
- FG is a tangent to the circle with centre E



(a) Prove that $\triangle DCE \parallel \triangle HEI$

(4)

In $\triangle DCE$ and $\triangle HEI$

$$\hat{C} = \hat{E}_2 \text{ Alt } \angle \text{ s } CD \parallel HE \checkmark$$

$$\hat{D} = \hat{E}_1 \text{ Tan from same pt isos } \triangle \checkmark$$

$$\hat{I}_1 = \hat{H} \text{ Radii isos } \triangle \checkmark$$

$$\therefore \hat{D} = \hat{E}_1 = \hat{I}_1 = \hat{H} \text{ Int } \angle \text{ s } \triangle$$

$$\triangle DCE \parallel \triangle HEI \text{ AAA } \checkmark$$

(b) Prove $DG = GE$ (2)

$$\hat{G}_1 = 90^\circ \text{ Rad } \perp \text{ Tan } \checkmark$$

$$DG = GE \text{ Line from centre } \perp \text{ chord } \checkmark$$

(c) Show that $2HE^2 = DC \times HI$ (5)

$$\frac{DC}{HE} = \frac{CE}{EI} = \frac{DE}{HI} \quad /// \checkmark$$

$$DC \times HI = HE \times DE \checkmark$$

$$DE = 2HE \text{ Proven } \checkmark$$

$$\therefore DC \times HI = HE \times 2HE \checkmark$$

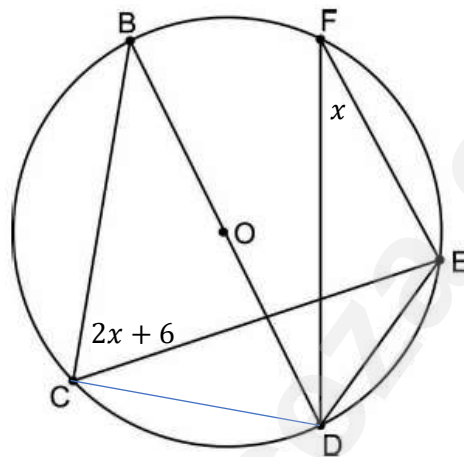
$$2HE^2 = DC \times HI \checkmark$$

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QUESTION 9

In the diagram below:

- O is the centre of the circle passing through B, C, D, E and F
- BD is a diameter
- $\angle FDE = x$ and $\angle BCE = 2x + 6^\circ$



Determine the value of x .

Draw CD ✓

$$\angle BCD = 90^\circ \quad \angle s \text{ in semi-circle} \quad \checkmark$$

$$\angle ECD = x \quad \angle s \text{ in same seg} \quad \checkmark$$

$$2x + 6 + x = 90^\circ \quad \checkmark$$

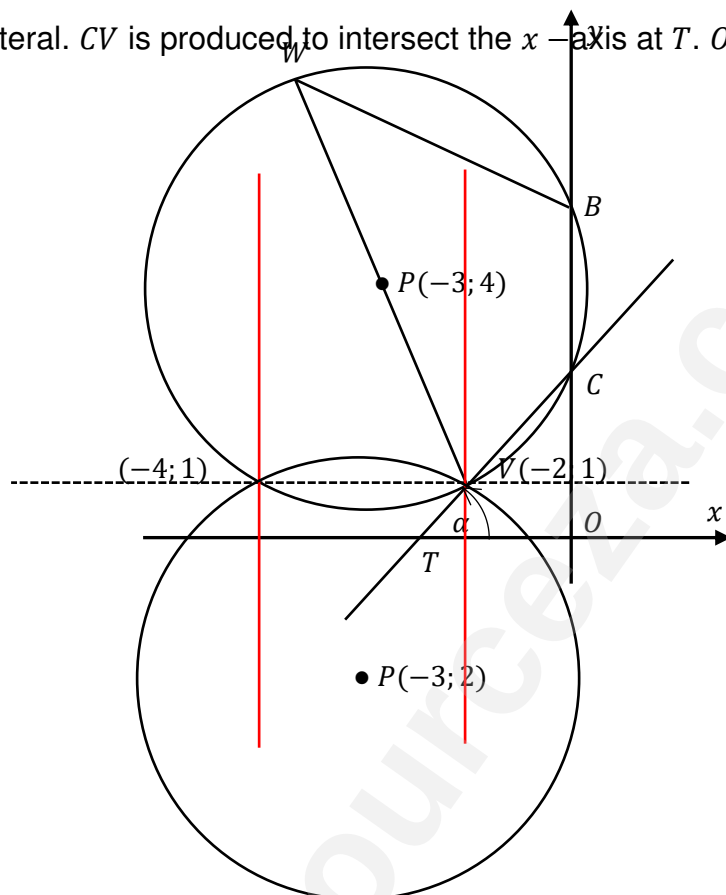
$$3x = 84^\circ \quad \checkmark$$

$$x = 28^\circ \quad \checkmark$$

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QUESTION 10

In the diagram, $P(-3; 4)$ is the centre of the circle. $V(k; 1)$ and W are the endpoints of a diameter. The circle intersects the y -axis at B and C . $BCVW$ is a cyclic quadrilateral. CV is produced to intersect the x -axis at T . $\widehat{OTC} = \alpha$.



- (a) The radius of the circle with centre P is $\sqrt{10}$. Determine the value of k if point V is to the right of point P . (4)

$$(-3 - k)^2 + (4 - 1)^2 = 10 \checkmark$$

$$9 + 6k + k^2 + 9 = 10 \checkmark$$

$$k^2 + 6k + 8 = 0 \checkmark$$

$$k = -2 \checkmark$$

- (b) Determine the length of BC . (5)

$$(x + 3)^2 + (y - 4)^2 = 10 \checkmark$$

$$\text{Let } x = 0$$

$$(0 + 3)^2 + (y - 4)^2 = 10 \checkmark$$

$$y^2 - 8y + 15 = 0 \checkmark$$

$$y = 5 \text{ or } y = 3 \checkmark$$

$$CB = 2 \text{ units } \checkmark$$

- (c) If $k = -2$, calculate the size of: (Round your answers off to one decimal place.)

- (1) α (2)

$$\tan \alpha = \frac{3 - 1}{0 - (-2)} = 1 \checkmark$$

$$\alpha = 45^\circ \checkmark$$

- (2) $\widehat{VWB}$ (2)

$$\angle TCO = 45^\circ \text{ Int } \angle s \Delta \checkmark$$

$$\widehat{VWB} = 45^\circ \text{ Ext } \angle c.q. \checkmark$$

- (d) A new circle is obtained by reflecting the given circle about the line $y = 1$.

Determine:

- (1) The coordinates of Q , the centre of the new circle. (2)

$$Q(-3; -2) \checkmark \checkmark$$

- (2) The equation of the new circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (2)

$$(x + 3)^2 + (y + 2)^2 = 10 \checkmark \checkmark$$

- (3) The equations of the lines parallel to the y -axis and passing through the points of intersection of the two circles. (2)

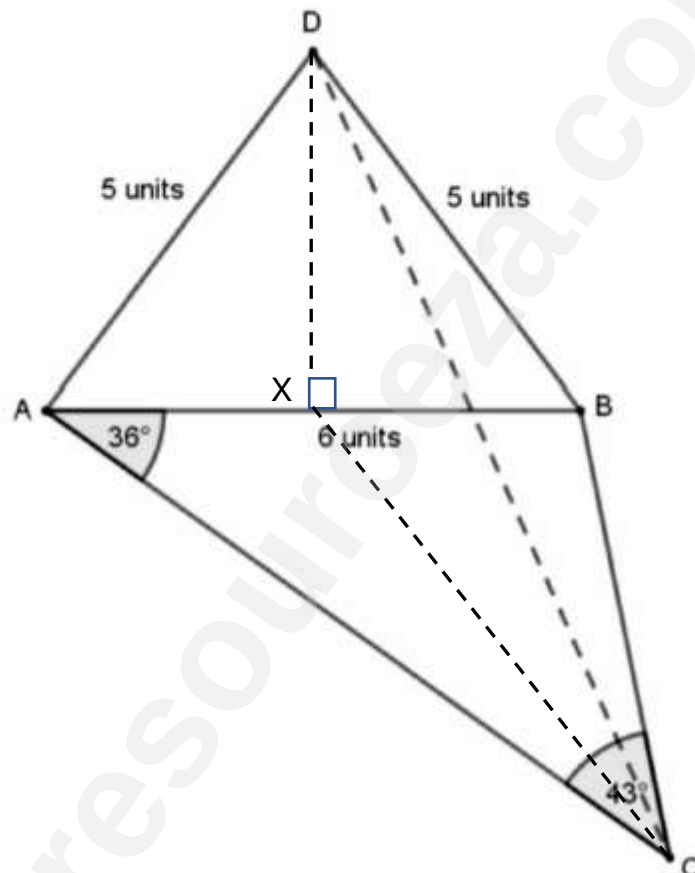
$$x = -2 \checkmark; x = -4 \checkmark$$

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QUESTION 11

In the diagram below, $\triangle ABC$ lies on the horizontal plane and $\triangle ABD$ lies on the vertical plane.

- $DB = DA = 5$ units
- $AB = 6$ units
- $\angle CAB = 36^\circ$ and $\angle BCA = 43^\circ$



- (a) Determine the length of DX. (2)

$$DX^2 = 5^2 - 3^2 \text{ Pythag } \checkmark$$

$$DX = 4 \text{ units } \checkmark$$

- (b) Determine the length of AC, correct to two decimal digits. (3)

$$\text{In } \triangle ABC: \angle BAC = 101^\circ \text{ Int } \angle \text{ s } \triangle \checkmark$$

$$\frac{\sin 43^\circ}{6} = \frac{\sin 101^\circ}{AC} \checkmark$$

$$AC = 8,64 \checkmark$$

- (c) Calculate the length of straight-line DC, correct to two decimal digits. (5)

Join CX ✓

$$\text{In } \triangle AXC: XC^2 = 3^2 + 8,64^2 - 2(3)(8,64) \cos 36^\circ \checkmark$$

$$XC = 6,458 \dots (A) \checkmark$$

$$\text{In } \triangle DXC: CD^2 = 4^2 + (A)^2 \text{ Pythag } \checkmark$$

$$CD = 7,6 \text{ units } \checkmark$$

- (d) If $\triangle ADB$ is folded back along line AB away from C so that D is on the same horizontal plane as A, B and C, then calculate the new straight-line distance from D to C. Give your answer correct to two decimal places. (6)

$$\cos D\hat{B}A = \frac{5^2 + 6^2 - 5^2}{2(5)(6)} \checkmark$$

$$D\hat{B}A = 53,13 \dots (B) \checkmark$$

$$A\hat{B}C = 101^\circ$$

$$D\hat{B}C = 154,13 \dots (C)$$

$$\frac{\sin 43^\circ}{6} = \frac{\sin 36^\circ}{BC} \checkmark$$

$$BC = 5,17 \dots (D) \checkmark$$

$$DC^2 = 5^2 + (D)^2 - 2(5)(D) \cos(C) \checkmark$$

$$DC = 9,91 \text{ units } \checkmark$$

or

$$\frac{\sin A\hat{X}C}{8,64} = \frac{\sin 36^\circ}{6,458} \checkmark \checkmark$$

$$A\hat{X}C = 180^\circ - 51,85^\circ \checkmark$$

$$B\hat{X}C = 51,85^\circ \checkmark$$

$$DC^2 = 4^2 + (6,458)^2 - 2(4)(6,458) \cos(90^\circ + 51,85^\circ) \checkmark$$

$$DC = 9,91 \text{ units } \checkmark$$

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