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MATHEMATICS PRELIMINARY EXAMINATION

PAPER I 2021

EXAMINATION NUMBER:

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<u>Question</u>	<u>Topic</u>	<u>Mark obtained</u>
1 [19]	Equations	
2 [10]	Sequences	
3 [8]	Calculus	
4 [8]	Finance	
5 [25]	Functions	
6 [5]	Probability	
Total Section A [75]		
7 [17]	Finance	
8 [7]	Counting methods	
9 [16]	Functions and inverses	
10 [10]	Sequences	
11 [18]	Calculus	
12 [7]	Maximising	
Total Section B [75]		

TOTAL: $\frac{\quad}{150} = \quad \quad \quad \%$

SECTION A

QUESTION 1

(a) Solve for x in the equations below. **Show all working detail.**

(1) $(x - 1)(2x + 5) = 8$ (round off to two decimal places) (3)

(2) $x - 3\sqrt{x + 7} = -9$ (4)

(3) $5^{x+1} - \sqrt{5^{x-2}} = 0$ (4)

(b) Given $f(x) = 2x^2 + 9x + 4$

(1) Determine the value(s) of x for which $\sqrt{f(x)}$ is defined. (4)

(2) For which values of k will $f(x) = 3x + k$ have non-real roots? (4)

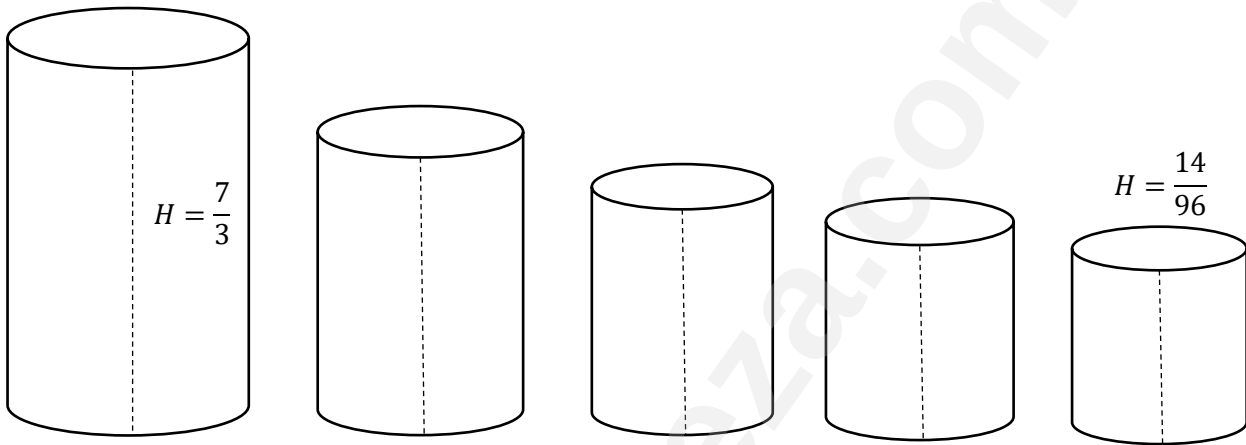
[19]

QUESTION 2

(a) Woolworths sells cylindrical vases whose heights follow a geometric sequence.

The height of the longest vase is $\frac{7}{3}$ m and the height of the fifth longest vase is

$\frac{14}{96}$ m.



(1) Determine a general expression for the height of the n th largest vase. (3)

(2) Hence, calculate the height of the 9th largest vase. (2)

(b) Abulele thinks of a sequence that has the following properties:

$$T_2 - T_1 = 4$$

$$T_3 - T_2 = 7$$

$$T_4 - T_3 = 10$$

(1) Write down the value of $T_5 - T_4$. (1)

(2) Determine an expression for $T_{n+1} - T_n$ where $n \geq 1$. (2)

(3) Hence, determine the value of $T_{112} - T_{111}$. (2)

[10]

QUESTION 3

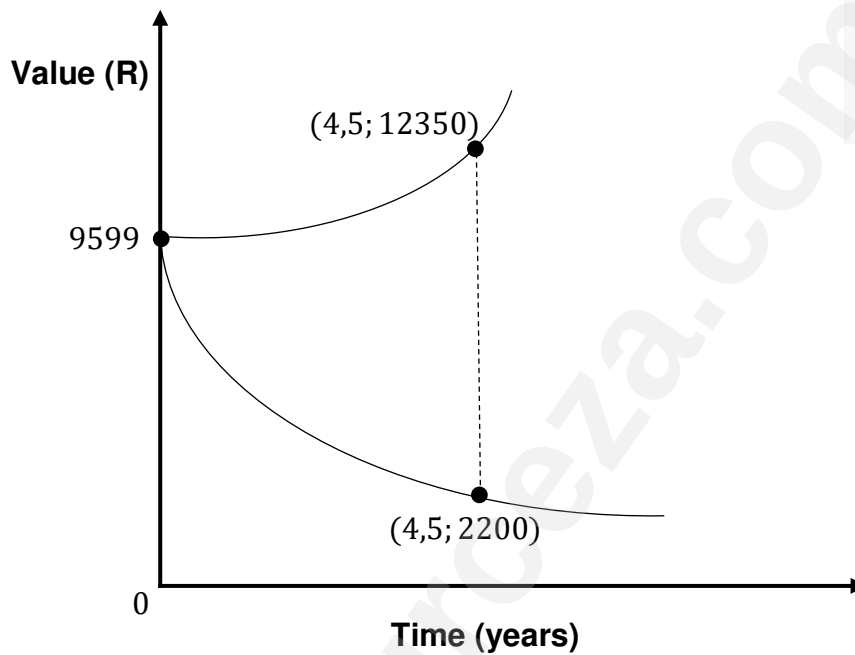
(a) If $f(x) = (5 - x)^2$, determine $f'(x)$ using first principles. (4)

(b) Determine: $D_x \left[\frac{-x^3+7}{x^2} + 6\sqrt[3]{x} \right]$ (4)

[8]

QUESTION 4

Mieke's mother bought her an iPad on the 1st of January 2021 for R9599. The graph below shows the value of a similar iPad in the future and the value of her iPad as it depreciates over time.

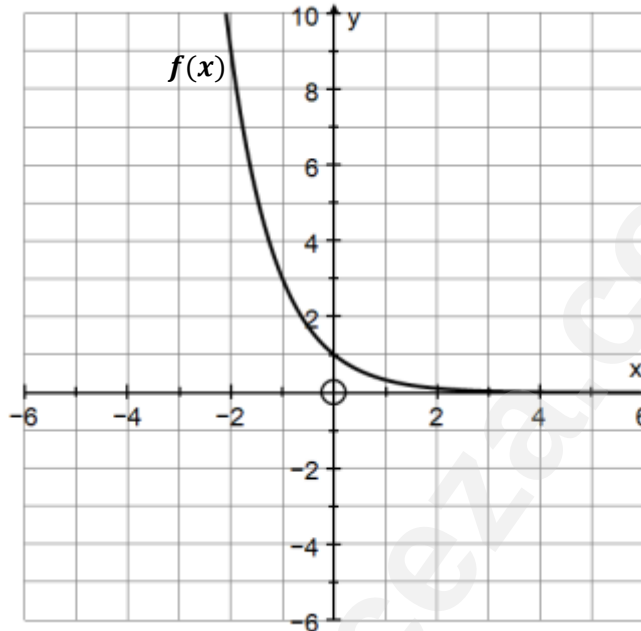


- (a) If the iPad depreciates using the reducing balance method, calculate the rate of depreciation. (Round your answer off to three decimal places.) (3)

- (b) Mieke decides to set up a sinking fund on the 1st of January 2021 to buy a new iPad on the 30th of June 2025. She will use the proceeds from selling her current iPad at the depreciated value and the money in her savings account.
- (1) Calculate how much Mieke will need to have in her sinking fund in order to have enough money to purchase the new iPad on the 30th of June 2025. (1)
- (2) How much does she need to save **per quarter** in her sinking fund if the bank offers her 1,75% per quarter? She starts saving immediately and makes the last payment on the 30th of June 2025. (4)

QUESTION 5

(a) The graph of $f(x) = \left(\frac{1}{3}\right)^x$ is sketched below.



(1) Write down the equation of $f^{-1}(x)$. (2)

(2) Sketch the graph of $f^{-1}(x)$ on the **same set of axes**. Clearly indicate any intercepts with the axes. (3)

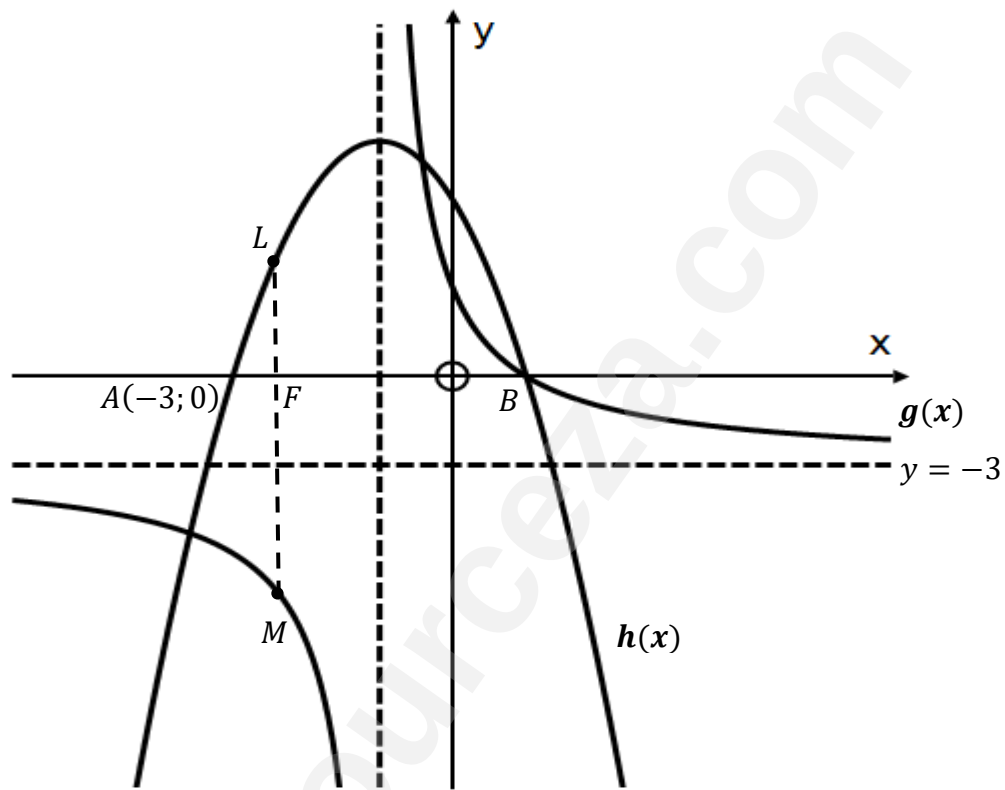
(3) Determine, algebraically, the value of x for which $f^{-1}(x) = -4$. (3)

SHOW ALL STEPS.

(4) Write down the domain of the graph of $f^{-1}(x + 5)$. (1)

(5) $f(x)$ is reflected about the y axis and shifted 6 units down to form $k(x)$.
Write down the equation of $k(x)$. (2)

- (b) The graph of $g(x) = \frac{k}{x-p} + q$ and $h(x) = -2(x+1)^2 + 8$ are sketched. The graph of $h(x)$ has x intercepts at points A and B . The graphs of $g(x)$ and $h(x)$ intersect at point B .



Determine:

- (1) the co-ordinates of point B . (1)

- (2) the equation of $g(x)$. (3)

(3) the axis of symmetry of $g(x)$ with a negative gradient. (2)

(4) the length of LM if $AF = 1$ unit. (4)

(5) the equation of the tangent to the graph of $h(x)$ at the point $(-5; -24)$. (4)

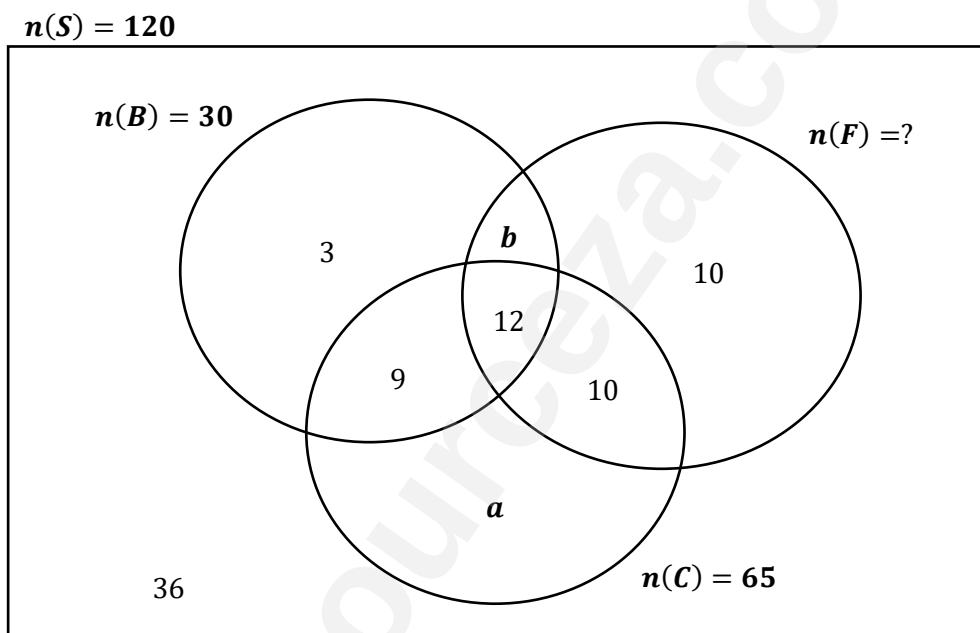
[25]

QUESTION 6

The Venn diagram below represents the group of Form V learners at St Mary's School.



- B represents the number of girls who play basketball.
- F represents the number of girls in Furse.
- C represents the number of girls who sing in the choir.



- (a) Calculate the value of b . (1)
- (b) Is playing basketball independent of being in Furse in the group of Form V girls?
Show all necessary workings. (4)

[5]

TOTAL SECTION A: 75 MARKS

SECTION B

QUESTION 7

- (a) Nnema wants to invest money at a bank that offers interest on investments at a rate of 7,5% per annum compounded monthly.
- (1) Calculate the annual effective interest rate equivalent to this nominal interest rate. Round off your answer to 2 decimal places. (2)
- (2) Nnema invests R8 557,00 at this bank at the same interest rate. After how many years will her investment be worth R12 450,84? (3)
- (b) Cath took out a home loan for R500 000 at an investment rate of 12% p.a. compounded monthly. She plans to repay this loan over 20 years and her first payment is made one month after the loan is granted.
- (1) Calculate Cath's monthly instalment. (4)

- (2) Ashley took out a loan for the same amount and at the same interest rate as Cath. Ashley decided to pay R6000 at the end of every month and she makes her first payment at the end of three months. Calculate her monthly payment. (4)

- (3) How much interest does Ashley pay in the first five years of the loan? (4)

[17]

QUESTION 8

Three items from four different departments of a major chain store will be featured in a one-page newspaper advert. The page layout for the advert is shown in the diagram below. One item is placed in each block.

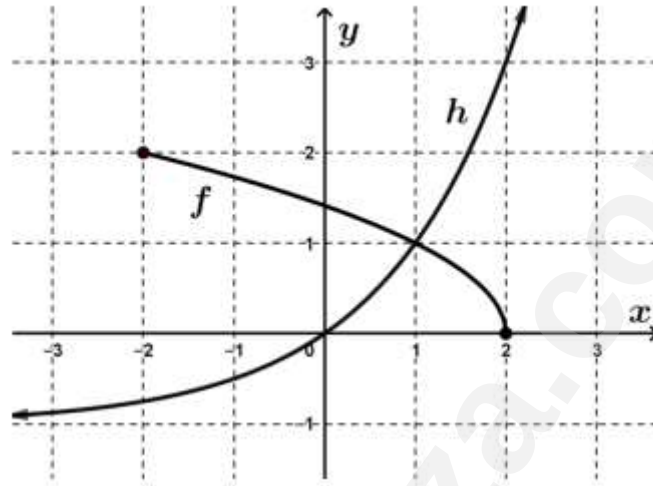
	Item 1	Item 2	Item 3
Department 1	A	B	C
Department 2	D	E	F
Department 3	G	H	I
Department 4	J	K	L

- (a) In how many different ways can all these items be arranged in the advert? (2)
- (b) In how many different ways can these items be arranged if specific items are to be placed in blocks A, F and J. (2)
- (c) In how many different ways can these items be arranged in the advert if the same items from the same department are grouped together in the same row? (3)

[7]

QUESTION 9

- (a) The graphs of $f(x) = \sqrt{-x+2}$ and $h(x) = 2^x - 1$ are shown below. The graph of $f(x)$ has a restricted domain.



- (1) For what value/s of x is:

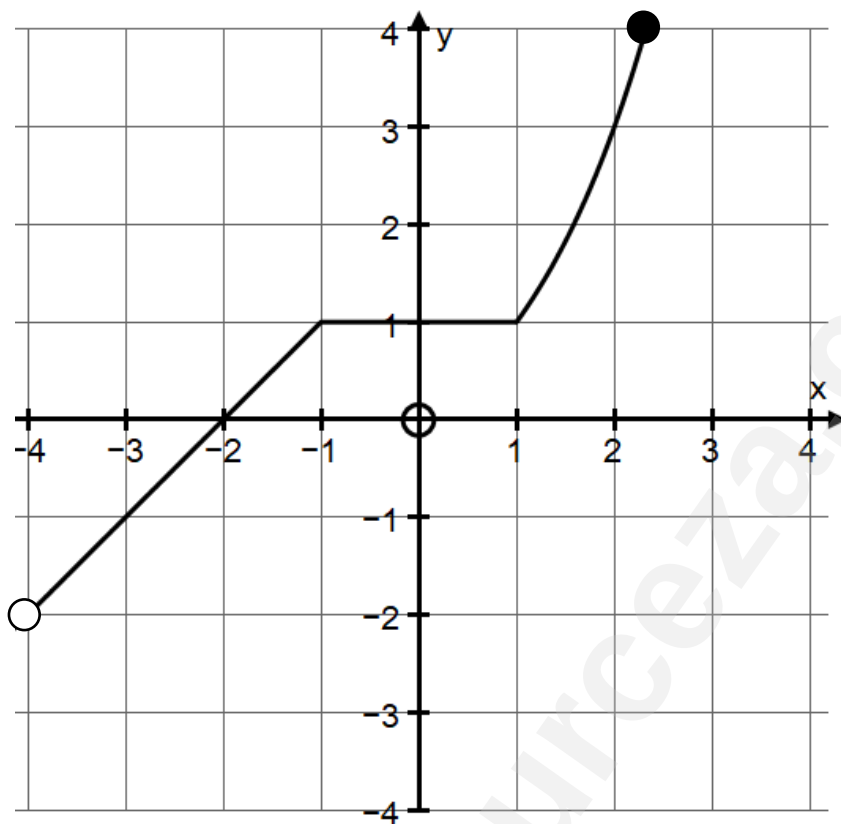
(i) $f(x) - h(x) = 0$ (1)

(ii) $h(x) \cdot f'(x) \leq 0$ (2)

- (2) Determine the equation of $f^{-1}(x)$. (3)

- (3) Give the domain of $f^{-1}(x)$. (3)

- (b) The graph of the function of $j(x) = \begin{cases} x + 2; & x \leq -1 \\ 1; & -1 < x < 1 \\ 2^x - 1; & x \geq 1 \end{cases}$ is shown below:



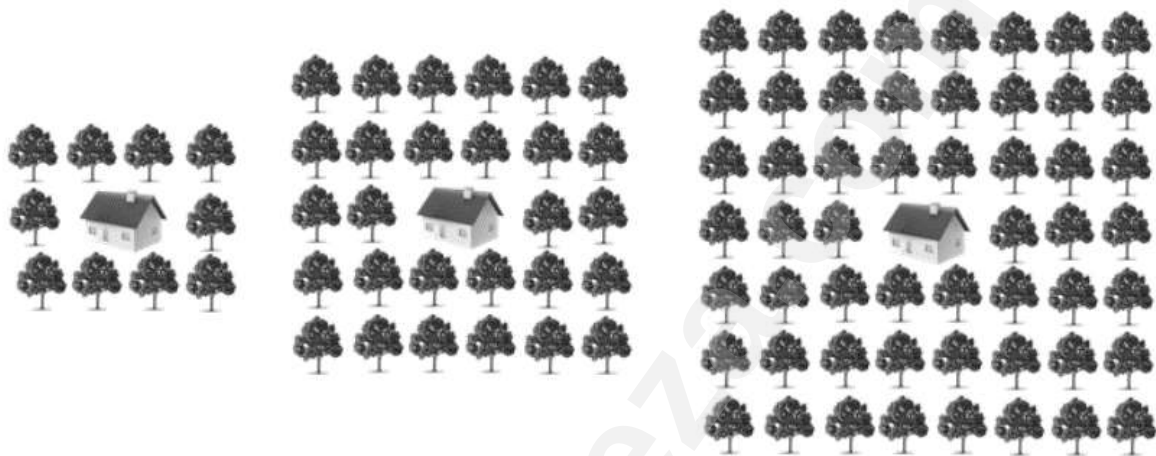
- (1) Find the domain of $j^{-1}(x)$. (3)

- (2) On the same set of axes, sketch the graph of $j^{-1}(x)$. (4)

[16]

QUESTION 10

A farmer wishes to develop a tree plantation. He starts by building a paper mill and planting trees in a rectangular fashion around it, so that the paper mill will be the centre of the plantation. The paper mill takes the space of two trees. Each month he adds another outer shell of trees as demonstrated in the figures



Month 1

Month 2

Month 3

(a) Determine how many trees there will be after

(1) 4 months (1)

(2) 5 months (1)

(3) n months. (4)

- (b) How long will it take him to have a forest of 550 trees? (4)

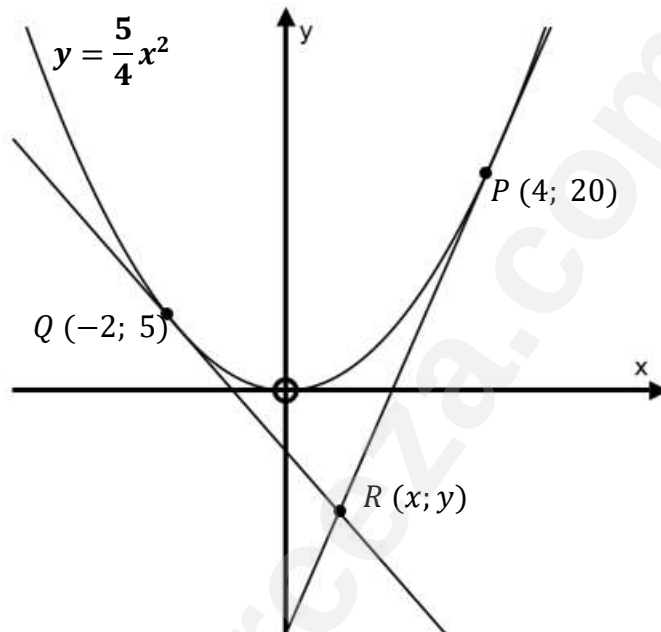
[10]

QUESTION 11

- (a) Given $p(x) = x^3 + 2x$. Show, by using relevant calculations, why it is **NOT** possible for a tangent drawn to the graph of p to have a negative gradient. (3)

- (b) In the diagram below, the parabola with the equation $y = \frac{5}{4}x^2$ is drawn and two points with coordinates $P(4; 20)$ and $Q(-2; 5)$ are shown.

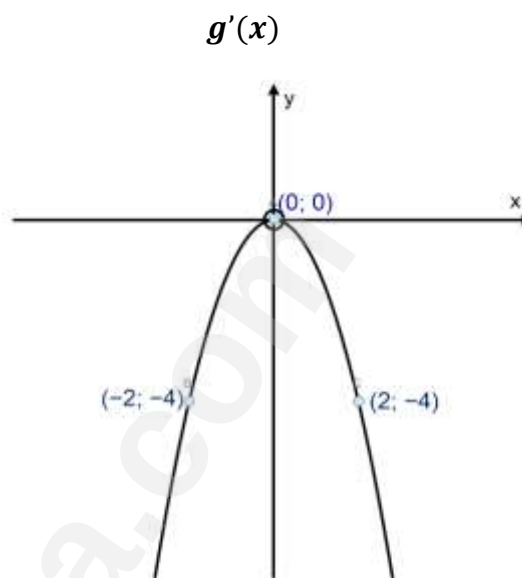
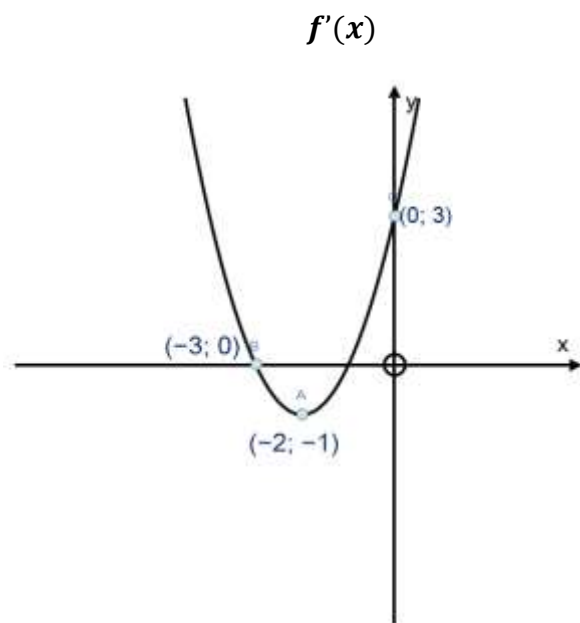
$R(x; y)$ is the point of intersection of the tangents to the parabola at P and Q .



Determine the coordinates of R .

(8)

(c) Study the graphs of $f'(x)$ and $g'(x)$ below, and answer the questions that follow:

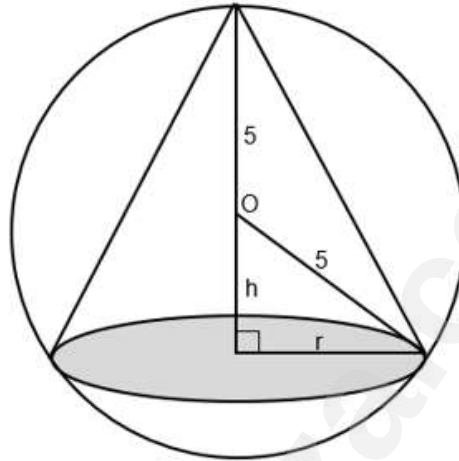


- (1) Does $f(x)$ or $g(x)$ have a point of inflection with a gradient of -1 ? (1)
- (2) What is the x -value at the point of inflection in (1) above? (1)
- (3) Which of the graphs, $f(x)$ or $g(x)$ has a point of inflection that is also a stationary point? (1)
- (4) What is the x -value at the point in (3) above? (1)
- (5) Which of the graphs, $f(x)$ or $g(x)$, is decreasing for all values of x ? Give a reason for your answer. (2)
- (6) What is the gradient of the tangent to $f(x)$ at $x = 0$? (1)

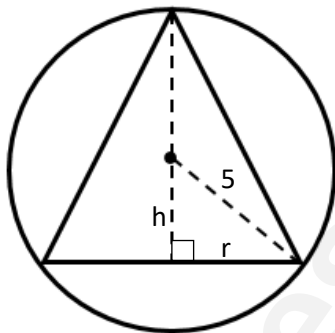
[18]

QUESTION 12

The vertex of a right circular cone and the circular edge of its base lie on the surface of a sphere. The sphere has a radius of 5 cm.



A cross section is show below:



Find the value of h that would maximise the volume of the cone. Round off your answer to 2 decimal places.

[Hint: write r in terms of h]

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 H$$

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[7]

TOTAL SECTION B: 75 MARKS

GRAND TOTAL: 150 MARKS