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SECTION A

QUESTION 1

(a) Showing all working detail. Solve for x :

$$(1) \quad (x - 3)(x + 5) = 9 \quad (3)$$

$$x^2 + 2x - 15 - 9\checkmark = 0$$

$$(x + 6)(x - 4) = 0$$

$$x = -6\checkmark \text{ or } x = 4\checkmark$$

$$(2) \quad 2x - 1 = \sqrt{2x + 1} \quad (4)$$

$$4x^2 - 4x + 1\checkmark = 2x + 1$$

$$4x^2 - 6x = 0\checkmark$$

$$2x(2x - 3) = 0$$

$$2x = 0 \text{ or } x = \frac{3}{2}\checkmark$$

$$x \neq 0\checkmark$$

$$(3) \quad 9 \cdot 3^{2x-1} = 81 \cdot 3^x \quad (4)$$

$$3^{2x-1} = 3^{x+2}\checkmark\checkmark$$

$$2x - 1 = x + 2\checkmark$$

$$x = 3\checkmark$$

(b) Solve for x and y

(6)

$$-2y + x = -1$$

$$x^2 - 7 - y^2 = -y$$

$$-2y + x = -1$$

$$x^2 - 7 - y^2 = -y$$

$$x = 2y - 1 \checkmark$$

$$(2y - 1)^2 - 7 - y^2 = -y$$

$$3y^2 - 3y - 6 = 0 \checkmark$$

$$y = 2 \checkmark \text{ or } y = -1 \checkmark$$

$$x = 3 \checkmark \text{ or } x = -3 \checkmark$$

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QUESTION 2

- (a) Determine the derivative from first principles of $f(x) = 4x^2 - 2x$. (6)

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ f'(x) &= \lim_{h \rightarrow 0} \frac{4(x+h)^2 - 2(x+h) - (4x^2 - 2x)}{h} \checkmark \checkmark \\ f'(x) &= \lim_{h \rightarrow 0} \frac{4(x^2 + 2xh + h^2) - 2x - 2h - 4x^2 + 2x}{h} \\ f'(x) &= \lim_{h \rightarrow 0} \frac{4x^2 + 8xh + 4h^2 - 2h - 4x^2}{h} \\ f'(x) &= \lim_{h \rightarrow 0} \frac{8xh + 4h^2 - 2h}{h} \checkmark \\ f'(x) &= \lim_{h \rightarrow 0} \frac{h(8x + 4h - 2)}{h} \checkmark \\ f'(x) &= 8x - 2 \checkmark \checkmark \end{aligned}$$

- (b) Determine $D_x \left[\frac{2x^2+3}{x} - 3x^{\frac{2}{3}} \right], x \neq 0$. (5)

$$\begin{aligned} D_x \left[\frac{2x^2}{x} + \frac{3}{x} - 3x^{\frac{2}{3}} \right] \checkmark \\ D_x \left[2x + 3x^{-1} - 3x^{\frac{2}{3}} \right] \checkmark \\ = 2 - 3x^{-2} - 2x^{-\left(\frac{1}{3}\right)} \checkmark \checkmark \checkmark \end{aligned}$$

- (c) A pool is being filled at a variable rate due to water pressure issues, and a leak. The height, h cm, of the water at a given time is defined by the function

$$h(t) = 8t + 24 - t^3 + t^2$$

- (1) Determine the average rate of change of the height of the water in between 5 minutes and 8 minutes. (3)

$$\begin{aligned} \bar{m} &= \frac{h(8) - h(5)}{8 - 5} \checkmark \\ \bar{m} &= \frac{-360 - (-36)}{8 - 5} \\ \bar{m} &= -108 \text{ cm per minute} \checkmark \checkmark \end{aligned}$$

- (2) Determine the time(s) for t , where the water level is neither increasing nor decreasing. (4)

$$h(t) = 8t + 24 - t^3 + t^2$$

$$h'(t) = -3t^2 + 2t + 8$$

$$-3t^2 + 2t + 8 = 0 \checkmark \checkmark$$

$$(t - 2)(3t + 4) = 0$$

$$t = -\frac{4}{3} \cup t = 2 \checkmark \checkmark$$

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QUESTION 3

Lerato establishes an earth moving company. The first earth moving machine bought cost R850 000 and it depreciates at 25% per annum on a reducing balance method. The price of a new earth moving machine appreciates at a rate of inflation of 12% per annum. Lerato plans to replace the earth moving machine in 3 years' time, using the current earth moving machine's book value as a trade-in and paying the remaining amount in cash. To save up for the cash payment he sets up a sinking fund making equal monthly instalments into an account earning interest of 8,5% per annum compounded monthly. The first payment is made at the end of the first month of the 3 year period and the last payment is at the end of the 3 years.

- (a) Calculate the trade in value of the earth moving machine after 3 years. (3)

$$F = 850000(1 - 0,25)^3 \checkmark \checkmark$$

$$F = R358593,75 \checkmark$$

- (b) Calculate the purchase price of the new earth moving machine in 3 years' time. (3)

$$F = 850000(1 + 0,12)^3 \checkmark \checkmark$$

$$F = 1194188,80 \checkmark$$

- (c) Calculate the monthly instalment Lerato pays into the sinking fund. (4)

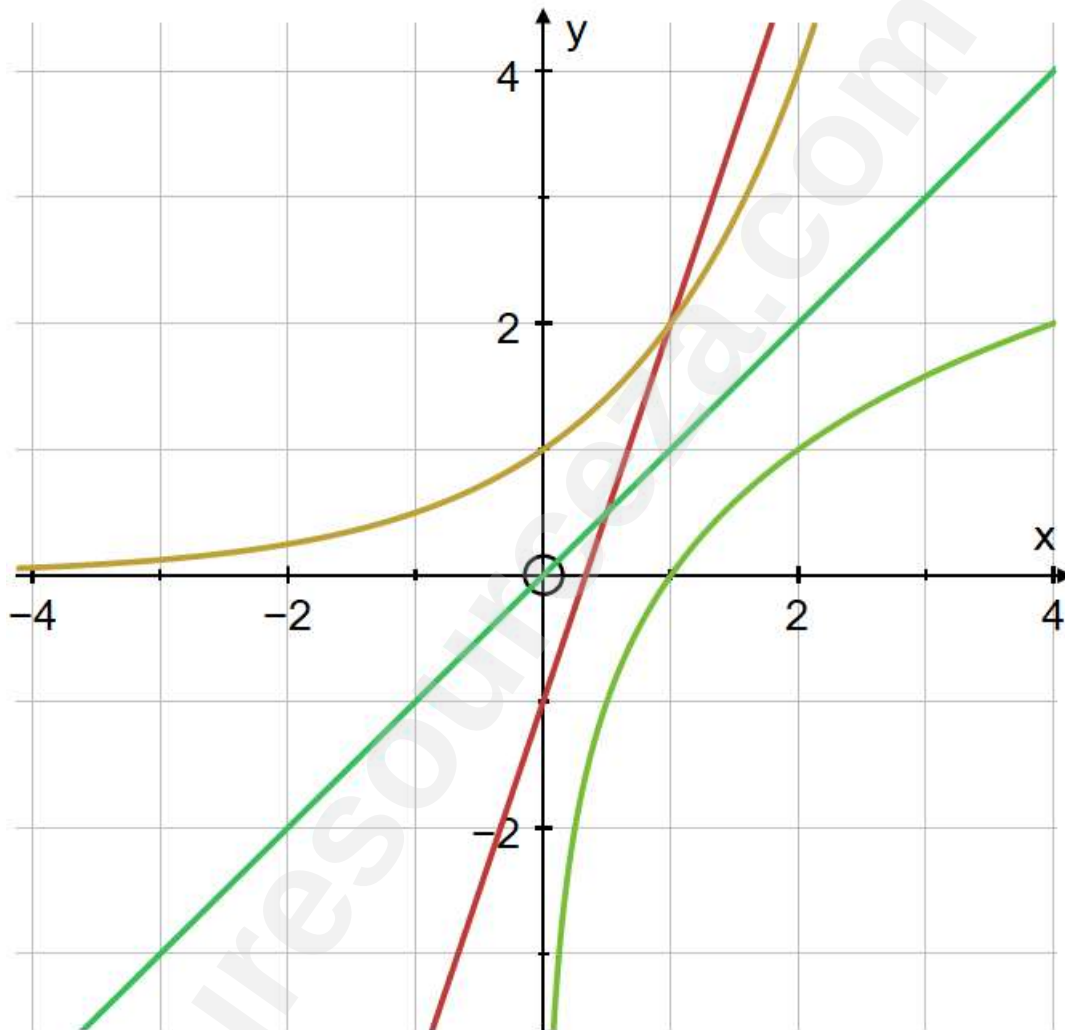
$$835595,05 \checkmark = \frac{x \left[\left(1 + \frac{0,085}{12} \right)^{3 \times 12} - 1 \right]}{\frac{0,085}{12}}$$

$$x = R20458,88 \checkmark$$

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QUESTION 4

Refer to the figure showing the graphs of $f(x) = 3x - 1$ and $g(x) = 2^x$ intersecting at $A(1; 2)$ and $B(3; 8)$.



(a) Determine the equation of :

(1) $y = f^{-1}(x)$ (2)

$$x = 3y - 1$$

$$y = \frac{x+1}{3} \checkmark \checkmark$$

(2) $y = g^{-1}(x)$ (2)

$$y = \log_2 x \checkmark; x > 0 \checkmark$$

(b) On the axes above sketch $y = g^{-1}(x)$ showing all intercepts with the axes and asymptotes. (2)

(c) Use the above graphs to determine the values of x for each of the following:

(1) $f(x) < g(x)$ (2)

$$x \in (-\infty; 1) \checkmark \cup (3; \infty) \checkmark$$

(2) $g^{-1}(x) < 0$ (2)

$$x \in (0; 1) \checkmark \checkmark$$

(3) $f^{-1}(x) = g^{-1}(x)$ (2)

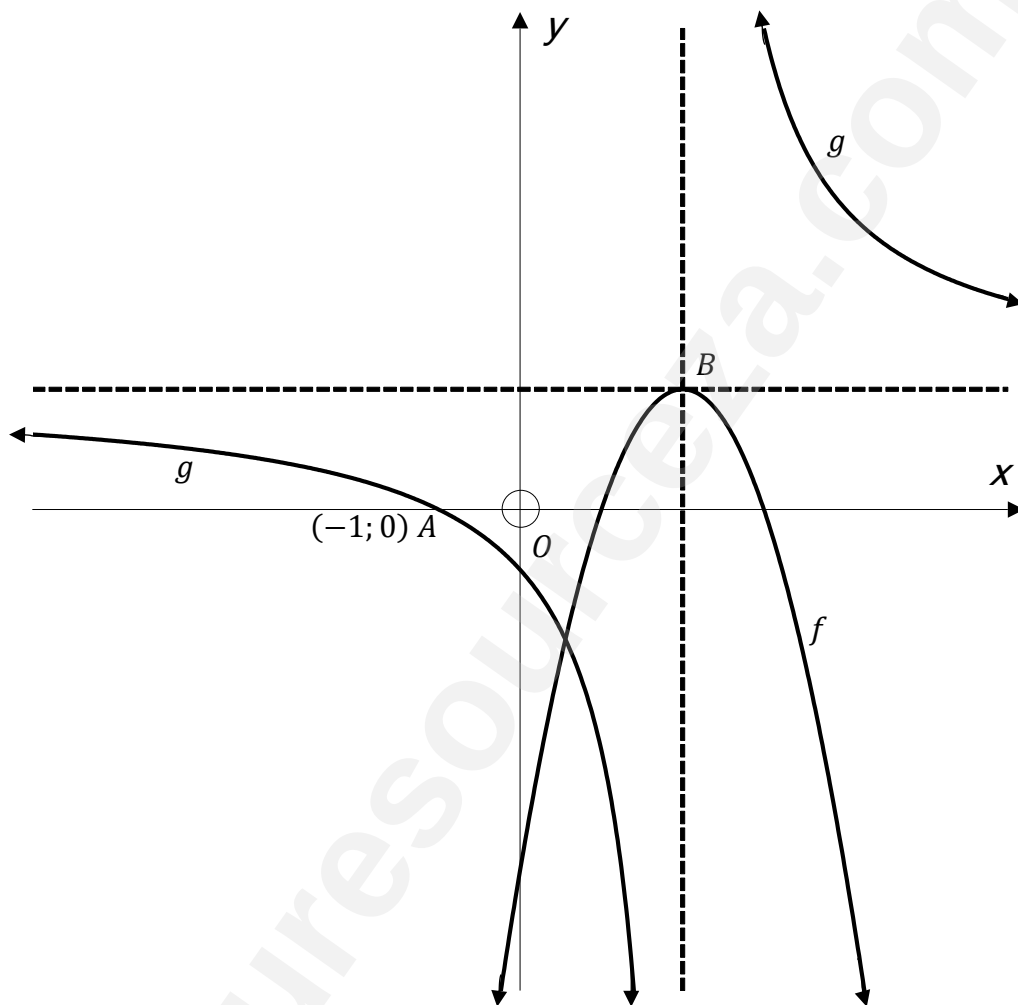
$$x = 2 \checkmark \text{ or } x = 8 \checkmark$$

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QUESTION 5

Sketched below is the parabola f , with the equation $f(x) = -x^2 + 4x - 3$ and a hyperbola g with equation $y = \frac{3}{x-p} + t$

- B , the turning point of f , lies at the point of intersection of the asymptotes of g .
- $A(-1; 0)$ is the x -intercept of g



- (a) Show the coordinates of B are $(2; 1)$. (2)

$$f'(x) = -2x + 4$$

$$0 = -2x + 4 \checkmark$$

$$x = 2$$

$$f(2) \checkmark = 1$$

- (b) Write down the range of f . (1)

$$y \leq 1 \checkmark$$

- (c) For which value(s) of x will $g(x) \geq 0$ (4)

$$x \in (-\infty; -1] \checkmark \checkmark \cup (2; \infty) \checkmark \checkmark$$

- (d) Determine the values of p and t . (4)

$$y = \frac{3}{x-p} + t$$

$$p = 2 \checkmark \checkmark$$

$$t = 1 \checkmark \checkmark$$

- (e) Determine the equation of the vertical asymptote of the graph of h
if $h(x) = g(x+4)$ (1)

$$x = -2 \checkmark$$

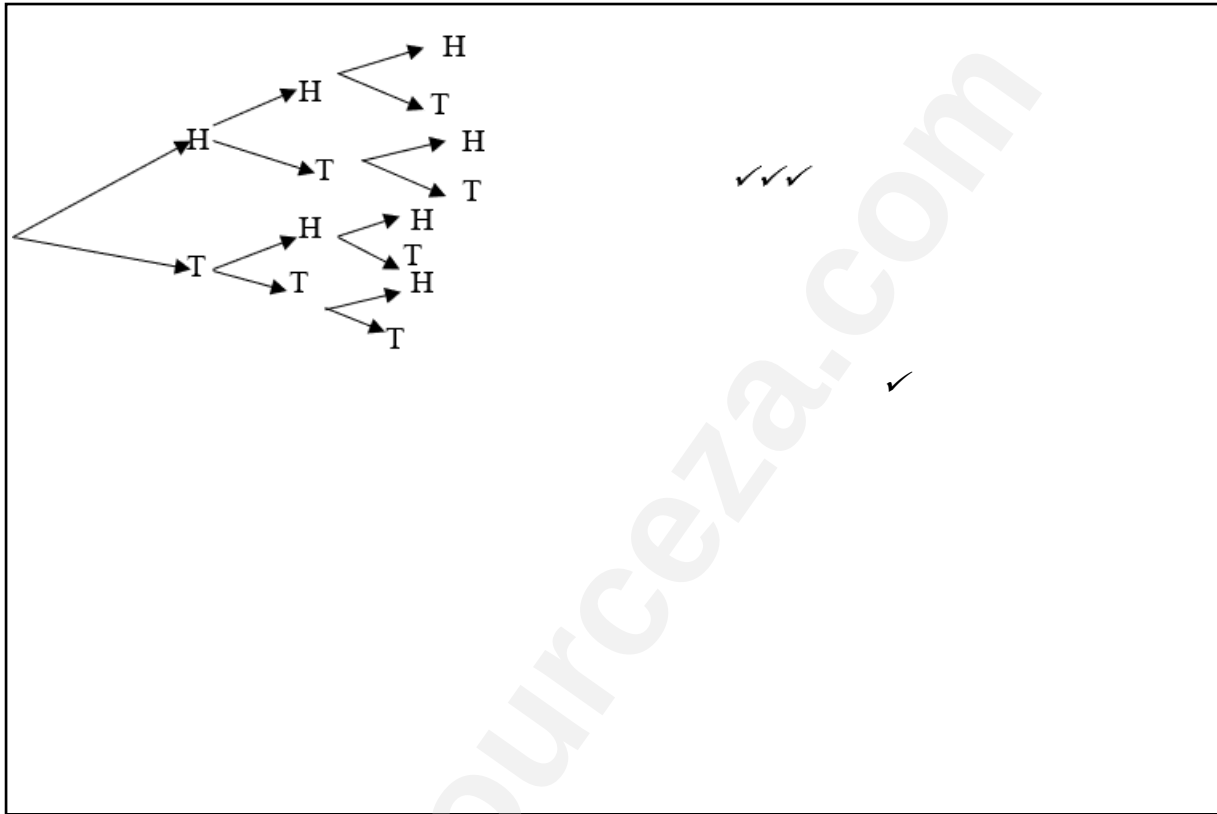
[12]

QUESTION 6

Three unbiased coins are tossed.

- (1) Represent all possible outcomes in a tree diagram.

(4)



- (2) Determine the probability of getting two tails and one head (in any order.) (2)

$$P(TTH) + P(THT) + P(HTT)$$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8} \quad \checkmark \checkmark$$

[6]

TOTAL SECTION A: 75 MARKS

SECTION B

QUESTION 7

When air travel is once again allowed and we are flying on LOCKDOWN AIRLINE, we will be allowed to choose exactly one free beverage from either tea, coffee, or fruit juice. For the service to run smoothly they will look at whether you are sitting on the right-hand or left-hand side of the central aisle. The results of the passengers' choice are shown in the table below.

	Left hand side	Right hand side	Total
Tea	20	40	60
Coffee	b	c	80
Fruit Juice	d	e	20
Total	60	100	a

- (a) (1) Write down the value of a . (1)

$$a = 160 \checkmark$$

- (2) What is the probability that a randomly chosen passenger is seated on the left-hand side of the plane. (2)

$$\frac{60}{160} = \frac{3}{8} \checkmark \checkmark$$

- (3) Given that the event of a passenger choosing coffee is independent of sitting on the left-hand side of the plane, calculate the value of b . (4)

$$P(C \cap L) = P(C) \times P(L) \checkmark$$

$$P(C \cap L) = \frac{80}{160} \checkmark \times \frac{60}{160} \checkmark$$

$$P(C \cap L) = \frac{3}{16}$$

$$\therefore b = 30 \checkmark$$

- (b) A LOCKDOWN AIRLINE airplane has 6 seats, that can hold 6 passengers in each row. Precious, Xoliswa and Anees and 3 other passengers are to be seated in a row.

- (1) How many possible arrangements are there for 6 people to sit in a row of 6 seats? (2)

$$6! = 720 \checkmark \checkmark$$

- (2) Xoliswa, Anees and the 4 other passengers sit in a certain row. In how many different ways can these 6 passengers be seated if Xoliswa and Anees must sit next to each other. (2)

$$2! \checkmark \times 5! \checkmark = 240$$

- (3) If the 6 passengers mentioned in b) are seated at random, what is the probability that Precious will be at either end of the row. (4)

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$$P = \frac{1 \times 5! \checkmark + 5! \times 1 \checkmark}{6! \checkmark}$$

$$P = \frac{1}{3} \checkmark$$

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QUESTION 8

Zanele buys a car for R380 000. She pays 15% of the purchase price in cash. The balance is paid through a bank loan where the interest rate is 20% p.a. compounded monthly. The loan is paid off in equal monthly payments over a period of 6 years.

- (a) Determine the equivalent effective annual interest rate. (Correct to two decimal places). (4)

$$1 + i_{eff} = \left(1 + \frac{0,2}{12}\right)^{12} \checkmark \text{ Formula} \checkmark$$

$$i_{eff} = 0,2193 \dots \checkmark$$

$$r = 21,94\% \text{ p.a.} \checkmark$$

- (b) Calculate her monthly payment. (5)

$$323000 \checkmark = \frac{x[1 - \left(1 + \frac{0,2}{12}\right)^{-12 \times 6} \checkmark]}{\frac{0,2}{12} \checkmark}$$

$$x = R7736,76 \text{ per month} \checkmark$$

Zanele gets a big bonus at the end of the 3rd year of paying off the car and decides to settle the loan.

- (c) Calculate the settlement amount at the end of year 3? (5)

$$B = \frac{7736,76 \checkmark \left[1 - \left(1 + \frac{0,2}{12}\right)^{-(72-36)} \checkmark\right]}{\frac{0,2}{12} \checkmark}$$

$$B = R208181,21 \checkmark$$

[14]

QUESTION 9

The function defined by $f(x) = 3x^2 - 2x^3$

- (a) Determine all intercepts with the axes. (3)

$$f(x) = 3x^2 - 2x^3$$

$$y \text{ intercept: } y = 0 \text{ (Let } x = 0)$$

$$x \text{ intercept: } 3x^2 - 2x^3 = 0 \text{ (Let } y = 0)$$

$$\therefore x^2(3 - 2x) = 0$$

$$x = 0 \cup x = \frac{3}{2}$$

$$(0; 0) \checkmark \text{ and } \left(\frac{3}{2}; 0\right) \checkmark \checkmark$$

- (b) Determine the coordinates of the stationary points and the nature thereof. (5)

$$f'(x) = 0$$

$$6x - 6x^2 = 0 \checkmark \checkmark$$

$$6x(1 - x) = 0 \checkmark$$

$$x = 0 \cup x = 1$$

$$(0; 0) \text{ local min and } (1; 1) \checkmark \checkmark \text{ local max}$$

- (c) The graph of f passes through the point $C(p; q)$ and $p > 0$. Determine the values of p and q if the tangent to the curve at C is parallel to the line

$$2y + 24x - 5 = 0. \quad (6)$$

$$y = -12x - \frac{5}{2} \checkmark$$

$$-12 = 6p - 6p^2 = f'(p) \checkmark$$

$$0 = -p^2 + p + 2$$

$$0 = p^2 - p - 2$$

$$0 = (p - 2)(p + 1) \checkmark$$

$$p = 2 \checkmark$$

$$f(2) = 3(2)^2 - 2(2)^3 \checkmark$$

$$f(2) = -4$$

$$\therefore q = -4 \checkmark$$

- (d) Determine the values of k for which the equation $3x^2 - 2x^3 = k$ will have 3 unequal roots. (3)

$$y = k$$

$$0 < k < 1 \checkmark \checkmark \checkmark$$

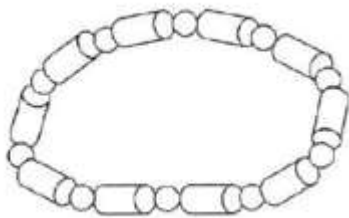
[17]

QUESTION 10

A necklace is made by using 10 wooden spheres and 10 wooden cylinders. The radii r of the spheres and the cylinders are exactly the same. The height of each cylinder is h .

The volume of the cylinder is 6cm^3 .

The wooden cylinders are to be painted. (Ignore the holes in the in the spheres and cylinders).



$$V = \pi r^2 h$$

$$S = 2\pi r^2 + 2\pi r h$$

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

(a) Show that the total surface area of all the painted surfaces of the necklace is

$$S = 60\pi r^2 + \frac{120}{r} \quad (7)$$

$$S = 60\pi r^2 + \frac{120}{r}$$

$$6\checkmark = \pi r^2 h$$

$$h = \frac{6}{\pi r^2} \checkmark$$

$$SA = 10[2\pi r^2 + 2\pi r h + 4\pi r^2]$$

$$SA = 10\checkmark[6\pi r^2 + 2\pi r h] \checkmark$$

$$SA = 10[6\pi r^2 + 2\pi r \left(\frac{6}{\pi r^2}\right)] \checkmark$$

$$SA = 10\left[6\pi r^2 + \frac{12}{r}\right] \checkmark$$

$$SA = 60\pi r^2 + \frac{120}{r} \checkmark$$

- (b) Determine the value of r so that the least amount of paint will be used.
(correct to 2 decimal places) (4)

$$S' = 120\pi r - \frac{120}{r^2}$$

$$0 \checkmark = 120\pi r \checkmark - \frac{120}{r^2} \checkmark$$

$$120 = 120\pi r^3$$

$$\pi r^3 = 1$$

$$r = 0,68 \text{ cm} \checkmark$$

[11]

QUESTION 11

0; 9; 16; 21 are the first four numbers of a quadratic sequence.

- (a) Find the equation of T_n for this sequence. (6)

0		9		16
9			7 ✓	
-2 ✓				

$$a = -1 \checkmark$$

$$3a + b = 9$$

$$b = 12 \checkmark$$

$$a + b + c = 0$$

$$c = -11 \checkmark$$

$$T_n = -n^2 + 12n - 11 \checkmark$$

- (b) At which term is the highest value of these numbers, and what is the highest value, before they will decline again? (5)

$$-n^2 + 12n - 11$$

$$= -(n^2 - 12n + 36 - 36 + 11) \checkmark$$

$$= -[(n - 6)^2 - 25] \checkmark$$

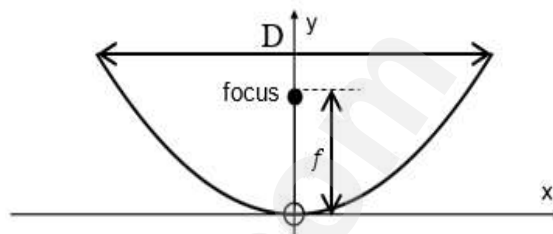
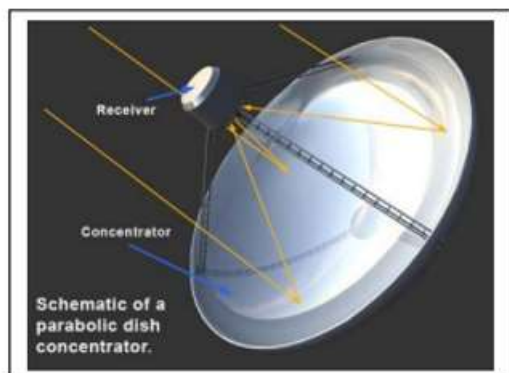
$$= -(n - 6)^2 + 25 \checkmark$$

$$\text{Max is 25 when the term number is 6} \checkmark \checkmark$$

[11]

QUESTION 12

Exploring Parabolas: The shape of a satellite dish



A very beautiful property of parabolas is that at a point called the FOCUS, all of the lines entering the parabola parallel to its **axis of symmetry** are “reflected” from the parabolic curve and intersect the focus. This property is used by astronomers to design telescopes, and by radio engineers to design satellite dishes.

The top figure to the left shows a satellite dish with a radio receiver located at the focus of the parabola. The radio rays are reflected from the parabolic surface and concentrated at the focus. This focussing and amplification property of parabolic reflectors is also used for solar heating and generating solar electricity.

The bottom figure defines the distance, f , of the focus from the bottom of the dish, and the diameter, D , of the dish.

Reference : Space Math <http://spacemath.gsfc.nasa.gov>

Given the general form of a parabola is $y = ax^2 + bx + c$. The y value of the focus is given as $y = \frac{1}{4a}$. The focus can be any point on the y -axis.

Using the above information, solve the following:

A bird watcher wants to record the bird sounds she is hearing. She finds a parabola shaped bowl in the kitchen. It has a diameter of 120 cm and a depth of 50 cm. How far below the edge of the dish would she need to mount the microphone for it to be at the focus point.

$$y = ax^2 \quad b = 0 \text{ and } c = 0 \quad \checkmark$$

$$50 \checkmark = a(60)^2 \checkmark$$

$$a = \frac{1}{72}$$

$$y = \frac{1}{72}x^2 \quad \checkmark$$

$$\text{FOCUS: } y = \frac{1}{4\left(\frac{1}{72}\right)} \checkmark = 18 \checkmark$$

$$\therefore 50\text{cm} - 18\text{cm} = 32\text{ cm} \quad \checkmark$$

[7]

TOTAL SECTION B: 75 MARKS

GRANDTOTAL: 150 MARKS