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MEMORANDUM PRELIM EXAM PAPER II - 2019

QUESTION 1

$$\begin{aligned}
 \text{(a)} \quad x_N &= \frac{9-3}{2} & y_N &= \frac{5+y}{2} \\
 &= 3 \quad \checkmark & 2 &= \frac{5+y}{2} \\
 & & y &= -1 \quad \checkmark
 \end{aligned}
 \tag{2}$$

$$\begin{aligned}
 \text{(b)} \quad (x-3)^2 + (y-2)^2 &= r^2 \\
 \text{Subs } (9;5): (9-3)^2 + (5-2)^2 &= r^2 \quad \checkmark \\
 r^2 &= 45 \quad \checkmark \\
 \therefore (x-3)^2 + (y-2)^2 &= 45 \quad \checkmark
 \end{aligned}
 \tag{3}$$

$$\begin{aligned}
 \text{(c)} \quad \tan 45^\circ &= m \quad \therefore m = 1 \quad \checkmark \\
 (9;5): y-5 &= 1(x-9) \quad \checkmark \\
 y &= x-4 \quad \checkmark
 \end{aligned}
 \tag{3}$$

$$\text{(d)} \quad C(0; -4) \quad \checkmark \tag{1}$$

$$\begin{aligned}
 \text{(e)} \quad m_{AC} &= 1 \\
 m_{BC} &= \frac{-4+1}{0+3} = -\frac{3}{3} = -1 \quad \checkmark \\
 m_{AC} \times m_{BC} &= -1 \quad \checkmark \therefore \hat{ACB} = 90^\circ \quad \checkmark \quad \text{Or proving Pythagoras}
 \end{aligned}
 \tag{3}$$

$$\begin{aligned}
 \text{(f)} \quad m_{AD} &= -2 \\
 \tan \theta &= -2 \quad \checkmark \\
 \theta &= -63,43 \dots^\circ + 180^\circ \\
 \theta &= 116,6^\circ \quad \checkmark \quad (\text{round off to 1 d.p. as per instructions on cover page})
 \end{aligned}
 \tag{2}$$

$$\begin{aligned}
 \text{(g)} \quad \hat{AFE} &= 180^\circ - 116,6^\circ = 63,4^\circ \quad (\text{Adj sup } \angle\text{s}) \quad \checkmark \\
 \hat{A}_2 &= 180^\circ - 45^\circ - 63,4^\circ = 71,6^\circ \quad (\angle\text{s of } \Delta) \quad \checkmark
 \end{aligned}
 \tag{2}$$

(h) $AC^2 = \sqrt{9^2 + (5 + 4)^2} \checkmark$

$$AC = \sqrt{162} \checkmark$$

$$\text{Area} = \frac{1}{2} 6\sqrt{5} \times \sqrt{162} \sin 71,6^\circ \checkmark$$

$$\text{Area} = 81 \text{ units}^2 \checkmark$$

(4)

[20]

QUESTION 2

(a)

$$\sin(180^\circ - x) \cos(90^\circ - x) - \frac{\tan(x) \sin(33^\circ)}{\cos(123^\circ) \tan(-x - 360^\circ)}$$

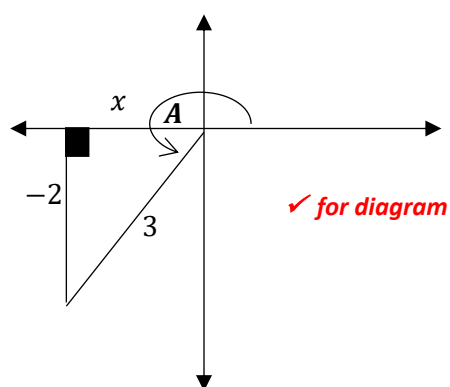
$$= \sin x \checkmark \cdot \sin x \checkmark - \frac{\tan x \cdot \sin 33^\circ}{(-\cos 57^\circ \checkmark)(-\tan x \checkmark)}$$

$$= \sin^2 x - 1 \checkmark$$

$$= -\cos^2 x \checkmark$$

(6)

(b)



$$x^2 = 9 - 4 \text{ (Pythag)}$$

$$x = -\sqrt{5} \checkmark$$

$$(1) \tan \theta = -\frac{2}{-\sqrt{5}} = \frac{2}{\sqrt{5}} \checkmark \quad \text{must be positive!}$$

(3)

$$(2) \cos(\theta + 60^\circ)$$

$$= \cos \theta \cdot \cos 60^\circ - \sin \theta \cdot \sin 60^\circ \checkmark$$

$$= \frac{-\sqrt{5}}{3} \cdot \frac{1}{2} - \left(-\frac{2}{3}\right) \left(\frac{\sqrt{3}}{2}\right) \checkmark \checkmark$$

$$= \frac{-\sqrt{5} + 2\sqrt{3}}{6} \checkmark$$

(4)

instructions say without a calculator so it must be in surd form!

$$(c) \text{ LHS : } \frac{1 - 2 + 4 \sin^2 x \checkmark}{2 \sin x \cos x - \cos x \checkmark}$$

$$= \frac{-1 + 4 \sin^2 x \checkmark}{\cos x (2 \sin x - 1) \checkmark}$$

$$= \frac{(2 \sin x - 1)(2 \sin x + 1) \checkmark}{\cos x (2 \sin x - 1) \checkmark}$$

$$= \frac{2 \sin x + 1}{\cos x} \checkmark = \text{RHS}$$

(5)

(d) $\sin(20^\circ - 3x) = \sin x$ ✓

$$20^\circ - 3x = x + k \cdot 360^\circ \quad \checkmark$$

$$-4x = -20 + k \cdot 360^\circ$$

$$x = 5^\circ + k \cdot 90^\circ \checkmark$$

OR

$$20^\circ - 3x = 180^\circ - x + k \cdot 360^\circ \quad \checkmark$$

$$-2x = 160^\circ + k \cdot 360^\circ$$

$$x = -80^\circ + k \cdot 180^\circ \quad k \in \mathbb{Z} \quad \checkmark$$

(5)

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QUESTION 3

(a) $r = 0,847$ ✓ **3 d.p as per instructions.**

Strong, positive ✓, linear ✓ correlations (3)

(b) $\hat{y} = 209,6 + 94,7x$ ✓ **one d.p as per front cover instructions.** (3)

(c) $\hat{y} = 209,6 + 94,7(13)$ ✓

$\hat{y} = R1440,70$ ✓ **MONEY must have 2 d.p** (2)

[8]

QUESTION 4

(a) (1) $D = 180^\circ - 115^\circ = 65^\circ$ ✓ (Opp $\angle$ s cyclic quad) ✓ (2)

(2) $\widehat{O_1} = 130^\circ$ ($\angle$ at centre = 2 x $\angle$ at circ) ✓ (1)

(b) $G_2 = 65^\circ$ (adj sup $\angle$ s or ext $\angle$ cyclic quad))

$\hat{E} = 65^\circ$ (Isos Δ) ✓

$\hat{F} = 50^\circ$ (Int $\angle$ s of Δ) ✓

$\widehat{O_1} + \hat{F} = 130^\circ + 50^\circ = 180^\circ$ ✓

$\therefore OEFH$ is a cyclic quad (conv. opp $\angle$ s cyclic quad sup) ✓ (4)

[7]

QUESTION 5

(a)

R.T.P. $\widehat{NKT} = \widehat{L}$

Construct The diameter MK ✓ and join M to N.

$$\widehat{MKT} = 90^\circ \text{ ✓ (Tan perp radius ✓)}$$

$$\widehat{N_1} + \widehat{N_2} = 90^\circ \text{ (< in semi - circle) ✓}$$

$$\widehat{MKN} + \widehat{M} = 90^\circ \text{ (<'s of a } \Delta \text{)}$$

$$\widehat{NKT} = \widehat{M}$$

$$\text{But } \widehat{M} = \widehat{L} \text{ (Angles in same segment) ✓}$$

$$\therefore \widehat{NKT} = \widehat{L} \text{ ✓}$$

(b)

$$\widehat{B_1} = \widehat{C_1} \text{ (alt < s BD//EF) ✓}$$

$$\widehat{C_1} = \widehat{D_2} \text{ (tan chord theorem) ✓}$$

$$\therefore \widehat{B_1} = \widehat{D_2}$$

$$\therefore BC = CD \text{ (Sides opp = angles) ✓}$$

(c) In $\triangle ADC$ and $\triangle ACE$

$$\text{i) } \widehat{A_2} = \widehat{A_1} \text{ (= chords subt = angles) ✓}$$

$$\text{ii) } \widehat{D_1} + \widehat{D_2} = \widehat{C_1} + \widehat{C_2} \text{ (proven) ✓}$$

$$\text{iii) } \widehat{C_3} = \widehat{E} \text{ (Int < s of } \Delta \text{) ✓}$$

$$\therefore \triangle ADC \text{ /// } \triangle ACE \text{ ✓ (AAA) ✓}$$

(5)

[13]

QUESTION 6

(a)

$$-4y = -3x - 8 \checkmark$$

$$y = -\frac{3}{4}x + 2$$

$$B(0; 2) \checkmark$$

(2)

$$(b) (x - 3)^2 + (y + 2)^2 = r^2 \checkmark$$

Subs (0; 2)

$$(0 - 3)^2 + (2 + 2)^2 = r^2$$

$$r^2 = 25 \checkmark$$

$$\therefore (x - 3)^2 + (y + 2)^2 = 25 \checkmark$$

$$\therefore x^2 - 6x + 9 + y^2 + 4y + 4 = 25$$

$$\therefore x^2 - 6x + y^2 + 4y - 12 = 0 \checkmark$$

(4)

$$(c) x = 3 - 5 \checkmark$$

$$\text{or } x^2 - 6x + (-2)^2 + 4(-2) - 12 = 0 \checkmark$$

$$\therefore x/q = -2 \checkmark$$

$$x^2 - 6x \pm 16 = 0$$

or

$$x = 8 \vee x = -2$$

$$x = 3 + 5 \checkmark$$

$$\therefore x/q = 8 \checkmark$$

(4)

$$(d) \quad (1) BD = BC = 5 \text{ units} \quad \text{no reason needed!}$$

(1)

$$(2) \quad y = \frac{3}{4}x + 2 \dots \dots \dots \text{eqn 1} \checkmark$$

$$\text{distance BD: } 5 = \sqrt{(0 - x)^2 + (2 - y)^2}$$

$$25 = x^2 + 4 - 4y + y^2 \dots \dots \dots \text{eqn 2} \checkmark$$

Subs 1 into 2

$$25 = x^2 + 4 - 4\left(\frac{3}{4}x + 2\right) + \left(\frac{3}{4}x + 2\right)^2 \checkmark$$

$$25x^2 = 400$$

$$x^2 = 16 \checkmark$$

$$x = -4n/a \vee x = 4 \checkmark$$

$$y = 5$$

$$\therefore D(4; 5) \checkmark$$

(6)

[17]

QUESTION 7

(a)

$$\begin{aligned}
 & \cos 73^\circ \cdot \cos 15^\circ + \sin 73^\circ \cdot \sin 15^\circ \\
 &= \cos(73^\circ - 15^\circ) \checkmark \\
 &= \cos 58^\circ \checkmark \\
 &= \cos 2(29^\circ) \checkmark \\
 &= 2 \cos^2 29^\circ - 1 \checkmark \quad \text{or} \quad = 1 - 2 \sin^2 29^\circ \\
 &= 2 \cos^2(90^\circ - 61^\circ) - 1 \quad = 1 - 2 \sin^2(90^\circ - 61^\circ) \\
 &= 2 \sin^2 61^\circ - 1 \checkmark \quad = 1 - 2 \cos^2 61^\circ \\
 &= 2k - 1 \checkmark \quad = 1 - 2(\sqrt{1-k})^2 \\
 & \quad \quad \quad = 1 - 2(1-k) \\
 & \quad \quad \quad = 2k - 1
 \end{aligned}$$

(6)

$$\begin{aligned}
 \text{(b) (1) } LHS &= \frac{\sin(2A+A)}{\sin A} \checkmark \\
 &= \frac{\sin 2A \cos A + \sin A \cos 2A}{\sin A} \checkmark \\
 &= \frac{2 \sin A \cos A \cdot \cos A + \sin A \cos 2A}{\sin A} \checkmark \\
 &= \frac{\sin A (2 \cos^2 A + \cos 2A)}{\sin A} \checkmark \\
 &= \frac{2 \cos^2 A + 2 \cos^2 A - 1}{1} \checkmark \\
 &= 4 \cos^2 A - 1
 \end{aligned}$$

(5)

(2) $\cos^2 A$ has a maximum of 1

$$\therefore 4 \cos^2 A - 1 \checkmark$$

$$\therefore 4(1) - 1 = 3 \checkmark$$

$\cos^2 A$ has a minimum of 0

$$\therefore 4 \cos^2 A - 1 \checkmark$$

$$\therefore 4(0) - 1 = -1 \checkmark$$

(4)

- (c) (1) $a = 3 \checkmark, b = -2 \checkmark, c = 2 \checkmark$ (3)
- (2) $x \in (0^\circ; 30^\circ) \checkmark \cup (150^\circ; 180^\circ) \checkmark$ (2)
- [20]

QUESTION 8

- (a) (1) $35 - 15 \checkmark \checkmark = 20 \text{ Students } \checkmark$ (3)
- (2) $50 - 35 \checkmark = 15 \text{ Students } \checkmark$ (2)
- (b) (1) B has the greater frequency at higher values $\checkmark$ (2)
- (2) A $\checkmark$ (1)
- (c) (1) Remains 13 $\checkmark$ (1)
- (2) 39 $\checkmark$ (1)
- [10]

QUESTION 9

(a)

In $\triangle KPL$ and $\triangle KQM$

$$\hat{K} = \hat{K} \quad (\text{common}) \checkmark$$

$$\hat{L}_1 = \hat{O}\hat{Q}\hat{M} \quad (\text{ext } \angle \text{ cyclic quad}) \checkmark$$

$$\hat{P} = \hat{M} \quad (3^{\text{rd}} \angle \text{ of } \triangle) \checkmark$$

$$\therefore \triangle KPL \sim \triangle KQM \quad (\text{AAA}) \checkmark \quad (4)$$

(b) $\frac{KP}{KM} = \frac{PL}{MQ} = \frac{KL}{KQ} \quad (\sim \triangle s) \checkmark$

$$\frac{9k}{75} = \frac{51}{17k} \checkmark$$

$$k^2 = 25$$

$$\therefore k = 5 \checkmark$$

$$\therefore r = 4(5) = 20 \checkmark \quad (4)$$

[8]

QUESTION 10

(a) In $\triangle PSQ$ and $\triangle PMO$

$$\hat{P} = \hat{P} \quad (\text{common}) \checkmark$$

$$\hat{S} = \hat{M}O \quad (\text{given}) \checkmark$$

$$P\hat{Q}S = P\hat{O}M \quad (3^{\text{rd}} \angle \text{ of } \triangle) \checkmark$$

$$\therefore \triangle PSQ \parallel \triangle PMO \quad (\text{AAA}) \quad (3)$$

(b) $PS = PO + OS \checkmark$

$$\therefore OP \cdot PS$$

$$= OP(PO + OS)$$

$$= OP^2 + OP \cdot OS \checkmark$$

$$\therefore OP \cdot PS = OP^2 + OP \cdot OS \quad (2)$$

(c) $\frac{PS}{PM} = \frac{PQ}{PO} \quad (\parallel \Delta s) \checkmark$

$$PS \cdot QO = PQ \cdot PM \quad (PO = OQ \text{ radii}) \checkmark$$

$$PS \cdot QO = PQ \cdot MQ \quad (PM = MQ \text{ line from centre perp to chord})$$

$$\therefore PS \cdot QO = 2MQ \cdot MQ \quad \checkmark$$

$$PS \cdot QO = 2MQ^2$$

$$(OQ + OS) \cdot QO = 2MQ^2 \checkmark$$

$$\therefore 2MQ^2 = QO^2 + QO \cdot OS$$

$$\therefore 2MQ^2 = QO^2 + OP \cdot OS \quad (OQ = OP \text{ radii}) \quad (4)$$

[9]

QUESTION 11

(a) $QS = SR$ (line perp. Isos Δ)

$$\therefore QR^2 = 10^2 + 10^2 \text{ (Pythag)} \checkmark$$

$$QR = 10\sqrt{2} \checkmark$$

$$QS = SR = 5\sqrt{2}$$

$$PS^2 = 10^2 - (5\sqrt{2})^2 \text{ (Pythag)} \checkmark$$

$$PS = 5\sqrt{2} \checkmark \quad (4)$$

(b) in ΔSQR : $\hat{Q} + \hat{R} = 180^\circ - \theta$ (Int $\angle$ s of Δ)

$$\therefore \hat{Q} = \hat{R} = 90^\circ - \frac{\theta}{2} \text{ (Isos } \Delta) \checkmark$$

$$\frac{QR}{\sin \theta} = \frac{5\sqrt{2}}{\sin(90^\circ - \frac{\theta}{2})} \checkmark \quad \text{using the sine rule or cos rule} \quad QR^2 = (5\sqrt{2})^2 + (5\sqrt{2})^2 - 2(5\sqrt{2})(5\sqrt{2})\cos \theta$$

$$\therefore QR = \frac{5\sqrt{2} \sin \theta}{\cos \frac{\theta}{2}} \checkmark \quad \text{or} \quad = 100 - 100 \cos \theta$$

$$\therefore QR = \frac{5\sqrt{2} \cdot 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{\cos \frac{\theta}{2}} \checkmark \quad \boxed{\text{double angle formula}} \quad = 100(1 - \cos \theta)$$

$$\begin{aligned} \therefore QR &= 10\sqrt{2} \sin \frac{\theta}{2} \checkmark \\ &= 100 \left[1 - \left(1 - 2 \sin^2 \frac{\theta}{2} \right) \right] \\ &= 100 \left(2 \sin^2 \frac{\theta}{2} \right) \\ &= 200 \sin^2 \frac{\theta}{2} \\ QR &= 10\sqrt{2} \sin \frac{\theta}{2} \end{aligned}$$

(5)

(c) $h^2 + \left(5\sqrt{2} \sin \frac{\theta}{2}\right)^2 = 10^2$ (pythag) ✓

$$h^2 + 50 \sin^2 \frac{\theta}{2} = 100 \quad \checkmark$$

$$h^2 = 100 - 50 \sin^2 \frac{\theta}{2}$$

$$h^2 = 75 + 25 - 50 \sin^2 \frac{\theta}{2} \quad \checkmark$$

$$h^2 = 75 + 25 \left(1 - 2 \sin^2 \frac{\theta}{2}\right) \quad \checkmark$$

$$h^2 = 75 + 25(\cos \theta) \quad \checkmark$$

$$h^2 = 25(3 + \cos \theta) \quad \checkmark$$

$$h = 5\sqrt{3 + \cos \theta}$$

or

$$h^2 = 100 - 50 \sin^2 \frac{\theta}{2}$$

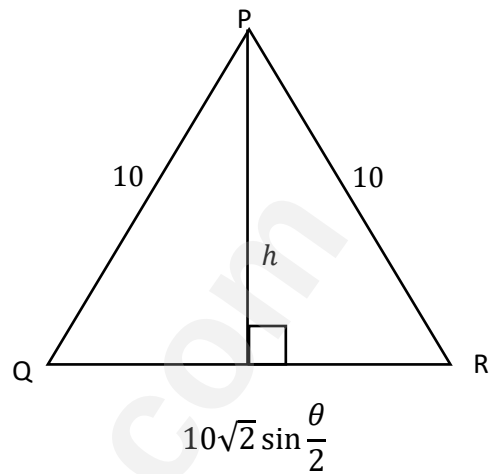
$$= 50 \left(2 - \sin^2 \frac{\theta}{2}\right)$$

$$= 50 \left[2 - \left(\frac{1}{2} - \frac{1}{2} \cos \theta\right)\right] \quad \leftarrow \text{double angle formula}$$

$$= 50 \left(\frac{3}{2} - \frac{1}{2} \cos \theta\right)$$

$$= 75 + 25 \cos \theta$$

$$\therefore h = 5\sqrt{3 + \cos \theta}$$



$$\cos \theta = 1 - 2 \sin^2 \frac{\theta}{2}$$

$$2 \sin^2 \frac{\theta}{2} = 1 - \cos \theta$$

$$\sin^2 \frac{\theta}{2} = \frac{1}{2} - \frac{1}{2} \cos \theta$$

(6)

[14]