

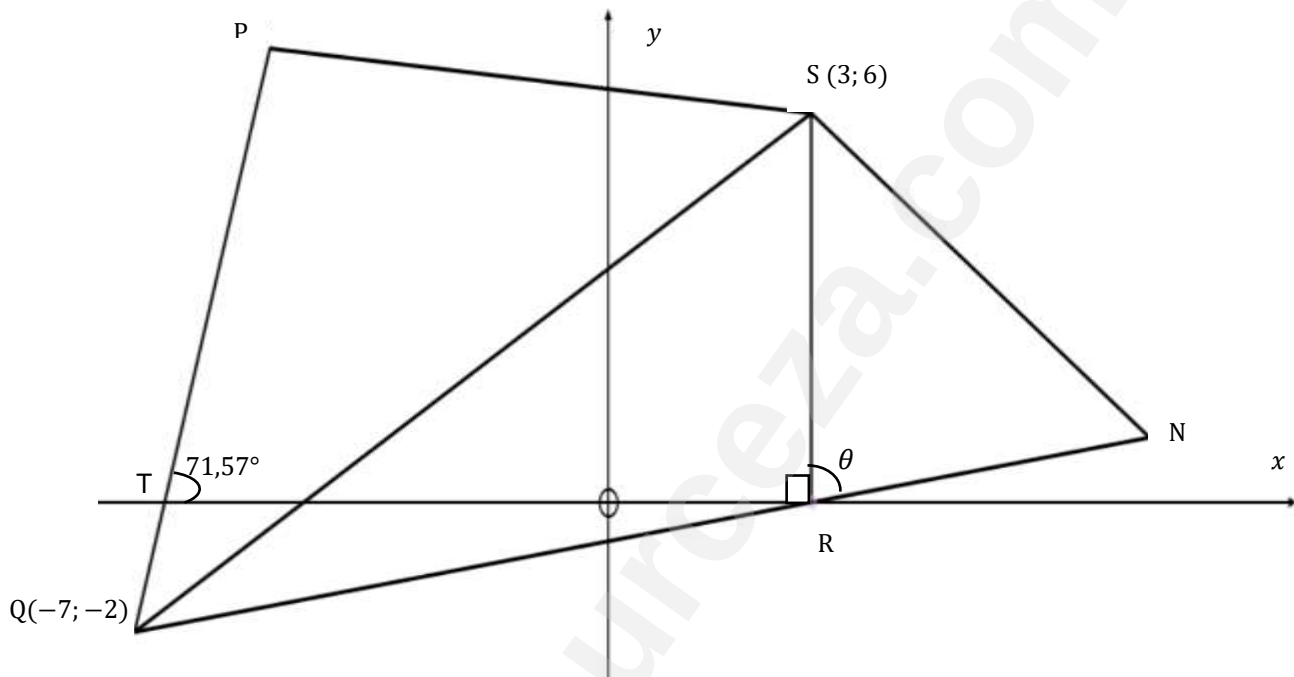


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SECTION A

QUESTION 1

In the diagram, P, Q(-7; -2), R and S(3; 6) are vertices of a quadrilateral. R is a point on the x -axis. QR is produced to N. SN is drawn. $\widehat{PTO} = 71,57^\circ$ and $\widehat{SRN} = \theta$. $SR \perp x$ -axis.



Determine:

- (a) The equation of SR. (1)

$$x = 3 \checkmark$$

- (b) The gradient of QP correct to the nearest integer. (2)

$$m_{QP} = \tan 71,57^\circ \checkmark$$

$$m_{QP} = 3 \checkmark$$

- (c) The equation of QP in the form $y = mx + c$. (2)

$$y - (-2) = 3(x - (-7)) \checkmark$$

$$y + 2 = 3x + 21$$

$$y = 3x + 19 \checkmark$$

- (d) (1) The angle of inclination of QN, β , correct to one decimal place. (3)

$$m_{QN} = \frac{2}{10} = \frac{1}{5} \checkmark$$

$$m_{QN} = \tan(\widehat{NRX})$$

$$\frac{1}{5} \checkmark = \tan(\widehat{NRX})$$

$$\beta = 11,3^\circ \checkmark$$

- (2) Hence, determine θ . (2)

$$\theta = 90^\circ - 11,3^\circ \checkmark$$

$$\theta = 78,7^\circ \checkmark$$

- (e) The length of QR. Leave your answer in simplest surd form. (2)

$$QR = \sqrt{(3 - (-7))^2 + (0 - (-2))^2} \checkmark$$

$$QR = 2\sqrt{26} \checkmark$$

- (f) If $QR = 2RN$ determine the area of $\triangle RSN$, correct to the nearest square unit. (4)

$$RN = \sqrt{26} \checkmark$$

$$\text{Area } \triangle RSN = \frac{1}{2} (6)(\sqrt{26}) \sin 78,7^\circ \checkmark \checkmark \checkmark$$

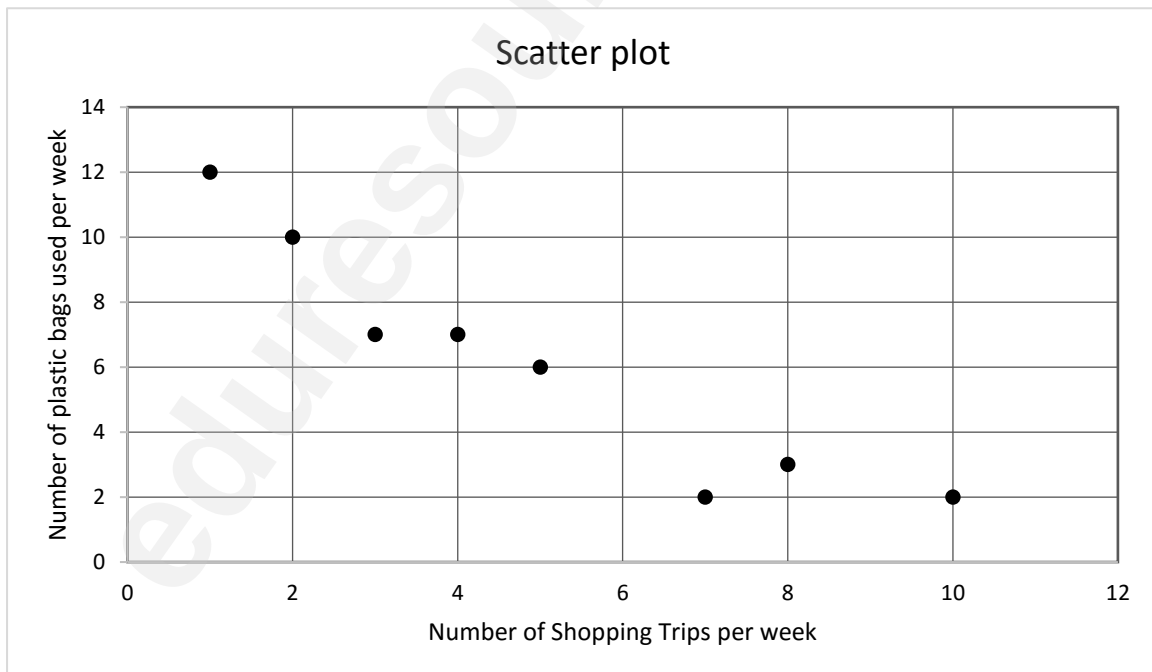
$$\text{Area } \triangle RSN = 15 \text{ units}^2$$

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QUESTION 2

A local supermarket is running a campaign for shoppers to use fewer single use plastic bags in their weekly shopping trips. The supermarket recorded the amount of single use plastic bags requested by shoppers as the campaign continued. The results are shown below:

Number of weeks in campaign	1	2	3	4	5	7	8	10
Number of plastic bags requested	12	10	7	7	6	2	3	2



- (a) Use the scatter plot to comment on the relationship between the number of plastic bags shoppers requested and the weeks the campaign was running. (2)

Strong ✓

Linear ✓

- (b) Use your calculator to determine the correlation coefficient of the data, correct to two decimal places. (1)

$$r = -0,95 \checkmark$$

- (c) Use your calculator to assist you in determining the equation of the least squares regression line of the data. Round a and b values to two decimal places. (3)

$$a = 11,71 \checkmark \text{ and } b = -1,12 \checkmark \text{ (calculator)}$$

$$\hat{y} = -1,12x + 11,71 \checkmark$$

- (d) Use your model from question 2(c) to estimate the average number of plastic bags a person who goes shopping in the 6th week of the campaign will request, correct to the nearest whole number. (2)

$$\hat{y} = -1,12(6) \checkmark + 11,71$$

$$\hat{y} \approx 5$$

$$\therefore 5 \text{ shopping bags } \checkmark$$

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QUESTION 3

- (a) Simplify $\sin(180^\circ - x) \cdot \cos(-x) + \cos(90^\circ + x) \cdot \cos(x - 180^\circ)$ to a single trigonometric ratio. (6)

$$\sin(180^\circ - x) \cdot \cos(-x) + \cos(90^\circ + x) \cdot \cos(x - 180^\circ)$$

$$= \sin x \cdot \cos x \checkmark \checkmark + (-\sin x)(-\cos x) \checkmark \checkmark$$

$$= 2\sin x \cos x \checkmark$$

$$= \sin 2x \checkmark$$

- (b) Determine the value of $\sin 3x \cdot \cos y + \cos 3x \cdot \sin y$ if $3x + y = 270^\circ$ (3)

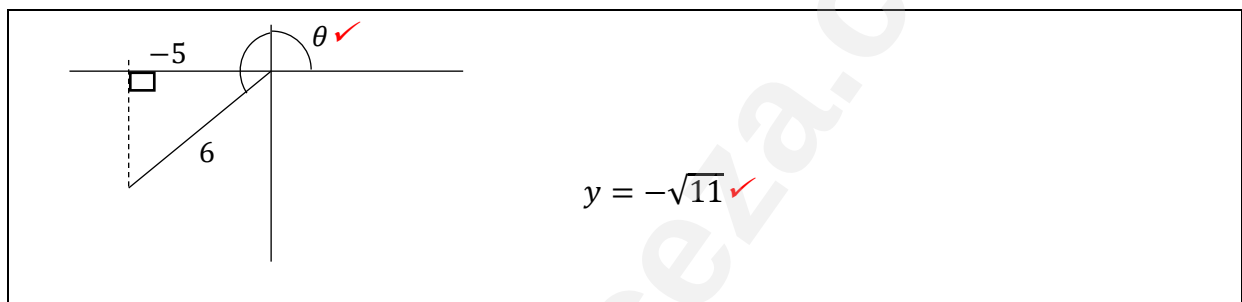
$$\sin 3x \cdot \cos y + \cos 3x \cdot \sin y$$

$$= \sin(3x + y) \checkmark$$

$$= \sin(270^\circ) \checkmark \text{subs}$$

$$= -1 \checkmark$$

- (c) If $\cos \theta = -\frac{5}{6}$, where $\theta \in [180^\circ; 270^\circ]$, calculate, without using a calculator and with the aid of a sketch, the values in simplest form of:



- (1) $\sin \theta$ (3)

$$\sin \theta = -\frac{\sqrt{11}}{6} \checkmark$$

- (2) $\sin 2\theta$ (3)

$$\sin 2\theta = 2\sin \theta \cos \theta \checkmark$$

$$\sin 2\theta = 2\left(-\frac{\sqrt{11}}{6}\right)\left(-\frac{5}{6}\right) \checkmark \text{subs}$$

$$\sin 2\theta = \frac{5\sqrt{11}}{18} \checkmark$$

(d) (1) Show that: $\frac{2\sin x}{2(1-\cos 2x)} = \frac{1}{2\sin x}$ (4)

LHS: $\frac{\sin x}{1-\cos 2x}$ ✓

$= \frac{\sin x}{1 - (1 - 2\sin^2 x)}$ ✓

$= \frac{\sin x}{2\sin^2 x}$ ✓

$= \frac{1}{2\sin x}$ ✓ = RHS

(2) For which values of x is the identity undefined. (3)

$\sin x = 0$

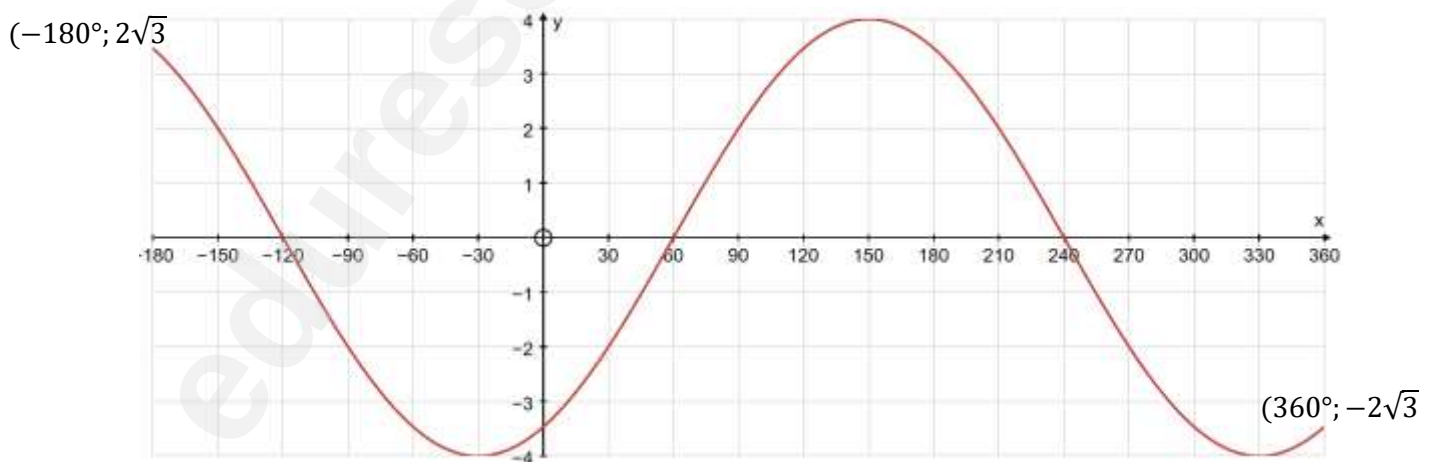
$1 - \cos 2x = 0$

$x = k180^\circ$ ✓

$2x = k360^\circ$ ✓ $\therefore x = k180^\circ$ ✓

(e) Consider: $g(x) = -4\cos(x + 30^\circ)$

(1) Draw a sketch of g on the system of axes below for $x \in [-180^\circ; 360^\circ]$ (6)



✓ ✓ end points ✓ maximum ✓ minimum

✓ ✓ x intercepts

(2) Determine the period of $g(3x)$. (2)

$$\frac{360^\circ}{3} = 120^\circ \checkmark \checkmark$$

(3) Determine the range of $g(x) + 1$ (1)

$$y \in [-3; 5] \checkmark$$

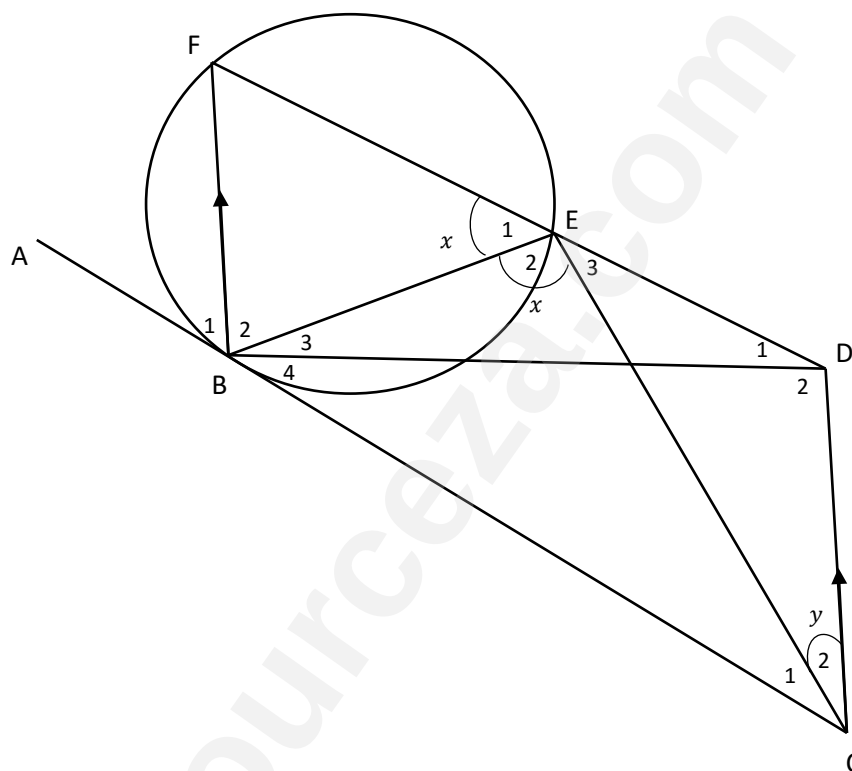
(4) The graph of g is shifted 60° to the right and then reflected about the x -axis to form a new graph h . Determine the equation of h in its simplest form. (2)

$$h(x) = 4 \checkmark \cos(x - 30^\circ) \checkmark$$

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QUESTION 4

- (a) ABC is a tangent to the circle BFE at B. From C a straight line is drawn parallel to BF to meet FE produced at D. EC and BD are drawn. $\hat{E}_1 = \hat{E}_2 = x$ and $\hat{C}_2 = y$.



(1) Give a reason why each of the following is **true**:

- (i) $\hat{B}_1 = x$ (1)

Tan chord theorem ✓

- (ii) $\hat{BCD} = \hat{B}_1$ (1)

corresponding angles $FB \parallel DC$ ✓

- (iii) Hence, show that BCDE is a cyclic quadrilateral. (2)

$$\hat{B}_1 = \hat{E}_1 = \hat{BCD} \quad \checkmark$$

$\therefore$ BCDE cyclic (exterior angle = interior opposite angle) ✓

- (2) Which two angles are each equal to x ? Give reasons for your answer. (2)

$$\hat{D}_2 = x \text{ (< in same seg)} \checkmark$$

$$\hat{B}_2 + \hat{B}_3 = x \text{ (alt <'s } FB \parallel DC) \checkmark$$

- (3) Prove that $\hat{B}_2 = \hat{C}_1$, giving reasons for your answer. (3)

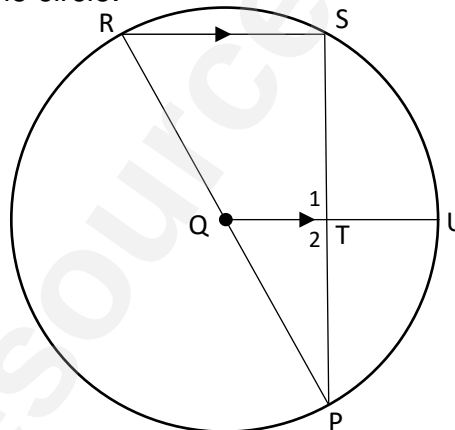
$$\hat{B}_2 + \hat{B}_3 = x \checkmark$$

$$\hat{C}_1 + \hat{C}_2 = x \checkmark$$

$$\text{But } \hat{B}_3 = \hat{C}_2 \text{ (<'s in same segment)} \checkmark$$

$$\therefore \hat{B}_2 = \hat{C}_1$$

- (b) WPR is a diameter of the circle PRSU. QU is drawn parallel to RS and meets SP in T. Q is the centre of the circle.



- (1) Determine the size of $\hat{S}$ giving a reason for your answer. (2)

$$\hat{S} = 90^\circ \checkmark \text{ (< in semi circle)} \checkmark$$

(2) If the diameter is 20 cm and $SP = 16$ cm:

- (i) Calculate the length of QT. Giving reasons for your answer. (5)

$$RS^2 = RP^2 - SP^2 \text{ (pythag) } \checkmark \checkmark$$

$$RS^2 = 400 - 16^2$$

$$RS = 12\text{cm} \checkmark$$

$$QT = \frac{1}{2}RS = 6\text{cm} \checkmark \text{ (midpoint th } \checkmark)$$

- (ii) Hence determine the length of TU. (1)

$$TU = 10 - 6 = 4\text{cm} \checkmark$$

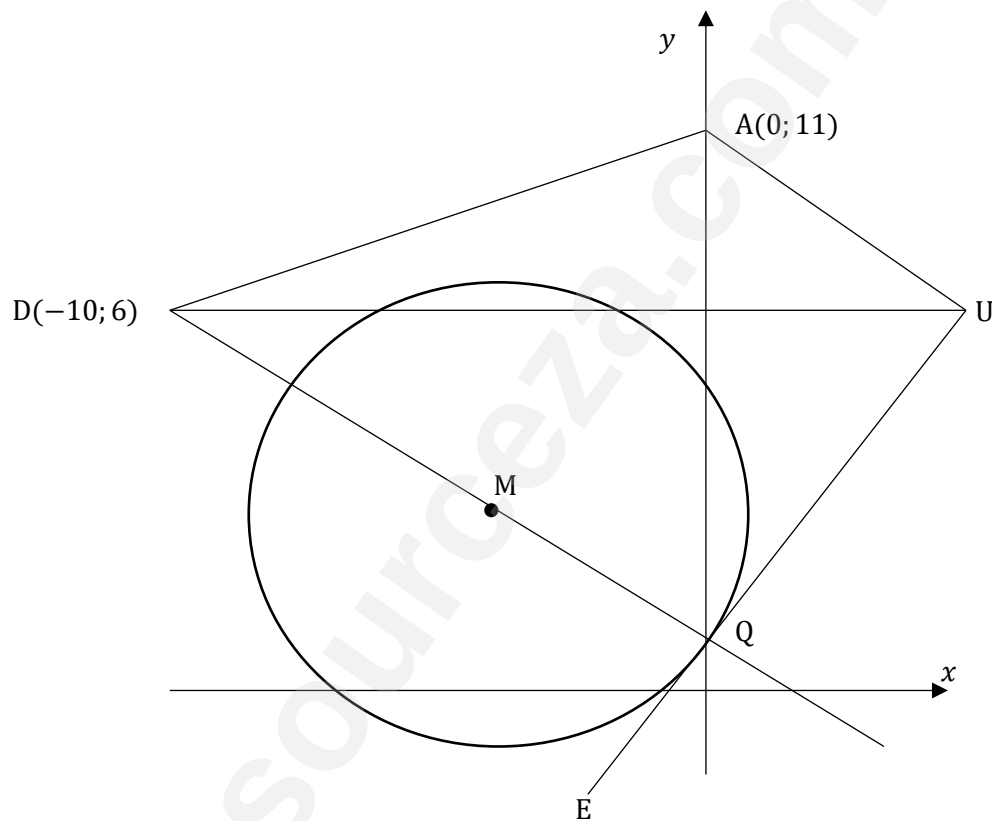
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SECTION B

QUESTION 5

- (a) $x^2 + y^2 + 8x - 6y = -5$, is the equation of the circle with centre M. UE is a tangent to the circle at Q. QMD, DA, AU and UQE are straight lines. DU is parallel to the x-axis.



- (1) Determine the coordinates of M, the centre of the circle. (4)

$$x^2 + y^2 + 8x - 6y = -5$$

$$x^2 + 8x + 16 + y^2 - 6y + 9 = -5 + 16 + 9$$

$$(x + 4)^2 + (y - 3)^2 = 20 \checkmark \checkmark$$

$$M(-4; 3) \checkmark \checkmark$$

(2) Calculate the coordinates of Q, if $y < 2$. (3)

$$(0 + 4)^2 + (y - 3)^2 = 20$$

$$(y - 3)^2 = 4$$

$$y = 3 \pm 2$$

$$\therefore y = 1 \therefore Q(0; 1)$$

(3) Determine the equation of the tangent UE. (4)

$$m_{rad} = \frac{3 - 1}{-4 - 0} = -\frac{1}{2}$$

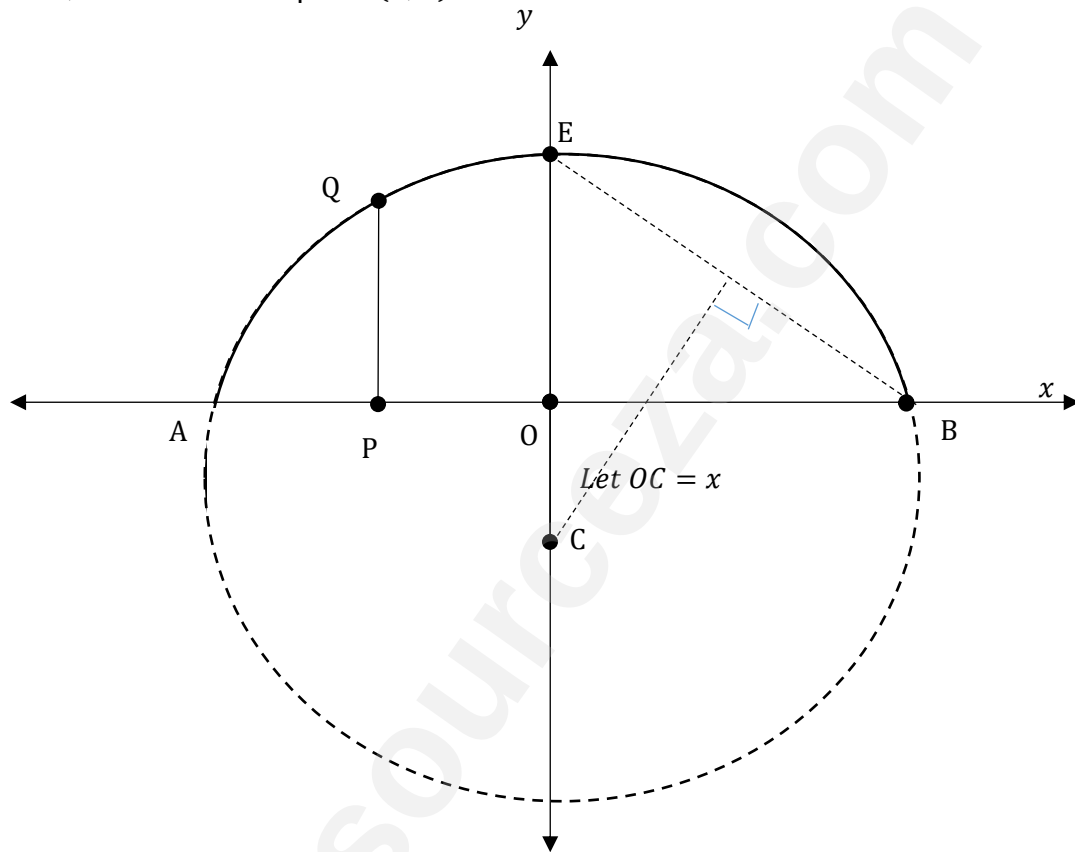
$$m_{tan} = 2 \text{ (tan } \perp \text{ radius)}$$

$$\therefore y - 1 = 2(x - 0)$$

$$y = 2x + 1$$

- (b) The diagram represents a model of a steel arch which is an arc of a circle centre C. The width, AB, of the arch is 16 metres and its maximum height, EO, is 2 metres.

One of the vertical pillars that supports the arch is at point P the midpoint of AO, where O is the point (0; 0).



- (1) Determine the equation of the circle with centre C in the form:

$$x^2 + y^2 + cx + dy + e = 0.$$

(5)

Join chord EB

Midpoint EB(4; 1)

$$m_{EB} = -\frac{2}{8} = -\frac{1}{4}$$

$$\therefore m_{\perp} = 4 \checkmark$$

$$(4; 1): y - 1 = 4(x - 4)$$

$$y = 4x - 15 \checkmark$$

$$\therefore C(0; -15)$$

$$(x - 0)^2 + (y + 15)^2 = r^2 \checkmark$$

Subs (0; 2)

$$(0 - 0)^2 + (2 + 15)^2 = r^2$$

$$\therefore r^2 = 17^2 \checkmark$$

$$\therefore (x - 0)^2 + (y + 15)^2 = 17^2$$

$$x^2 + y^2 + 30y - 64 = 0 \checkmark$$

Or

Construct AC Let $OC = x$

$$EC = x + 2 \text{ (radii)}$$

$OA = 8$ units (line from centre $\perp$ chord AB)

$$(x + 2)^2 = x^2 + 8^2 \text{ pythag}$$

$$x = 15 \text{ units}$$

$$C(0; -15) \text{ radius} = 17 \text{ units} \checkmark \checkmark$$

$$\text{Sub } (x - 0)^2 + (y + 15)^2 = 17^2$$

$$x^2 + y^2 + 30y \checkmark - 64 \checkmark = 0 \dots 0x \text{ implied} \checkmark$$

- (2) Determine the height of the pillar PQ correct to one decimal digit given that Q is a point on the arc. (3)

$$P(-4; 0) \checkmark$$

$$Q(-4; y)$$

$$(-4)^2 + (y + 15)^2 = (17)^2 \checkmark$$

$$y = 1,5 \dots \checkmark$$

$$\text{Pillar} = 1,5 \text{ metres}$$

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QUESTION 6

(a) Given: $2\cos x = 3\tan x$

(1) Show that the equation can be rewritten as $2\sin^2 x + 3\sin x - 2 = 0$ (4)

$$2\cos x = \frac{3\sin x}{\cos x}$$

$$2\cos^2 x = 3\sin x \checkmark$$

$$2(1 - \sin^2 x) = 3\sin x$$

$$0 = 3\sin x - 2(1 - \sin^2 x) \checkmark$$

$$0 = 2\sin^2 x + 3\sin x - 2 \checkmark$$

(2) Hence, determine the general solution of x if $2\cos x = 3\tan x$ (4)

$$0 = 2\sin^2 x + 3\sin x - 2$$

$$0 = (2\sin x - 1)(\sin x + 2) \checkmark$$

$$\sin x = \frac{1}{2} \checkmark \quad \sin x = -2 \text{ NA}$$

$$x = 30^\circ + k360^\circ \quad (k \in \mathbb{Z}) \checkmark$$

$$x = 150^\circ + k360^\circ \checkmark$$

(3) Hence, determine two values of y , $y \in [150^\circ; 180^\circ]$, that are solutions of $2\cos 5y = 3\tan 5y$. (4)

$$5y = 30^\circ + k360^\circ$$

$$y = 6^\circ + k72^\circ \checkmark \quad (k \in \mathbb{Z})$$

$$5y = 150^\circ + k360^\circ$$

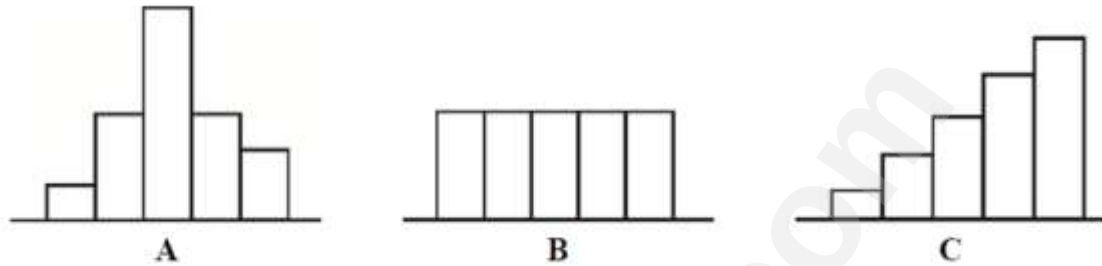
$$y = 30^\circ + k72^\circ \checkmark$$

$$y = \{150^\circ; 174^\circ\} \checkmark \checkmark$$

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QUESTION 7

Study the three histograms given below and answer the questions that follow.



- (a) Which set of data (A, B, or C) has the smallest standard deviation? Explain your answer. (2)

Data set "B" ✓ – all the bars are the same height and therefore no SD ✓

- (b) For data set A the mean is 3,5 and the standard deviation is 1,2. (2)

The values of the data set are doubled.

Determine the mean and standard deviation of the new data set.

Mean – 7 ✓

Standard deviation – 2,4 ✓

- (c) The table below represents data set C. (2)

CLASS INTERVAL	FREQUENCY
$0 \leq x < 2$	a
$2 \leq x < 4$	b
$4 \leq x < 6$	c
$6 \leq x < 8$	d
$8 \leq x < 10$	e

It is given that the mean is p and the standard deviation is q .

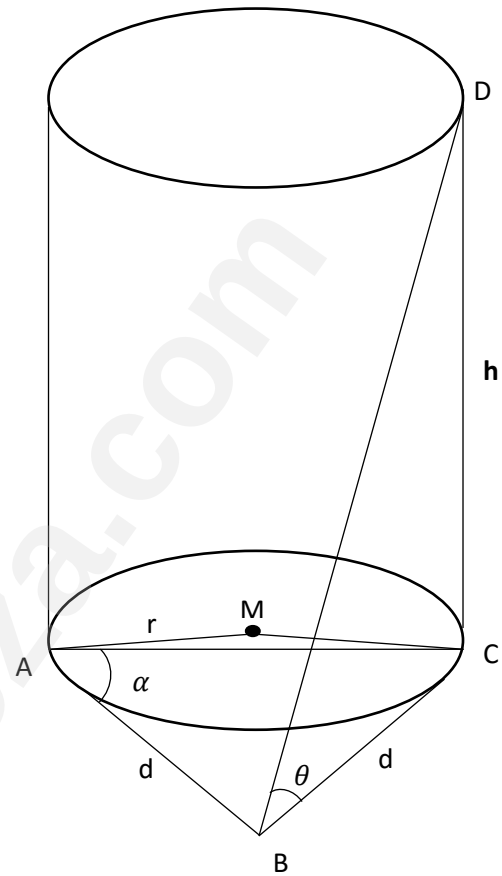
If it is given that the frequency of each class interval is doubled, write down the mean and standard deviation of the new data set in terms of p and q .

Mean – “ p ” ✓

Standard deviation – “ q ” ✓

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QUESTION 8



The diagram, represents one of the right cylindrical silos above. M is the centre of the circular base with BA and BC tangents to the base at A and C. MA, the radius of the circular base is r . A, B and C lie on the same horizontal plane. DC represents the vertical height of the cylindrical part of the silo.

$\widehat{BAC} = \alpha$ and $\widehat{DBC} = \theta$. $DC = h$ and $AB = BC = d$.

(a) **Prove that $AC = d\sqrt{2 + 2\cos 2\alpha}$** (5)

$$\widehat{ACB} = \alpha \text{ (tan from same point) } \checkmark$$

$$\widehat{ABC} = 180^\circ - 2\alpha \text{ (int } < \text{ of } \Delta ABC) \checkmark$$

$$AC^2 = d^2 + d^2 - 2d^2(\cos(180^\circ - 2\alpha)) \checkmark$$

$$AC^2 = d^2 + d^2 + 2d^2(\cos 2\alpha) \checkmark$$

$$AC = d\sqrt{2 + 2\cos 2\alpha} \checkmark$$

(b) **Hence, prove that $\frac{AC \tan \theta}{\sqrt{2 + 2\cos 2\alpha}} = h$** (4)

$$\tan \theta = \frac{h}{d} \checkmark$$

$$d = \frac{AC}{\sqrt{2 + 2\cos 2\alpha}} \checkmark$$

$$\tan \theta = \frac{\frac{h}{AC}}{\frac{AC}{\sqrt{2 + 2\cos 2\alpha}}} \checkmark$$

$$h = \frac{AC \tan \theta}{\sqrt{2 + 2\cos 2\alpha}} \checkmark$$

(c) If $h = 36$ metres, $d = 10,3$ metres and $\alpha = 54^\circ$:

(1) **Prove that $AC = 12,22$ metres.** (2)

$$AC = (10,3)\sqrt{2 + 2\cos(108^\circ)} \checkmark$$

$$AC = 12,22 \text{ metres} \checkmark$$

(2) **Find $\hat{A}BC$ correct to two decimal places.** (2)

$$\hat{B} = 180^\circ - 2(54^\circ) \checkmark$$

$$\hat{B} = 72^\circ \checkmark$$

(3) **If $AMCB$ is a cyclic quadrilateral determine the size of $\hat{M}$.** (1)

$$\hat{M} = 107,23^\circ \checkmark \text{ (opp } \angle\text{s of cyclic quad supplementary)}$$

(4) **Show that $r = 7,59$ metres.** (4)

In $\triangle AMC$

$$\hat{A} = \frac{180^\circ - 108^\circ}{2} = 36^\circ \checkmark$$

$$\frac{r}{\sin 36^\circ} \checkmark = \frac{12,22}{\sin 108^\circ} \checkmark \checkmark$$

$$r = 7,55 \text{ metres}$$

(5) **Find the volume of the silo, correct to the nearest cubic metre.**

(2)

$$V = \pi(7,59)^2 \times 36 \checkmark$$

$$V = 6515\text{m}^3 \checkmark$$

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QUESTION 9

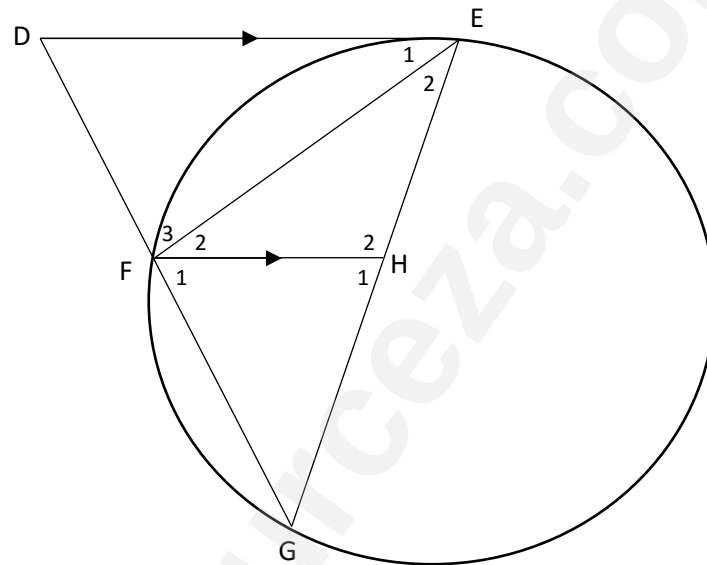
- (a) Complete the theorem statement: (2)

A line drawn parallel to one side of a triangle divides

the other two sides ✓ proportionally ✓.

- (b) In the accompanying diagram DE is a tangent to the circle at E. DFG is a secant.

$DE = EF = FG$; $FH \parallel DE$ and $\hat{E}_1 = \theta$



- (1) State, with reasons, three angles each equal to θ . (3)

$\hat{G} = \theta$ (tan chord th) ✓

$\hat{F}_2 = \theta$ ($DE \parallel FH$; alt $\angle$'s) ✓

$\hat{E}_2 = \theta$ (isos Δ) ✓

(2) Prove that:

$$(i) \quad FE = \frac{GH \cdot DF}{HE} \quad (4)$$

$$\frac{FG}{DF} = \frac{HG}{EH} \text{ (|| lines cut in same proportion) } \checkmark$$

$$FG = \frac{HG \cdot DF}{EH} \checkmark$$

$$\text{But } FG = EF \text{ (given) } \checkmark$$

$$FE = \frac{GH \cdot DF}{HE} \checkmark$$

$$(ii) \quad DE^2 = DF \cdot DG \quad (5)$$

In $\triangle DEF$ and $\triangle DEG$

- $\hat{D}$ is common $\checkmark$
- $\hat{E}_1 = \hat{G} = \theta$ (proven) $\checkmark$
- $\hat{F}_3 = \hat{E}_1 + \hat{E}_2$ (3^{rd} $\angle$ of triangle) $\checkmark$

$\triangle DEF \parallel \triangle DEG$ (AAA)

$$\frac{DE}{DG} = \frac{DF}{DE} \checkmark$$

$$DE^2 = DF \cdot DG \checkmark$$

(3) If $\frac{DF}{DE} = y$, show that $y^2 + y = 1$ (5)

LHS:

$$= \left(\frac{DF}{DE}\right)^2 + \frac{DF}{DE} \checkmark$$

$$= \frac{DF^2 + DF \cdot DE}{DE^2} \checkmark$$

$$= \frac{DF(DF+DE)}{DG \cdot DF} \checkmark \dots \text{denominator from b(2)}$$

$$= \frac{DF + DE}{DG} \checkmark$$

$$= \frac{DF + FG}{DG} \checkmark = \frac{DG}{DG}$$

$$= 1 = \text{RHS}$$

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TOTAL: 150 MARKS