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ST BENEDICT'S

SUBJECT
GRADE
EXAMINER
YOUR
NAME
TEACHER
NAME

Mathematics
12
Ms Brown
<i>MB</i>

PAPER
DATE
MARKS
MODERATOR

DURATION

1
July 2023
150
Gr 12 Educators
3 Hours

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
	Section A		
1	Algebra	26	
2	Sequences	8	
3	Sequences	15	
4	Finance	10	
5	Functions	9	
6	Calculus	11	
		79	
	Section B		
7	Calculus and Functions	23	
8	Probability	13	
9	Calculus	6	
10	Functions	20	
11	Calculus	9	
		71	
TOTAL		150	

Marj: Qn 3 + 4 + 6
= 15 + 10 + 11 = 36 marks

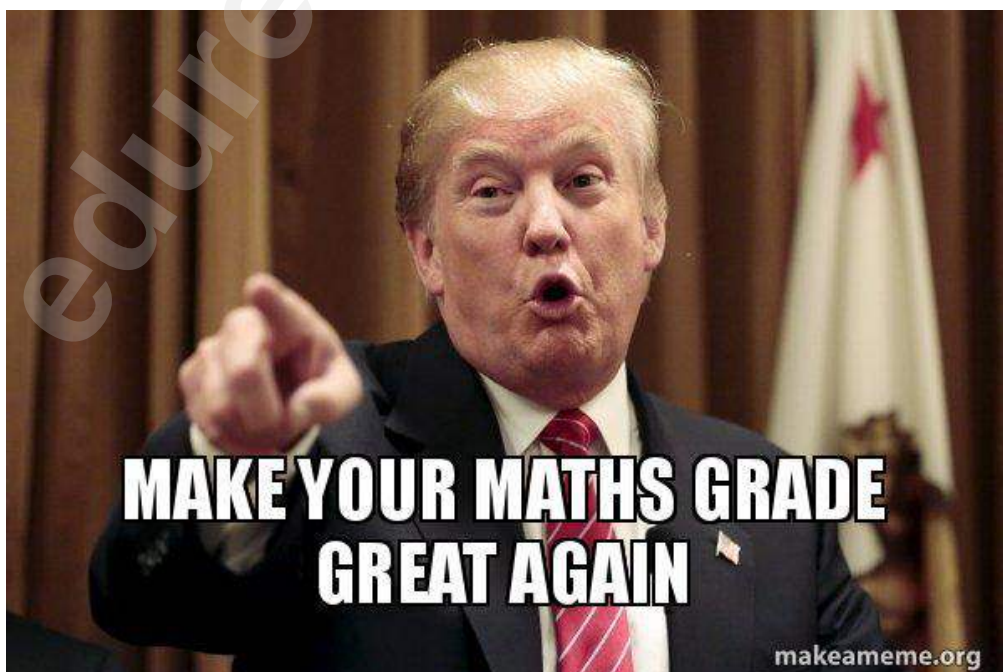
Grant: Qn 7 + 8
= 23 + 13 = 36 marks

Kete: Qn 5 + 10 + 11
= 9 + 20 + 9 = 38 marks

Mads: Qn 1 + 2 + 9
= 26 + 8 + 6 = 40 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This paper consists of 11 questions and 24 pages, and an Information Sheet of 2 pages. Please check that your question paper is complete.
2. The last page can be used for additional working, if necessary. If this space is used, make sure that you indicate clearly which question is being answered.
3. Read the questions carefully.
4. Answer all the questions and answer in the spaces provided on the question paper.
5. Diagrams are not necessarily drawn to scale.
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. Ensure your calculator is in DEGREE mode.
8. All the necessary working details must be clearly shown. Answers only will not necessarily be awarded full marks.
9. It is in your interest to write legibly and to present your work neatly.
10. Round off to **ONE** decimal digit, unless otherwise stated.



QUESTION 1

26 MARKS

a) Solve for x :

1) $3x^2 + 2x + 6 = 10$ (correct to two decimal digits) (4)

$$3x^2 + 2x - 4 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(3)(-4)}}{6}$$

$$x = 0,87$$

$$x = -1,54$$

2) $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 28 = 0$ (4)

$$(x^{\frac{1}{4}} + 7)(x^{\frac{1}{4}} - 4) = 0$$

$$x^{\frac{1}{4}} = -7 \quad \text{or} \quad x^{\frac{1}{4}} = 4$$

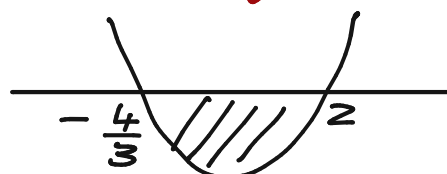
$$N/A$$

$$x = 256$$

3) $x(3x + 4) - 2(3x + 4) \leq 0$ (3)

$$(3x + 4)(x - 2) \leq 0$$

$$-\frac{4}{3} \leq x \leq 2$$



4) $\sqrt{2-x} = x-2$

(5)

$$\begin{aligned}
 2-x &= x^2 - 4x + 4 \quad \checkmark^m \text{ (squaring)} \\
 0 &= x^2 - 3x + 2 \quad \checkmark^A \\
 0 &= (x-2)(x-1) \quad \checkmark^m \text{ (factoring)} \\
 x &= 2 \text{ or } x \neq 1 \quad \checkmark^A \text{ (answers)} \\
 &\text{valid} \quad \checkmark^A \text{ (checking)}
 \end{aligned}$$

b) Solve simultaneously for x and y :

$$2y + x + 3 = 0 \quad \text{and} \quad x^2 + y^2 + 2xy = 1$$

(6)

$$\begin{aligned}
 x &= -3-2y \quad \checkmark^A \\
 (-3-2y)^2 + y^2 + 2(-3-2y)y &= 1 \quad \checkmark^m \\
 9 + 12y + 4y^2 + y^2 - 6y - 4y^2 &= 1 \\
 y^2 + 6y + 8 &= 0 \\
 (y+4)(y+2) &= 0 \\
 y &= -4 \quad \checkmark^A \quad \text{or} \quad y = -2. \quad \checkmark^A \\
 x &= 5 \quad \checkmark^A \quad \quad x = 1 \quad \checkmark^A
 \end{aligned}$$

c) Given: $3x^2 + kx - 3x - k = 0$

For which values of k will the equation have real roots?

(4)

$$\Delta \geq 0 \quad \checkmark \text{ A}$$

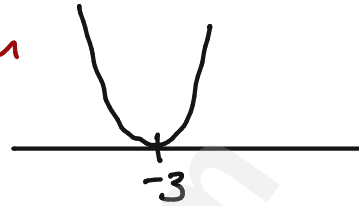
$$(k-3)^2 - 4(3)(-k) \geq 0 \quad \checkmark \text{ M}$$

$$k^2 - 6k + 9 + 12k \geq 0$$

$$k^2 + 6k + 9 \geq 0$$

$$(k+3)^2 \geq 0 \quad \checkmark \text{ A.}$$

$$k \in \mathbb{R} \quad \checkmark \text{ CA}$$



QUESTION 2

8 MARKS

The first five terms of an arithmetic progression are given as:

$$10 ; a ; 24 ; b ; 38 ; \dots$$

a) Show that $a = 17$ and $b = 31$.

(2)

$$24 - a = a - 10 \quad \checkmark \text{ A}$$

$$34 = 2a$$

$$a = 17$$

$$b - 24 = 38 - b \quad \checkmark \text{ A.}$$

$$2b = 62$$

$$b = 31.$$

b) Calculate the sum of the first 67 terms of the sequence.

(2)

$$S_{67} = \frac{67}{2} [2(10) + 66(7)] \quad \checkmark \text{ M}$$

$$= 16147 \quad \checkmark \text{ A}$$

- c) Given that there are 67 terms in the sequence, determine the sum of all the terms in the even positions of this sequence. (4)

$$T_2 + T_4 + T_6 \dots \dots \dots + T_{66}$$

$$17 + 31 + \dots \dots \dots \checkmark \text{A.}$$

$$a = 17 \quad d = 14 \quad n = 33$$

$$S_{33} = \frac{33}{2} [2(17) + 32(14)] \checkmark \text{A.} \checkmark \text{m}$$

$$= 7953 \checkmark \text{A.}$$

QUESTION 3

15 MARKS

- a) A convergent geometric series is given by: $\frac{5(x+1)}{2} + \frac{5(x+1)^2}{2} + \frac{5(x+1)^3}{2} + \dots$
- 1) Calculate the possible value(s) of x , showing all working. (2)

$$-1 < r < 1$$

$$-1 < x+1 < 1 \checkmark \text{m}$$

$$-2 < x < 0 \checkmark \text{A.}$$

- 2) If $x = -1,5$, calculate the sum of infinity (S_{∞}). (3)

$$r = -1,5 + 1 = -0,5 \quad a = \frac{5}{2} \left(-\frac{1}{2}\right) = -\frac{5}{4} \checkmark \text{A.}$$

$$\therefore S_{\infty} = -\frac{5}{4} \div \left(1 + \frac{1}{2}\right) \checkmark \text{m}$$

$$\therefore -\frac{5}{6} \checkmark \text{A.}$$

- b) Expand and calculate, showing all working, the value of:

(3)

$$\sum_{r=4}^6 \frac{2^r}{r}$$

$$\frac{2^4}{4} + \frac{2^5}{5} + \frac{2^6}{6} \quad \checkmark \text{ M A}$$

$$= 4 + \frac{32}{5} + \frac{32}{3}$$

$$= \frac{316}{15} \quad \checkmark \text{ CA}$$

- c) The first four terms of a quadratic pattern are: 11 ; 20 ; 33 ; 50 ;

- 1) Determine the general term of this pattern in the form:

$$T_n = an^2 + bn + c.$$

(4)

$$\begin{array}{ccccccc} 11 & & 20 & & 33 & & 50 \\ & \underbrace{\hspace{1cm}} & & \underbrace{\hspace{1cm}} & & \underbrace{\hspace{1cm}} & \\ & 9 & & 13 & & 17 & \\ & & \underbrace{\hspace{1cm}} & & \underbrace{\hspace{1cm}} & & \\ & & 4 & & 4 & & \end{array} \quad \checkmark \text{ M}$$

$$2a = 4$$

$$a = 2 \quad \checkmark \text{ A}$$

$$6 + b = 9$$

$$b = 3 \quad \checkmark \text{ CA}$$

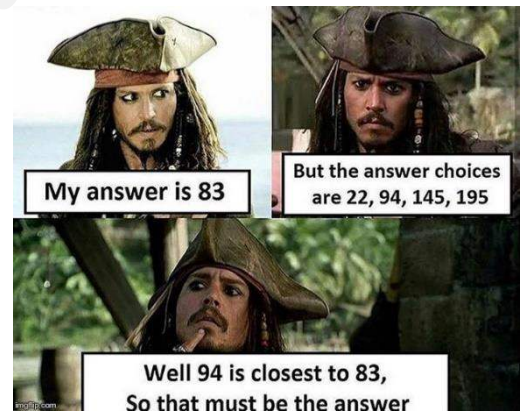
$$2 + 3 + c = 11$$

$$c = 6 \quad \checkmark \text{ CA}$$

$$\therefore T_n = 2n^2 + 3n + 6$$

- 2) Prove that the sum of the first n first-differences of this quadratic pattern can be given by $S_n = 2n^2 + 7n$. (3)

$$\begin{aligned}
 & 9 + 13 + 17 \dots \quad a = 9 \\
 \therefore S_n &= \frac{n}{2} [2(a) + (n-1)(d)] \quad \checkmark \quad d = 4 \quad \checkmark \quad A. \\
 &= \frac{n}{2} (18 + 4n - 4) \\
 &= \frac{n}{2} (4n + 14) \quad \checkmark \quad P. \\
 &= 2n^2 + 7n.
 \end{aligned}$$



QUESTION 4

10 MARKS

- a) Exactly six years ago, Lesego bought a new car which is worth one third of its original price now. If the car depreciated by a fixed annual rate according to the reducing-balance method, calculate the annual rate of depreciation. (4)

$$\begin{aligned}
 n &= 6 \\
 \frac{1}{3}x &= x(1-i)^6 \quad \checkmark \quad A. \quad \checkmark \quad \text{formula} \\
 \frac{1}{3} &= (1-i)^6 \\
 \sqrt[6]{\frac{1}{3}} &= 1-i \quad \checkmark \quad A. \\
 -0,167\dots &= -i \quad \checkmark \quad A. \\
 i &= 16,7\%
 \end{aligned}$$

- b) Sekwala needs to save money to buy a cellphone at the end of 7 years. He invests R3 000 in a bank account. Two years later, his mother gives him R4 000 to add to his investment. The interest rate for the first 3 years is 7% per annum compounded quarterly, but changes to 12% per annum compounded every 6 months. Will he have enough money at the end of the 7-year period to buy a cellphone worth R15 000? Show all your calculations, using a timeline to assist you. (6)

Timeline:

```

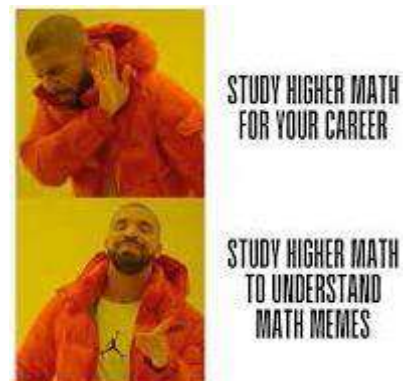
    0          2          3          7
    |-----|-----|-----|
    | 7% m=4  |-----| 12% m=2  |
    
```

3000 + 4000

$$A = 3000 \left(1 + \frac{0,07}{4}\right)^{3 \times 4} \left(1 + \frac{0,12}{2}\right)^{4 \times 2} + 4000 \left(1 + \frac{0,07}{4}\right)^{1 \times 4} \left(1 + \frac{0,12}{2}\right)^{4 \times 2}$$

$$= 12721,70$$

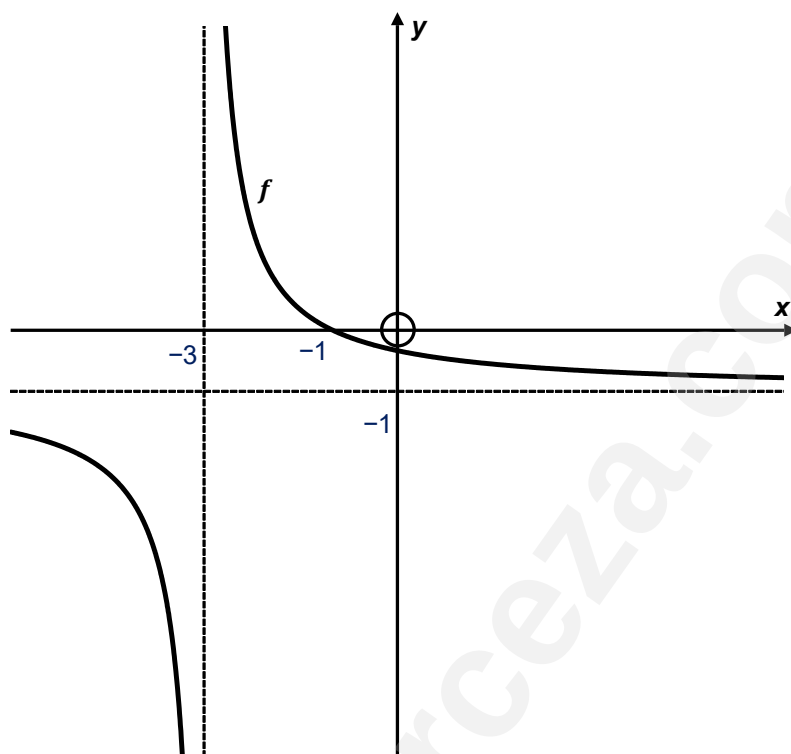
$\therefore$ No, he will not have enough money.



QUESTION 5

9 MARKS

The graph of $f(x) = \frac{2}{x-p} + q$ is sketched below.



- a) Write down the values of p and q . (2)

$$p = -3 \quad \checkmark \quad \text{A.}$$

$$q = -1 \quad \checkmark \quad \text{A.}$$

- b) The equation $g(x) = -x + k$ is one of the axes of symmetry of the graph of f . Determine the value of k . (2)

$$y = \frac{2}{x+3} - 1 \quad \checkmark \quad \text{M}$$

$$g(x) = -x - 4$$

$$\therefore k = -4 \quad \checkmark \quad \text{A.}$$

c) Simplify the following:

$$g^{-1}(3) + g'(x) + \left[\frac{1}{g(-6)} \right]^2 \quad (5)$$

$$g(x) = -x - 4$$

$$\therefore x = -y - 4$$

$$-(x+4) = y$$

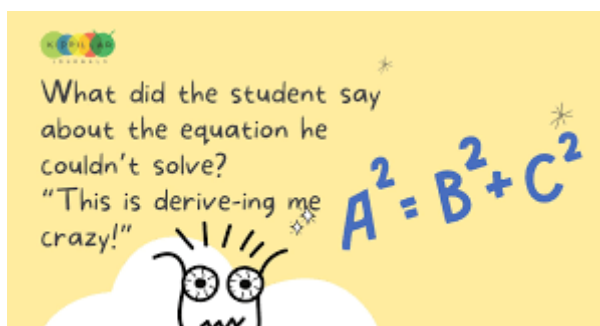
$$\therefore g^{-1}(3) = -7. \quad \checkmark$$

$$g'(x) = -1 \quad \checkmark$$

$$\left(\frac{1}{g(-6)} \right) = \left(\frac{1}{6-4} \right)^2 = \left(\frac{1}{2} \right)^2 = \frac{1}{4} \quad \checkmark$$

$$\therefore -7 - 1 + \frac{1}{4} \quad \checkmark$$

$$= -7\frac{3}{4} \quad \checkmark$$



QUESTION 6

11 MARKS

a) Given: $f(x) = 2x - x^2$

Determine f' from FIRST PRINCIPLES.

(5)

$$f(x+h) = 2(x+h) - (x+h)^2 = 2x + 2h - x^2 - 2xh - h^2$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2x+2h-x^2-2xh-h^2 - (2x-x^2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2 - 2x - h)}{h}$$

$$= 2 - 2x.$$

b) Determine, leaving your answers with positive exponents:

1) $\frac{dy}{dx}$ if $xy = 3$.

(2)

$$y = \frac{3}{x}$$

$$y = 3x^{-1}$$

$$\therefore \frac{dy}{dx} = -3x^{-2}$$

$$= -\frac{3}{x^2}$$

$$2) \quad D_x \left[\frac{\sqrt[3]{x^2 - 2x + 1}}{x^2} \right] \quad (4)$$

$$= D_x \left[\frac{x^{\frac{2}{3}}}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2} \right] \quad \checkmark^A$$

$$= D_x \left[x^{-\frac{4}{3}} - 2x^{-1} + x^{-2} \right] \quad \checkmark^A$$

$$= -\frac{4}{3}x^{-\frac{7}{3}} + 2x^{-2} - 2x^{-3} \quad \checkmark^A$$

$$= -\frac{4}{3x^{\frac{7}{3}}} + \frac{2}{x^2} - \frac{2}{x^3} \quad \checkmark^A$$

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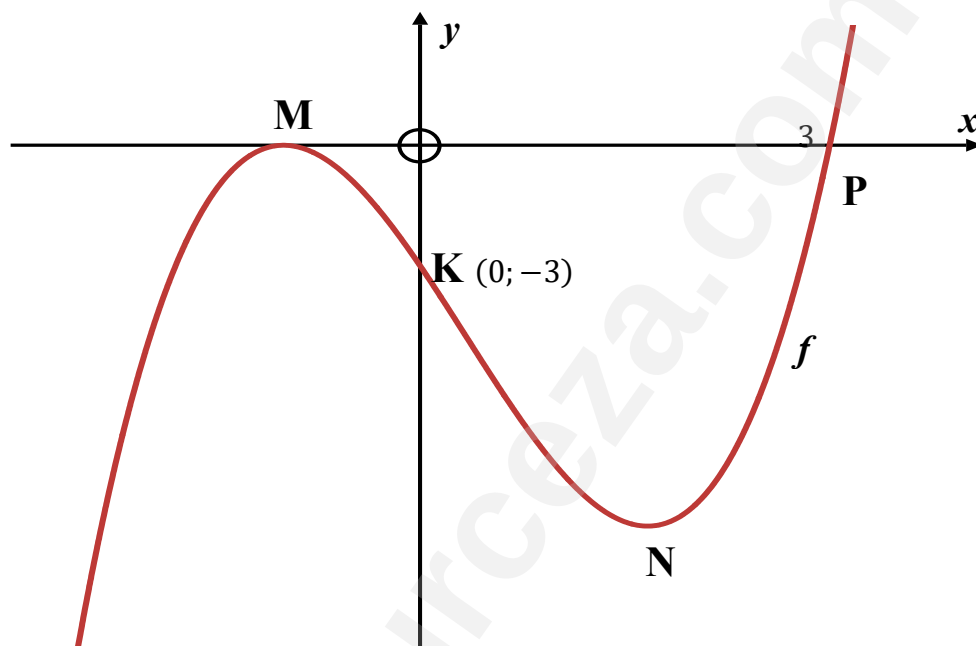
QUESTION 7

23 MARKS

Sketched below is the graph of $f(x) = x^3 + ax^2 + bx + c$.

The x -intercepts of f are at $P(3; 0)$ and M . $K(0; -3)$ is the y -intercept of f .

M and N are the turning points of f .



- a) Show that the equation of f is given by $f(x) = x^3 - x^2 - 5x - 3$

$y = 1(x-3)(x-x_1)^2$ (x - root method) $\rightarrow (\)^2$

$= 1(x-3)(x-x_1)^2$

$-3 = (-3)(-x_1)^2$ m (sub (0|-3))

$x_1^2 = 1$

$\therefore x_1 = \pm 1$

$x_1 = -1$ CA

$\therefore y = (x-3)(x+1)^2$

$\therefore y = (x-3)(x^2 + 2x + 1)$ m multiplying out

$= x^3 + 2x^2 + x - 3x^2 - 6x - 3$

$= x^3 - x^2 - 5x - 3$

b) Calculate the coordinates of N .

(5)

$$f'(x) = 3x^2 - 2x - 5 \quad \checkmark \text{A.}$$

$$0 = (3x - 5)(x + 1) \quad \checkmark \text{M (=0)}$$

$$x = \frac{5}{3} \quad \text{or} \quad x = -1 \quad \checkmark \text{CA.}$$

$$f\left(\frac{5}{3}\right) = -\frac{256}{27} \quad \checkmark \text{CA.}$$

$$\left(\frac{5}{3}, -\frac{256}{27}\right) \quad \checkmark \text{CA.}$$

c) Determine the equation of the tangent to f at P .

(4)

$$f'(3) = 3(3)^2 - 2(3) - 5 \quad \checkmark \text{M}$$

$$= 16 \quad \checkmark \text{CA.}$$

$$\therefore y = 16x + c$$

$$0 = 16(3) + c \quad \checkmark \text{sub (3;0)}$$

$$c = -48 \quad \checkmark \text{CA.}$$

$$y = 16x - 48$$

A second copy of the graph has been provided for you at the bottom of the page.

d) For which values of x will:

1) $f(x) < 0$.

(2)

$$x < 3 \quad ; \quad x \neq -1$$

2) f be concave up.

(2)

$$f''(x) > 0$$

$$6x - 2 > 0$$

$$x > \frac{1}{3}$$

3) f' be decreasing.

(2)

$$x \in (-\infty; \frac{1}{3}) \quad \text{or} \quad x < \frac{1}{3}$$

e) If $g(x) = (x+2)^3 - (x+2)^2 - 5(x+2) - 3$

Determine the

1) new coordinate of M .

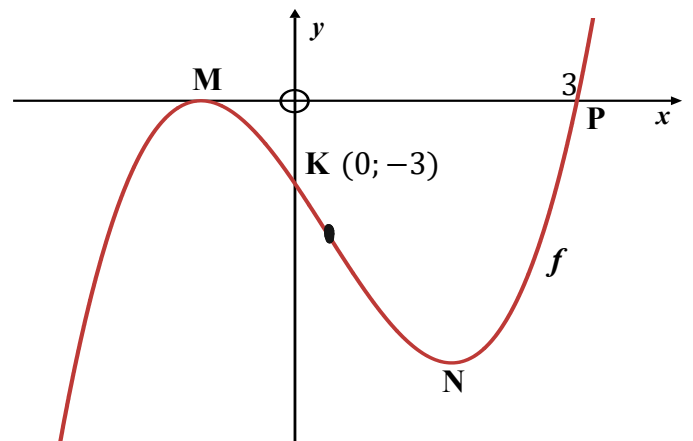
(1)

$$M(-1; 0) \rightarrow \text{moved 2 left} \rightarrow (-3; 0)$$

2) values of k , such that $g(x) = k$ has 3 distinct roots.

(2)

$$-\frac{256}{27} < k < 0$$



QUESTION 8

13 MARKS

- a) A and B are two independent events.

$$P(A) = 0,4 \text{ and } P(A \text{ or } B) = 0,88.$$

Calculate $P(B)$.

(3)

$$\begin{aligned}
 P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\
 0,88 &= 0,4 + P(B) - P(A \text{ and } B) \\
 0,88 &= 0,4 + P(B) - 0,4 \times P(B) \\
 0,48 &= P(B) (0,6) \\
 P(B) &= 0,8
 \end{aligned}$$

- b) Flags from four African and three European countries were displayed in a row during the 2021 Olympics.

Determine:

- 1) the total number of possible ways all 7 flags from these countries could be displayed.

(2)

$$\begin{aligned}
 &7! \\
 &= 5040
 \end{aligned}$$

- 2) the probability that the flags from the African countries were displayed next to each other.

(3)

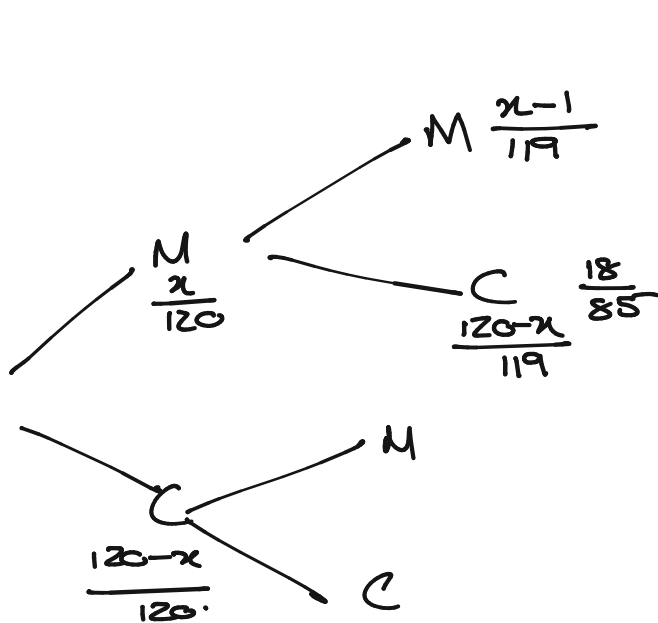
$$\begin{aligned}
 &\frac{4! 4!}{7!} \\
 &= 0,11
 \end{aligned}$$



- c) There are 120 passengers on board an aeroplane. Passengers have a choice between a meat sandwich or a cheese sandwich, but more passengers will choose a meat sandwich. There are only 120 sandwiches available to choose from. The probability that the first passenger chooses a meat sandwich and the second passenger chooses a cheese sandwich is $\frac{18}{85}$.



(5)



(+tree) ✓

$$P(M \cap C) = \frac{18}{85}$$

$$\frac{x}{120} \times \frac{120-x}{119} = \frac{18}{85}$$

$$\frac{120x - x^2}{14280} = \frac{18}{85}$$

$$120x - x^2 = 3024$$

$$0 = x^2 - 120x + 3024$$

$$x = \frac{120 \pm \sqrt{(-120)^2 - 4(3024)}}{2}$$

$$x = 84 \quad \text{or} \quad x = 36$$

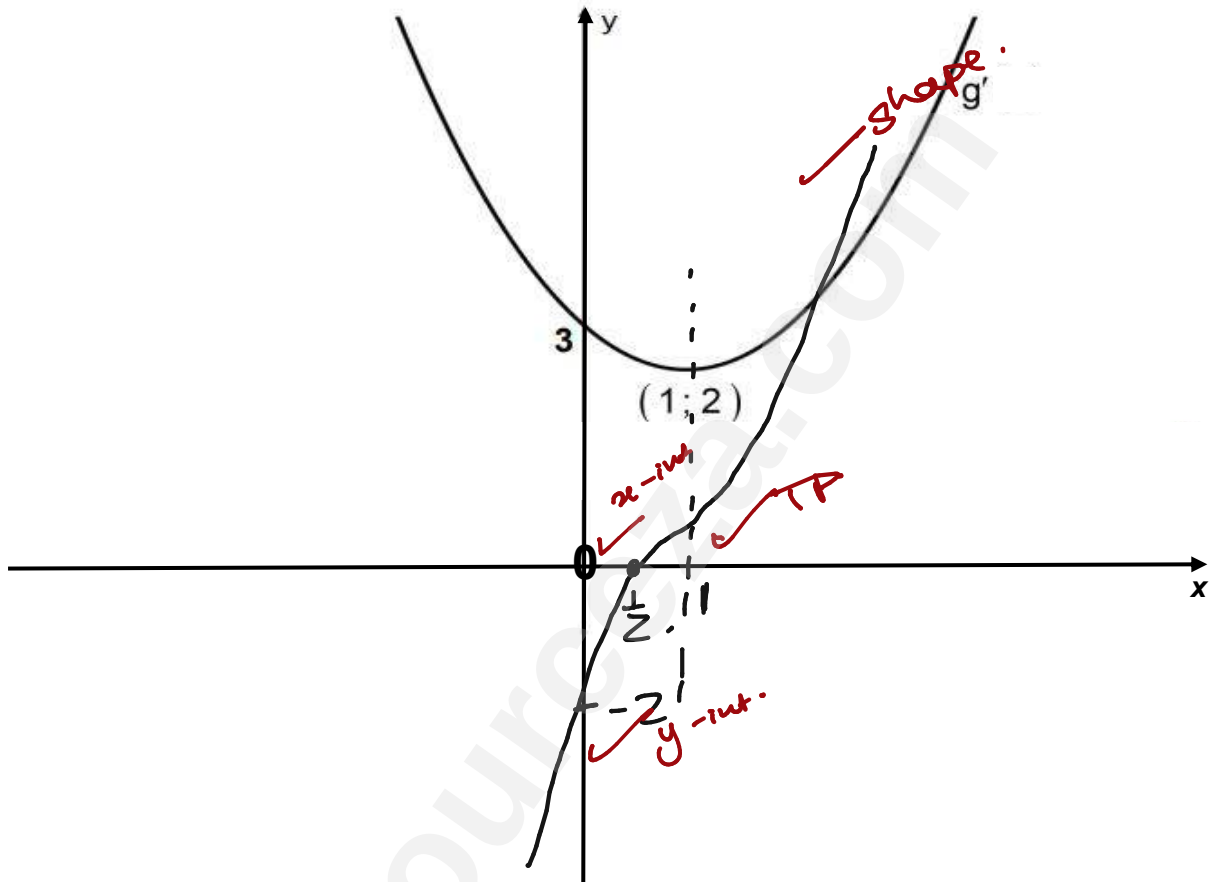
$$\therefore x = 84$$

$\therefore$ the probability that the first person chooses a cheese sandwich is $\frac{36}{120} = \frac{3}{10}$

QUESTION 9

6 MARKS

The sketch shows the graph of $y = g'(x)$. The turning point of g' is $(1; 2)$ and the y -intercept is $(0; 3)$.



- a) For which values of x is g increasing? (1)

$$x \in \mathbb{R}. \checkmark$$

- b) What is the gradient of the tangent to g at the point where $x = 1$? (1)

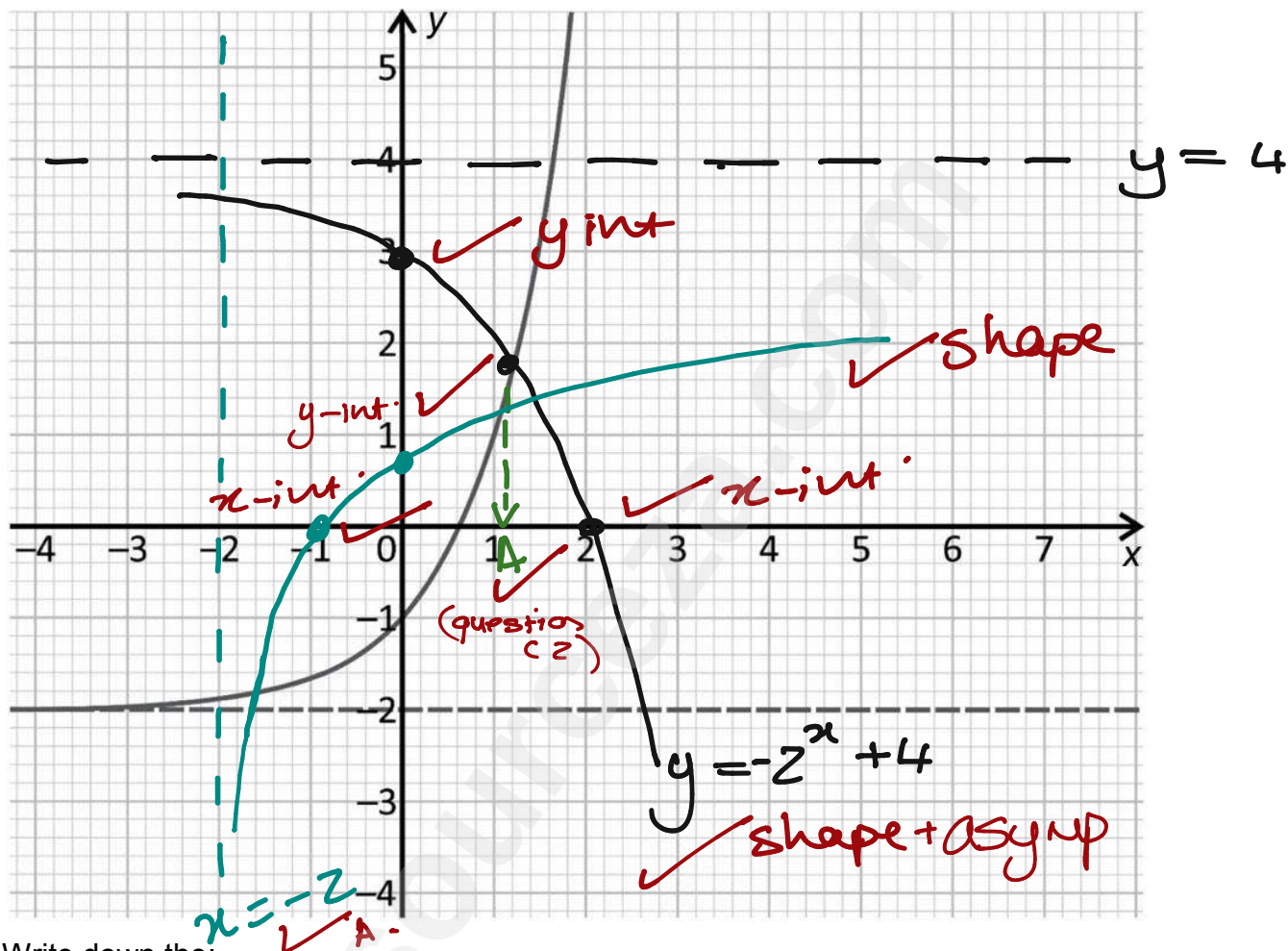
$$2 \checkmark$$

- c) It is given that $g(0) = -2$ and $g\left(\frac{1}{2}\right) = 0$.

Sketch the graph of g on the same set of axes as above. Label all intercepts, stationary points and points of inflection, if any. (4)

on graph.

The graph of $f(x) = 3^x - 2$ has been drawn on the set of axes below.



a) Write down the:

- 1) domain of f . (1)

$x \in \mathbb{R}$ ✓ ~~A~~

- 2) range of f . (1)

$$y > -2 \quad \checkmark \rightarrow A-$$

- 3) equation of the graph that is a reflection of f in the x -axis. (1)

$$y = -3^x + 2 \quad \checkmark \text{ A.}$$

- b) Determine the values of x for which $f(x) \geq 0$. (3)

$$3^x - 2 = 0 \quad \checkmark^m$$

$$3^x = 2$$

$$x = 0,63 \quad \checkmark^A$$

$$\therefore x > 0,63 \quad \checkmark^{CA}$$

- c) 1) On the same set of axes on the page before sketch g , if $g(x) = -2^x + 4$. (3)

$$y\text{-int} : (0, 3) \quad \checkmark^A$$

$$x\text{-int} : 2^x = 4$$

$$x = 2 \quad (2, 0) \quad \checkmark^A$$

- 2) Show on your graph where you would read off the solution to $3^x + 2^x = 6$. Indicate this with letter A. (2)

$$3^x - 2 = -2^x + 4 \quad \checkmark^m \text{ (understanding its an intercept)}$$

- d) Determine the equation of:

- 1) h if h is a reflection of g about the y -axis. (1)

$$g(-x) = -2^{-x} + 4$$

$$\therefore h(x) = -\frac{1}{2^x} + 4$$

- 2) h^{-1} . $x = -\frac{1}{2^y} + 4$

$$x - 4 = -\frac{1}{2^y}$$

$$-x + 4 = 2^{-y}$$

$$-y = \log_2(4 - x)^{(2)}$$

$$y = -\log_2(4 - x)$$

- e) Draw the graph of f^{-1} on the axes on the previous page and hence determine for which values of x will $0 \leq \log_3(x + 2) \leq 2$. (6)

$$x = 3^y - 2$$

$$x + 2 = 3^y$$

$$y = \log_3(x + 2)$$

$$x\text{-int} (-1, 0) \quad \checkmark^A$$

$$y\text{-int} (0, 0,63) \quad \checkmark^A$$

$$y = 3^0 - 2 = -1$$

$$y = 3^2 - 2 = 7$$

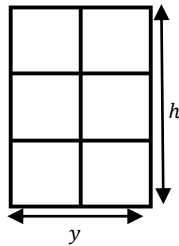
shape asymptote

$$-1 \leq x \leq 7$$

QUESTION 11

9 MARKS

A cottage pane window is illustrated below where the window frame has dimensions y metres by h metres. The frame consists of six smaller congruent sections.



- a) The entire frame is made with 12 metres of wood.

Show that $y = \frac{1}{4}(12 - 3h)$. (2)

$$4y + 3h = 12$$

$$y = \frac{(12 - 3h)}{4}$$

- b) Show that the Area of the window is given by: $A = 3h - \frac{3}{4}h^2$ (3)

$$A = y \times h$$

$$= \left(\frac{12 - 3h}{4} \right) h$$

$$= \frac{12h}{4} - \frac{3h^2}{4} = 3h - \frac{3h^2}{4}$$

- c) Find the dimensions of y and h of the frame so that the area of the frame is a maximum. (4)

$$\frac{dA}{dh} = 3 - \frac{6}{4}h$$

$$0 = 3 - \frac{3}{2}h$$

$$6 = 3h$$

$$h = 2$$

$$\therefore y = \frac{3}{2}$$

EXTRA SPACE

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