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ST BENEDICT'S

SUBJECT	Mathematics	PAPER	1
GRADE	12	DATE	16 September 2021
EXAMINER	Ms Brown	MARKS	150
TEACHER		MODERATOR	Gr 12 Educators
NAME		DURATION	3 Hours

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
	Section A		
1	Algebra	20	
2	Sequences	16	
3	Finance	15	
4	Functions	12	
5	Differentiation	13	
		76	
	Section B		
6	Functions	8	
7	Probability	15	
8	Functions and Calculus	22	
9	Calculus application	13	
10	Sequences	8	
11	Calculus application	8	
		74	
TOTAL		150	

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This paper consists of 11 questions and 22 pages, and an Information Sheet of 2 pages. Please check that your question paper is complete.
2. The last page can be used for additional working, if necessary. If this space is used, make sure that you indicate clearly which question is being answered.
3. Read the questions carefully.
4. Answer all the questions on the question paper.
5. Diagrams are not necessarily drawn to scale.
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. Ensure your calculator is in **DEGREE** mode.
8. All the necessary working details must be clearly shown. Answers only will not necessarily be awarded full marks.
9. It is in your interest to write legibly and to present your work neatly.
10. Round off to **TWO** decimal digits, unless otherwise stated.



QUESTION 1

20 MARKS

a) Solve for the unknown variable:

1) i) $2k^2 = k + 6$ (3)

ii) Hence, or otherwise, solve for x :
 $2(x^2 - x)^2 = x^2 - x + 6$ (3)

2) $x - 3 - \sqrt{x - 1} = 0$ (4)

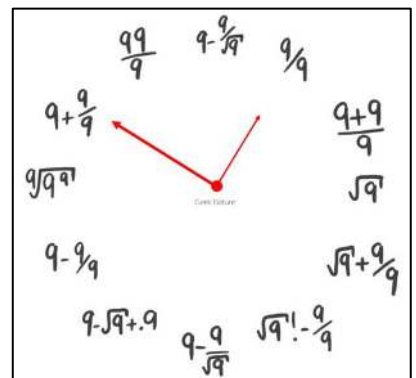
$$3) \quad \frac{35^x}{7^x} = 15 \quad (3)$$

b) Fully simplify:

$$\frac{18^{x+1} \cdot \sqrt{25^x}}{15^{x+1} \cdot 6^x} \quad (3)$$

c) Given: $\frac{14}{\sqrt{63}-\sqrt{28}} = a\sqrt{b}$

Without using a calculator, determine the values of a and b . (4)



QUESTION 2

16 MARKS

a) Consider the following sequence: 2 ; 5 ; 8 ; 11 ;

1) Find the value of the 10th term of the sequence. (2)

2) Calculate the sum of the first 50 terms of the sequence. (3)

b) A geometric series is given as: $\left(-\frac{3}{4}\right) + m + \left(-\frac{1}{3}\right) + \frac{2}{9} + \dots$

Determine the values of m , and show that the series converges. (5)

- c) Calculate the value of n :

(6)

$$\sum_{r=1}^n (5r - 2) = 255$$

QUESTION 3

15 MARKS

Ayanda wants to buy a new car that costs R225 000. He has not saved up enough money to pay a deposit and will need to take out a bank loan for the whole amount. The bank agrees to give Ayanda an interest rate of 9,6 % per annum compounded monthly over 6 years.

- a) Calculate what Ayanda's monthly instalments will be?

(4)

- b) If Ayanda decides to pay off his loan by making monthly payments of R5 000 starting in one month's time, how many full payments of R5 000 does he make to finish paying off his loan? (5)

- c) After considering all this, Ayanda decides to rather save his money until he has enough money to buy the car cash.

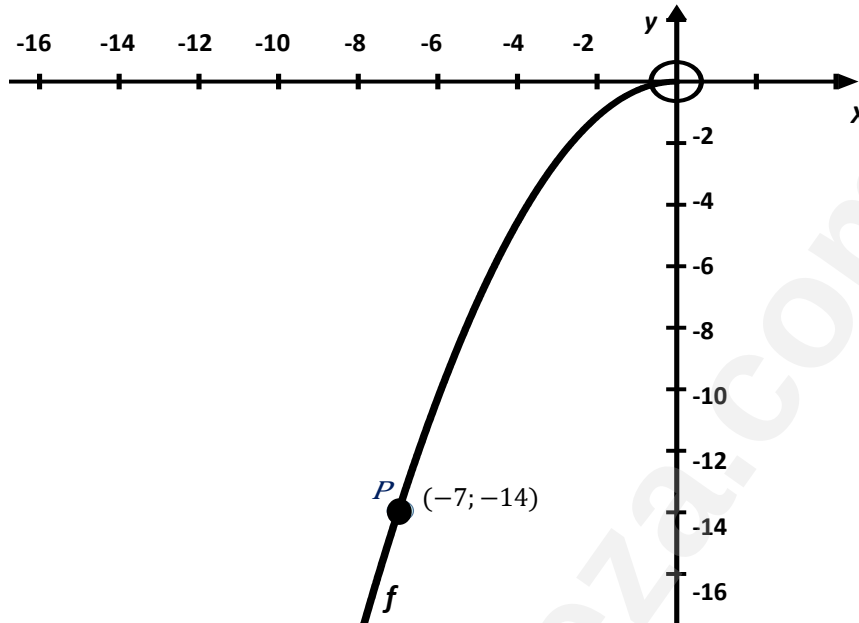


- 1) The current inflation rate is 4,6%p.a.. Calculate the price of the car in four year's time, if the rate of inflation stays the same for the next four years. (3)
- 2) An investment company offers Ayanda a savings plan of 10,4% per annum compounded quarterly. How much should Ayanda's quarterly deposits into the savings plan be if he would like to buy the car in four years' time? His first payment will be made in 3 months time. (3)

QUESTION 4

12 MARKS

- a) In the diagram below, the graph of $f(x) = ax^2$ is drawn in the interval $x < 0$. $P(-7; -14)$ is a point of f . Answer the questions that follow.

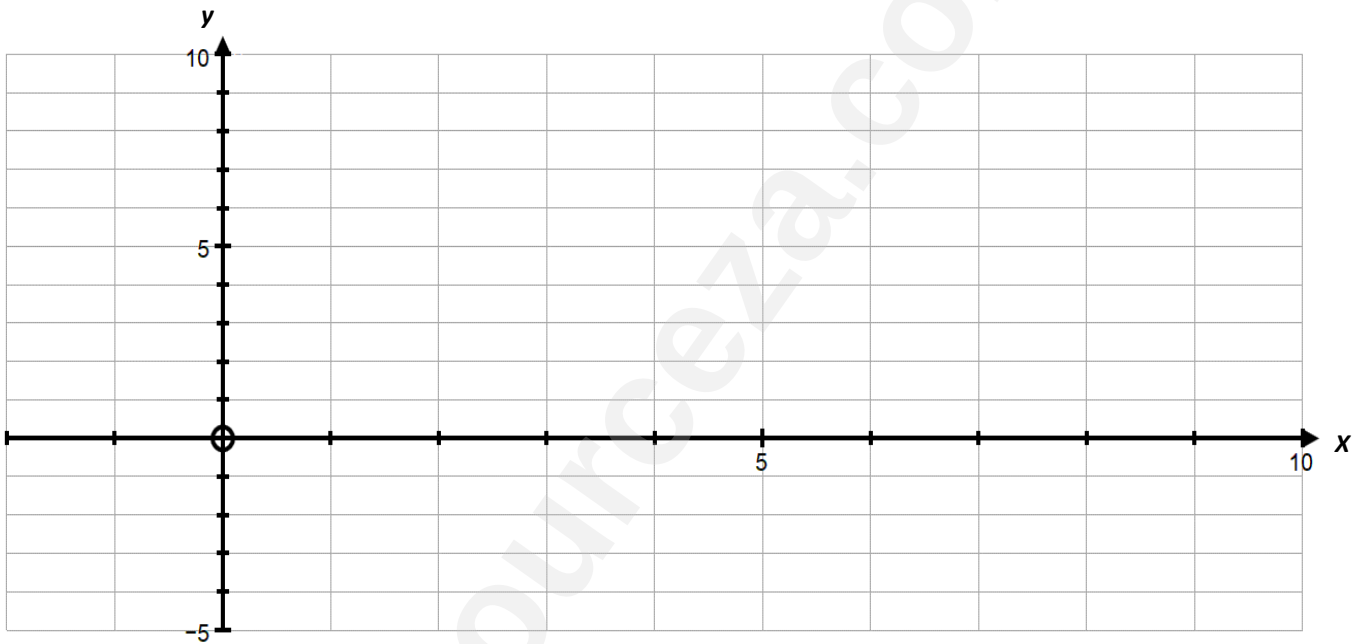


- 1) Calculate the value of a . (2)
- 2) If R is the reflection of P in the line $y = x$, write down the coordinates of R . (1)
- 3) Sketch the graph of f^{-1} on the same set of axes as f above. (2)
- 4) Write down the equation of f^{-1} in the form $y = \dots\dots$ (3)

b) A cubic function has the following properties:

- $g(0) = 8$
- $g(4) = g(1) = 0$
- $g'(3) = g'(1) = 0$
- $g(3) = 8$

- 1) Sketch of the graph of g on the grid below, clearly indicating the coordinates of the turning point(s) and the points of intersections of the graph with the axes. (3)



- 2) Determine the value(s) of x for which graph of g is concave down. (1)

QUESTION 5

13 MARKS

- a) Given: $f(x) = 1 - 3x^2$
Determine $f'(x)$ from first principles.

(5)

- b) Determine, leaving your answers with **positive exponents**:

1) $D_x[\sqrt[3]{x^2} + \pi + 2x]$

(4)

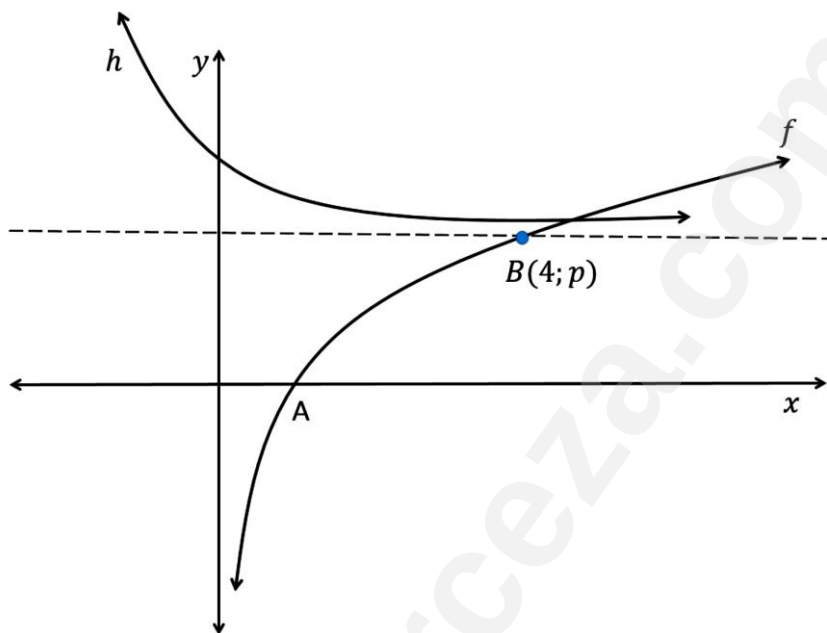
2) $\frac{dy}{dx}$ where $y = \left(x^2 - \frac{2}{x}\right)\left(x^2 + \frac{2}{x}\right)$ (4)

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QUESTION 6

8 MARKS

Sketched below are the graphs of $h(x) = b^x + q$ and $f(x) = \log_2 x$.
Graph f and the asymptote of h intersect at $B(4; p)$.



- a) Write down the coordinates of A , the x -intercept of f . (1)
- b) Determine the domain of f . (1)
- c) Determine the equation of the asymptote of h . (1)

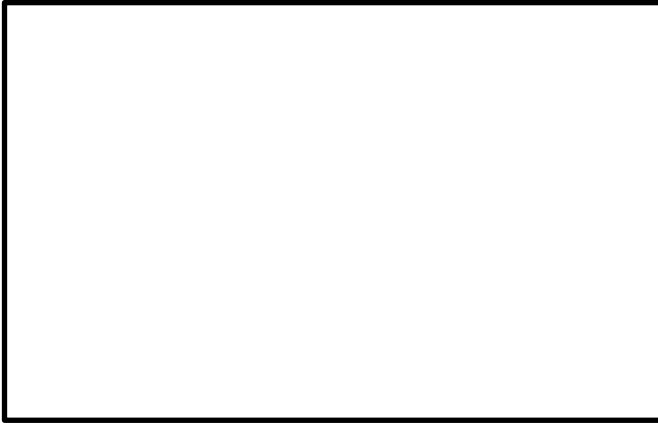
- d) It is given that $h(-x) - q = f^{-1}(x)$. Find the value of b . (2)

- e) Determine the largest integer value of x where $f^{-1}(x) < 295$. (3)

QUESTION 7

15 MARKS

- a) The probability that a boy from Grade 12 is in the rugby team is 0,4 and the probability that he is in the chess team is 0,5. The probability that a boy in Grade 12 is in neither team is 0,2.
- 1) With the aid of a Venn diagram, prove that the events 'In the rugby team' and 'In the chess team' are not mutually exclusive. Show your working. (4)



- 2) Are the events 'In the rugby team.' and 'In the chess team.' independent? Show your working. (3)

- 3) Calculate the probability that a boy selected at random is in the rugby team, but not the chess team. (1)





- b) The 11 boys on the chess team decide to have a waffle evening, and each person is asked to bring a different topping.

- 1) If each boy is allowed to put four different toppings on his waffle, and if they can use each topping only once, how many different ways can the toppings be arranged in layers on the waffle? (2)

- 2) Tony chooses 7 different coloured Astros, 5 different coloured Smarties and 3 different coloured Jelly Tots to go on his waffle and he places them in a straight line.

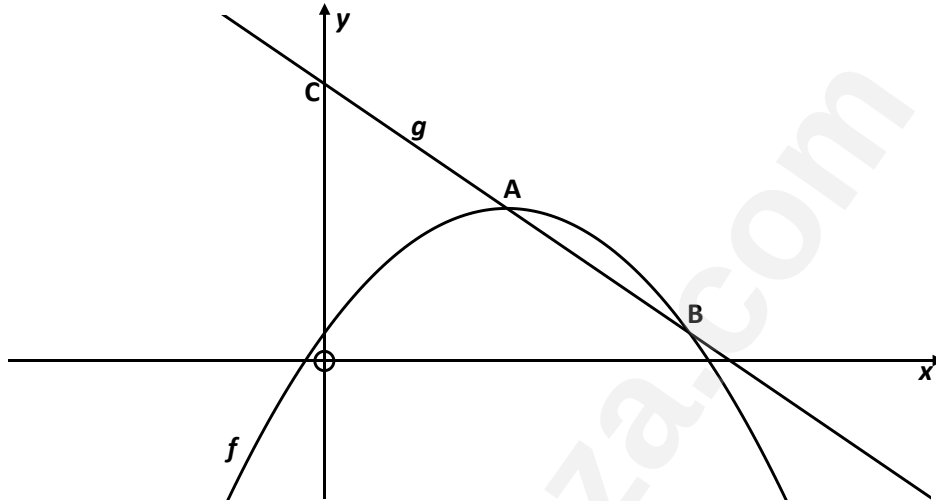
- i) How many ways can he arrange these sweets in a straight line on his waffle, so that all the same type of sweets must stay together? (3)

- ii) What is the probability that Tony arranges the sweets so that the same sweets stay together? Give your answer to seven decimal digits. (2)

QUESTION 8

22 MARKS

Sketched below are the graphs of $g(x) = -3x + 20$ and $f(x) = -x^2 + 6x + 2$. Graph f has a turning point at A and a y -intercept at C . f and g have an intersection point at A and B .



- a) Algebraically, showing all working, solve for x for where $f(x) < g(x)$. (4)

- b) Hence, determine the coordinates of A and B . (2)

- c) Using the graph, describe the nature of the roots for h if $h(x) = f(x) - 11$. (2)
- e) Using the graph, determine the value(s) of x for which $f'(x) \cdot g'(x) > 0$. (2)
- d) Algebraically, determine the values of k such that the equation of $g(x) + k = f(x)$ has no real roots. (6)
- f) The graph of f is the derivative graph for the cubic function p , such that $f(x) = p'(x)$ where it is given that $p(x) = ax^3 + bx^2 + cx - 5$.
- 1) Determine the values of a , b and c in the equation of function p . (3)

- 2) Determine the coordinates of the point of inflection of p . (3)

QUESTION 9**13 MARKS**

- a) Evaluate the limit: $\lim_{x \rightarrow \frac{3}{2}} \frac{8x^3 - 27}{2x - 3}$ (3)

- b) t minutes after a kettle starts to boil, the height of the water in the kettle is given by $d = 86 - \frac{1}{8}t - \frac{1}{4}t^3$, where d is measured in millimetres.

- 1) Calculate the height of the water level in the kettle just before it starts to boil. (2)



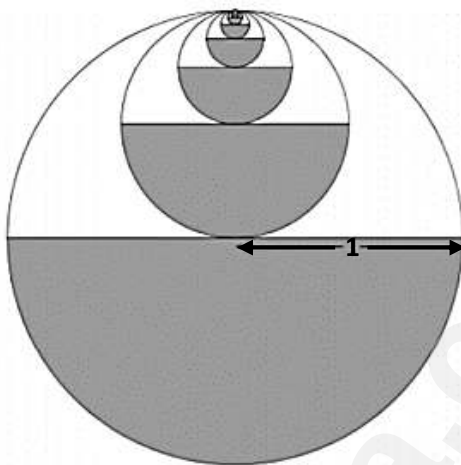
- 2) Due to vaporisation, as the water boils, the water level in the kettle decreases slightly. Determine the rate at which the water level is decreasing when $t = 2$ minutes. (4)

- 3) How many minutes after the kettle starts to boil will the water level be decreasing at a rate of $12\frac{1}{8}$ mm per minute? (4)

QUESTION 10

8 MARKS

- a) The diagram represents a sketch of circles. Half of the area of each circle is shaded. The radius of the largest circle is 1 unit. The diameter of each smaller circle is equal to the radius of the previous circle.



Determine the sum of the shaded areas of all the circles if this process is going on forever. (Give your answer in terms of π .) (5)

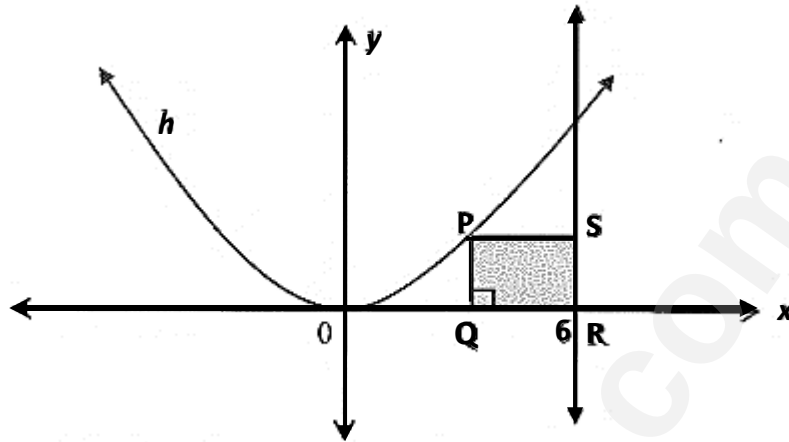
- b) After the age of 40, an average man loses 5% of his hair every year. At what age should an average man expect to have half his hair left? (3)



QUESTION 11

8 MARKS

PQRS is a rectangle with P on the curve $h(x) = x^2$ and with the x -axis and the line $x = 6$ as boundaries. It is given that Q is the point $(x; 0)$.



a) 1) Write down the coordinates of S in terms of x . (1)

2) Hence, show that the area of the rectangle PQRS can be expressed as:
 $A = 6x^2 - x^3$ (3)

b) Determine the largest possible area for the rectangle PQRS.
 Show all your calculations. (4)

ADDITIONAL WORKING:

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