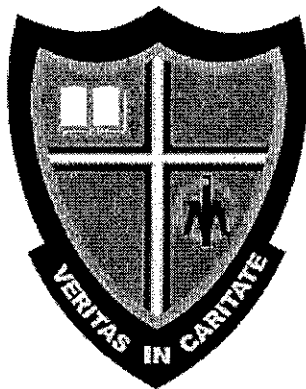




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Final & Memo

ST BENEDICT'S

SUBJECT
GRADE
EXAMINER
NAME
TEACHER

Mathematics
12
Gr 12 Teachers

PAPER
DATE
MARKS
MODERATOR
DURATION

Prelim 2020: Paper 1
14 September
150
Gr 12 Teachers
3 hrs

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1	Algebra	10	
2	Number Patterns	14	
3	Finance	10	
4	Calculus	11	
5	Functions	8	
6	Probability	7	
7	Algebra (Section A: 75)	15	
8	Probability	7	
9	Finance	6	
10	Calculus / Functions	18	
11	Number Patterns	11	
12	Functions	10	
13	Calculus	10	
14	Calculus (Section B: 75)	13	
	TOTAL:	150	

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SECTION A**72 MARKS****QUESTION 1****10 MARKS**Solve for x :

a) $6x^2 - 12x = -1$ (3)

$$6x^2 - 12x + 1 = 0 \quad \checkmark$$

$$x = \frac{12 \pm \sqrt{144 - 4(6)(1)}}{2(6)}$$

$$= \frac{12 \pm \sqrt{120}}{12} \quad 3$$

$$x = 1,91 \quad \checkmark \quad \text{or} \quad x = 0,09 \quad \checkmark$$

b) $\sqrt{2x-1} - 1 + 2x = 0$ (4)

$$(\sqrt{2x-1})^2 = (1-2x)^2 \quad \checkmark$$

$$2x-1 = 1-4x+4x^2 \quad \checkmark$$

$$0 = 4x^2 - 6x + 2$$

$$0 = 2x^2 - 3x + 1$$

$$0 = (2x-1)(x-1)$$

$$x = \frac{1}{2} \quad \checkmark \quad \text{or} \quad x = 1 \quad \checkmark$$

N/A

4.

c) $x^2 - 3x - 28 \geq 0$

(3)

$$(x - 7)(x + 4) \geq 0$$



$$\therefore x \leq -4 \quad \text{or} \quad x \geq 7$$

QUESTION 2

14 MARKS

a) Given the arithmetic sequence: 3 ; 7 ; 11 ;

1) Calculate the sum of the first twenty terms.

(2)

$$S_{20} = \frac{20}{2} [2(3) + 19(4)]$$

$$= 10 \times 82$$

$$= \underline{820}$$

2) The n^{th} term is 191. Determine the value of n .

(3)

$$T_n = 4n - 1$$

$$(T_n = 3 + (n-1)4)$$

$$191 = 4n - 1$$

$$4n = 192$$

$$n = 48$$

b) Consider the series:

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots$$

1) Determine the following in the form $\frac{a}{b}$:

i) $S_2 =$ (1)

$\frac{2}{3}$ ✓	/

ii) $S_3 =$ (1)

$\frac{3}{4}$ ✓	/

iii) $S_4 =$ (1)

$\frac{4}{5}$ ✓	/

2) Hence, determine S_n . (1)

$\frac{n}{n+1}$ ✓	/

3) $T_n = \frac{1}{2015 \times 2016}$. Determine S_n . (1)

$S_n = \frac{2015}{2016}$ ✓	/

c) Find the value of a in the following:

(4)

$$\sum_{r=0}^{\infty} \frac{a}{4^r} = 16$$

$a + \frac{a}{4} + \frac{a}{16} + \dots = 16$	$r = \frac{1}{4}$
$S_{\infty} = \frac{a}{1-r}$	
$16 = \frac{a}{1-\frac{1}{4}}$	4.
$16 = \frac{a}{\frac{3}{4}}$	
$16 \times \frac{3}{4} = a$	
$a = 12$	

QUESTION 3

10 MARKS

Anna wants to buy a house in 10 years' time. Currently the value of the house she is looking at is R800 000.

a) Anna is informed that the property market grows at an average rate of 8% p.a.

What will this house cost in 10 years' time?

(2)

$A = P(1+i)^n$	
$= 800000 (1 + 0.08)^{10}$	
$= \underline{R1727140}$	2

- b) Anna deposits R10 000 into a savings account. One month later she deposits a further R10 000 in the account and she continues to do so for 10 years. Interest on the account is 6% per annum compounded monthly.

Will she be able to buy the house at the end of the 10 year period?

(4)

$$F = \frac{x [(1+i)^n - 1]}{i}$$

$$= \frac{10000 \left[\left(1 + \frac{0.06}{12}\right)^{120} - 1 \right]}{\frac{0.06}{12}}$$

$$= R1656987,44$$

No, she will have not saved enough

- c) After how many months will Anna's savings account balance exceed R1 000 000 for the first time?

(4)

$$F = \frac{x [(1+i)^n - 1]}{i}$$

$$1000000 = \frac{10000 \left[\left(1 + \frac{0.06}{12}\right)^n - 1 \right]}{\frac{0.06}{12}}$$

$$1 + \frac{1}{2} = \left(1 + \frac{0.06}{12}\right)^n$$

$$\frac{3}{2} = \left(\frac{201}{200}\right)^n$$

$$\log_{\frac{201}{200}} \left(\frac{3}{2}\right) = n$$

$$n = 81,29 \dots$$

$\therefore$ 82 payments $\therefore$ 81 months

QUESTION 4

11 MARKS

a) If $f(x) = 2 - 2x^2$, determine f' from first principles.

(4)

$$f(x+h) = 2 - 2(x+h)^2$$

$$= 2 - 2(x^2 + 2xh + h^2)$$

$$= 2 - 2x^2 - 4xh - 2h^2$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2 - 2x^2 - 4xh - 2h^2 - (2 - 2x^2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h}$$

$$= -4x$$

4.

b) Determine (leaving all answers with positive exponents):

1) $D_x \left[\frac{-2x^3 + \sqrt{x^3}}{x^2} \right]$ (4)

$$= D_x \left[\frac{-2x^3 + x^{3/2}}{x^2} \right]$$

$$= D_x \left[-2x + x^{-1/2} \right]$$

$$= -2 - \frac{1}{2} x^{-3/2}$$

$$= -2 - \frac{1}{2x^{3/2}} \quad \text{or} \quad -2 - \frac{1}{2\sqrt{x^3}}$$

2) $\frac{dy}{dx}$ if $y = \frac{8x^3 - 125}{2x - 5}$ (3)

$$y = \frac{(2x-5)(4x^2+10x+25)}{(2x-5)}$$

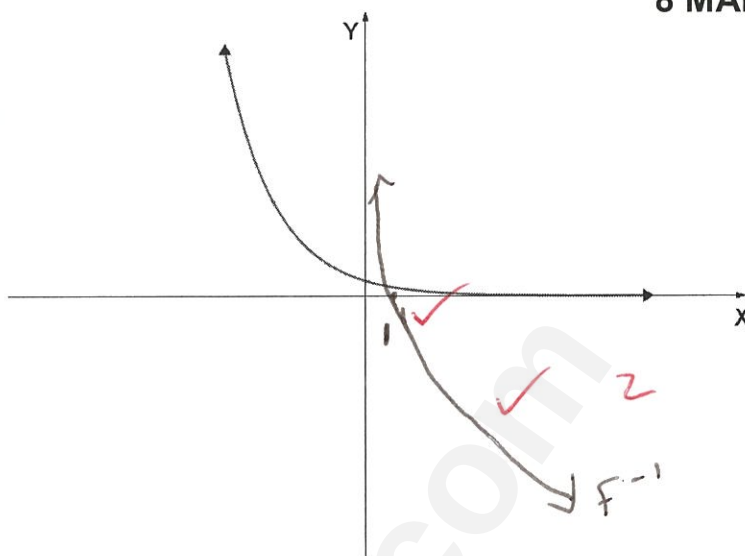
$$y = 4x^2 + 10x + 25$$

$$\frac{dy}{dx} = 8x + 10$$

QUESTION 5

8 MARKS

The graph of $f(x) = 2^{-x}$ is sketched alongside.



- a) Determine the equation of the inverse of f in the form $f^{-1}(x) = \dots\dots\dots$ (2)

$f: y = 2^{-x}$
$f^{-1}: x = 2^{-y}$
$x = \left(\frac{1}{2}\right)^y$
$\log_{\frac{1}{2}} x = y$
$\therefore f^{-1}(x) = \log_{\frac{1}{2}} x$

- b) Draw the graph of f^{-1} on the set of axes, clearly showing all intercepts with the axes. (2)

- c) State the range of the graph of f^{-1} . (1)

$y \in \mathbb{R}$

- d) Calculate the values of x if $f(x) > 8$. (3)

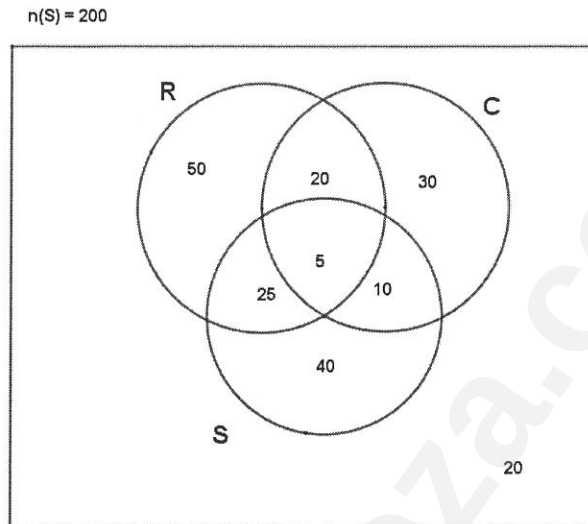
$2^{-x} > 8$
$2^{-x} > 2^3$
$-x > 3$
$x < -3$

QUESTION 6

7 MARKS

A group of 200 people were asked about the kind of sports they watch on television.

180 People watched rugby, cricket or soccer. The information was collected and represented in the Venn diagram where R represents Rugby, S represents Soccer and C represents Cricket.



- a) How many people watched Cricket or Rugby? (1)

140

- b) Determine $P(R \text{ or } S \text{ or } C)$. (2)

$$\frac{20}{200} = 0,1$$

- c) Determine whether "watching Cricket" and "watching Rugby" are independent events?(4)

$$P(C) = \frac{65}{200}$$

$$P(C \text{ and } R) = \frac{25}{200}$$

$$P(R) = \frac{100}{200}$$

$$= 0,125$$

$$\therefore P(C) \times P(R) = \frac{65}{200} \times \frac{100}{200}$$

$$= \frac{13}{80}$$

$$= 0,1625$$

$$P(C) \times P(R) \neq P(C \cap R)$$

$\therefore$ Not independent

QUESTION 7

15 MARKS

a) $f(x) = 2x^3 - 5x^2 + ax + 18$, where a is a constant.

Given that $(x - 3)$ is a factor of f :

1) Show that $a = -9$.

(2)

$f(3) = 0$
$2(3)^3 - 5(3)^2 + a(3) + 18 = 0$ ✓
$54 - 45 + 3a + 18 = 0$
$3a = -27$
$a = -9$ ✓

2) Factorise f completely.

(4)

$2x^3 - 5x^2 + ax + 18$
$= (x-3)(2x^2 + 1x - 6)$ ✓
$= (x-3)(2x^2 + x - 6)$ ✓
$= (x-3)(2x-3)(x+2)$ ✓

3) Given that $g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18$, find the values of y that satisfy $g(y) = 0$, giving your answer to 2 decimals digits where appropriate.

(5)

let $3^y = x$
$\therefore 3^y = 3$ ✓ or $3^y = \frac{3}{2}$ ✓ or $3^y = -2$ ✓
$y = 1$ ✓ $y = \log_3(\frac{3}{2})$ N/A
$= 0,369$
$\approx 0,4$ ✓

SECTION B**70 MARKS****QUESTION 8****7 MARKS**

The word HELKHANAH is given.

- a) In how many ways can these letters be arranged?

(3)

$\frac{9!}{3! \cdot 2!} = 30240$	✓
	✓
	✓
	✓

- b) What is the probability that if a random combination is chosen, it will start and end with vowel (a, e, i, o, u)?

(4)

A _ _ _ _ A	$\frac{7!}{3!}$	✓
A _ _ _ _ E	$\frac{7!}{3!} \times 2$	✓
E _ _ _ _ A		
Now of ways = $\frac{7!}{3!} + \frac{7!}{3!} \times 2 = 2520$		
$\therefore \text{Prob} = \frac{2520}{30240} = \frac{1}{12} = 0,08$		
		4.

b) Prove that $\frac{x^4+1}{x^4} = \frac{1}{2}$ has no real roots.

(4)

$$2x^4 + 2 = x^4 \quad \checkmark$$

$$x^4 = -2 \quad \checkmark$$

$$x = \sqrt[4]{-2} \quad \checkmark$$

No real roots $\checkmark$

4.

QUESTION 9

6 MARKS

Mavis is a farmer and needs to buy new farming equipment. The new equipment will cost R350 000. She is granted a loan by the bank for the full amount under the following conditions:

The loan needs to be paid off in 5 years

The interest rate for the loan is 7,5% p.a. compounded quarterly

Mavis needs to start making payments in 3 months' time.

Determine Mavis' instalment.

(6)

$P = x \frac{[1 - (1+i)^n]}{i}$
$350000 = x \frac{[1 - (1 + \frac{0.075}{4})^{-20}]}{\frac{0.075}{4}}$
$350000 \times 0.075 = x [1 - (1 + \frac{0.075}{4})^{-20}]$
$\frac{13125}{2} = x [0,3103 \dots]$
$x = R21147,52$
$\therefore$ Her installment is R21147,52

QUESTION 10

18 MARKS

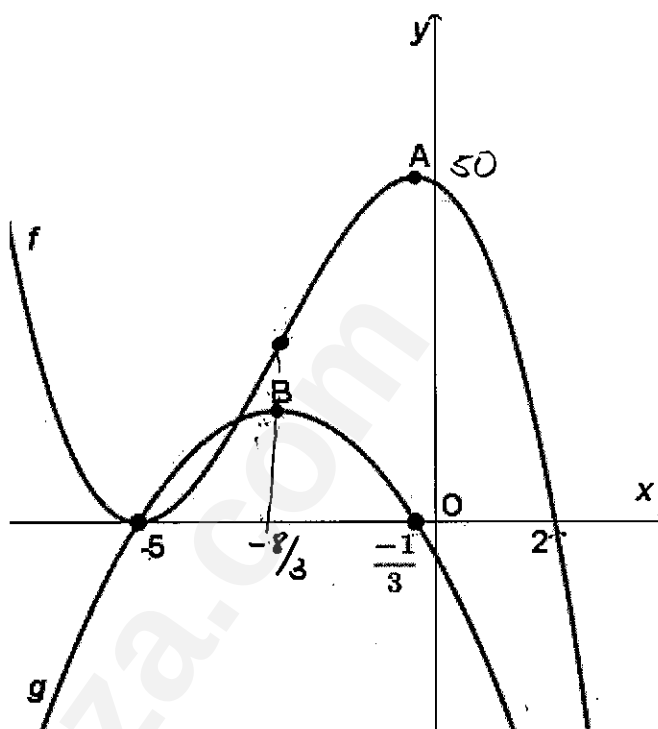
The sketch shows the graphs of the following functions:

$$f(x) = ax^3 + bx^2 - 5x + 50.$$

$$g(x) = f'(x) = px^2 + qx + t$$

The x -intercepts of f are $(-5; 0)$ and $(2; 0)$.

The x -intercept of g are $(-5; 0)$ and $(-\frac{1}{3}; 0)$.



a) Find the values of a and b .

(4)

$$y = a(x+5)^2(x-2) \quad \checkmark$$

$$\text{Sub in } (0, 50)$$

$$50 = a(0+5)^2(0-2) \quad \checkmark$$

$$50 = -50a$$

$$a = -1 \quad \checkmark$$

4

$$\therefore y = -(x+5)^2(x-2)$$

$$y = -(x^2 + 10x + 25)(x-2)$$

$$y = -x^3 - 8x^2 - 5x + 50 \quad \checkmark$$

$$\therefore a = -1 \quad \text{and} \quad b = -8$$

b) Find the values of p , q and t .

(2)

$$g(x) = -3x^2 - 16x - 5$$

$$\therefore p = -3 \quad q = -16 \quad t = -5$$

c) Determine the coordinates of A.

(2)

$$f'(x) = 0$$

$$-3x^2 - 16x - 5 = 0$$

$$x = -\frac{1}{3}$$

$$y = -\left(-\frac{1}{3}\right)^3 - 8\left(-\frac{1}{3}\right)^2 - 5\left(-\frac{1}{3}\right) + 50$$

$$= \frac{1372}{27} \approx 50,8$$

$$\therefore A\left(-\frac{1}{3}; \frac{1372}{27}\right) \quad \text{or} \quad A\left(-\frac{1}{3}; 50,8\right)$$

d) 1) Determine the coordinates of B.

(2)

$$g'(x) = 0$$

$$-6x - 16 = 0$$

$$-6x = 16$$

$$x = -\frac{8}{3}$$

$$y = -3\left(-\frac{8}{3}\right)^2 - 16\left(-\frac{8}{3}\right) - 5$$

$$= \frac{49}{3} = 16,3$$

$$\therefore B\left(-\frac{8}{3}; \frac{49}{3}\right) \quad \text{or} \quad B\left(-\frac{8}{3}; 16,3\right)$$

2) Hence, determine the values of x for which f is concave down.

(1)

$$x > -\frac{8}{3}$$

e) Determine the value of $g[f(2)]$.

(2)

$f(2) = 0$
$g[f(2)] = g(0) = -5$

f) Determine for which value(s) of x will $f(x) \cdot g(x) < 0$

(3)

$x < -5$ or $-\frac{1}{3} < x < 2$

g) If $f(x) = ax^3 + bx^2 - 5x + m$, determine the value(s) of m if f has only one real root.

(2)

$m > 50$

QUESTION 11

11 MARKS

A little boy, standing on the side of a still lake, picks up a flat stone and throws it horizontally across the water. The stone flies 30 metres through the air then bounces several times across the surface of the water, before finally coming to rest and sinking.



The distance (d) in **metres** that the stone travels after each bounce (n) on the water is given by the equation: $d_n = 18 \left(\frac{1}{2}\right)^{n-1}$

a) How far has the stone travelled as it hits the water for the third time? (3)

$S_2 = 30 + 18 + 9$	
$= 57m$	

b) Between which two bounces does the stone travel a distance of 56,25 cm? (4)

$0,5625 = 18 \left(\frac{1}{2}\right)^{n-1}$	
$\frac{1}{32} = \left(\frac{1}{2}\right)^{n-1}$	
$n-1 = 5$	
$n = 6$	
$\therefore$ between bounce 6 and 7	4.

c) What is the total distance travelled by the stone as it sinks?

(4)

$$S_{\infty} = \frac{a}{1-r} \quad \checkmark$$

$$= 18 \quad \checkmark$$

$$1 - \frac{1}{2} \quad \checkmark$$

$$= 36m$$

4.

$$\therefore \text{Total} = 30 + 36 \quad \checkmark$$

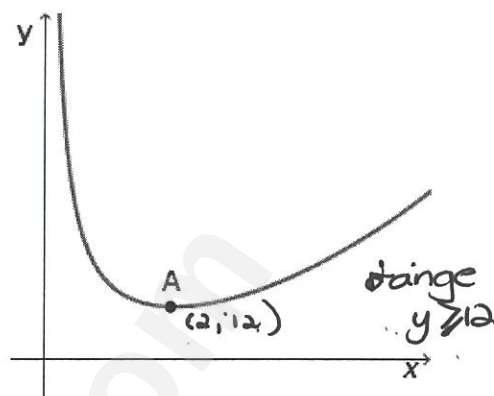
$$= 66m$$

QUESTION 12

10 MARKS

Given $f(x) = x^2 + \frac{16}{x}$ and $x > 0, x \in \mathbb{R}$.

The curve has a minimum turning point at A.



a) Determine the coordinates of A.

(6)

$f'(x) = 0$
$2x - 16x^{-2} = 0$ ✓
$2x = \frac{16}{x^2}$
$2x^3 = 16$ ✓
$x^3 = 8$
$x = 2$ ✓
$y = 4 + 8 = 12$ ✓
$\therefore A(2; 12)$ ✓

b) Give the domain of f^{-1} .

(2)

$x \geq 12$ ✓ ✓

c) Using your answer in (a), write down the coordinates of the turning point of the curve with equation:

1) $f(x - 2)$

(1)

$(4; 12)$ ✓

2) $\frac{1}{2}f(x)$

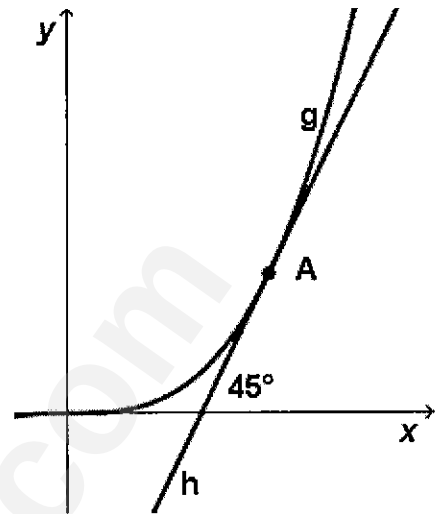
(1)

$(2; 6)$ ✓

QUESTION 13

10 MARKS

- a) The function $g(x) = 3x^3$ (where $x > 0$) is given. The tangent h (at the point A) forms an angle of 45° with the x -axis.



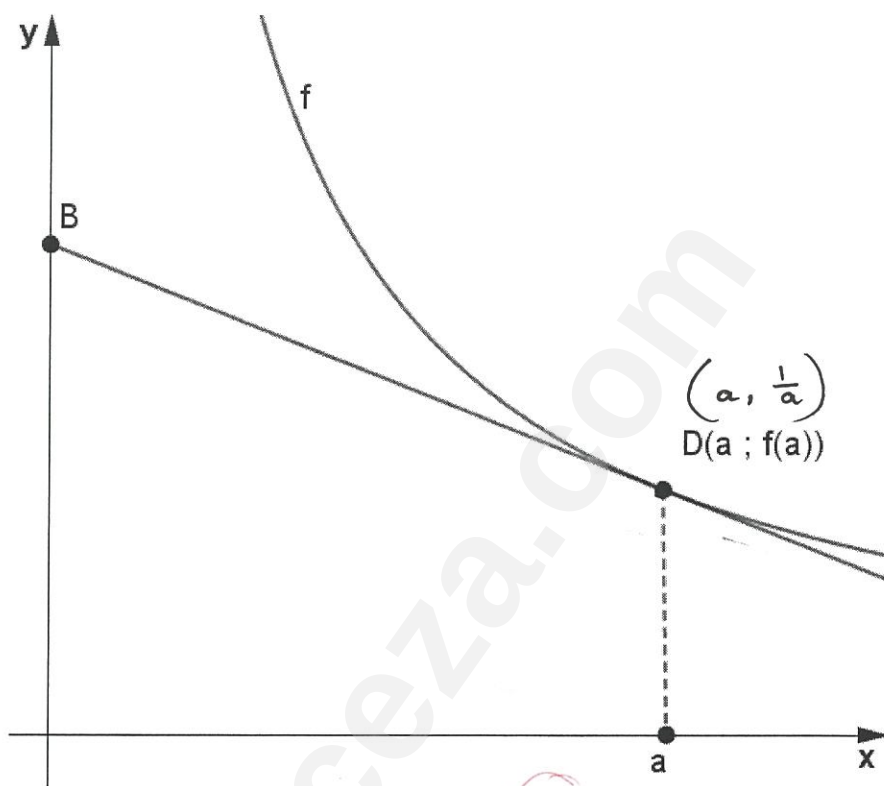
Determine the coordinates of A.

(4)

$g(x) = 3x^3$	
$g'(x) = 9x^2$ ✓	$\tan 45^\circ = 1$
$\therefore 9x^2 = 1$ ✓	
$x^2 = \frac{1}{9}$	
$x = \frac{1}{3}$ ✓ (quad 1)	
$y = 3\left(\frac{1}{3}\right)^3$	
$= \frac{1}{9}$ ✓	4.
$\therefore A \left(\frac{1}{3}; \frac{1}{9}\right)$	

b) Given: $f(x) = \frac{1}{x}$

The tangent AB is drawn and the point of tangency is $D(a; f(a))$



Show that the equation of the tangent is $x + a^2y = 2a$.

(6)

$$f(x) = \frac{1}{x}$$

$$f'(x) = -\frac{1}{x^2}$$

$$f'(a) = -\frac{1}{a^2}$$

eqn of tang:

$$y - f(a) = -\frac{1}{a^2}(x - a)$$

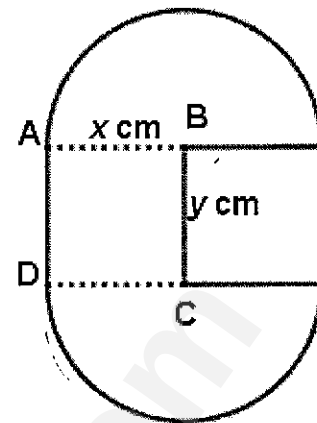
$$y - \frac{1}{a} = -\frac{1}{a^2}x + \frac{1}{a}$$

$$y = -\frac{1}{a^2}x + \frac{1}{a} + \frac{1}{a}$$

$$a^2y = -x + a + a$$

$$\therefore x + a^2y = 2a$$

The diagram shows a badge made of thin sheet of metal, consisting of two semi-circular pieces with centres B and C with radius x . The semi-circles are connected by a rectangular section ABCD such that $AB = x$ and $BC = y$.



- a) Given that the area of the badge is 20 cm^2 , show that the perimeter, P of the badge is given by : (7)

$$P = 2x + \frac{40}{x}$$

$$r = x$$

$$P = 2\pi r + 2y + 2x$$

$$\text{but } \pi r^2 + xy = 20$$

$$= 2\pi x + 2\left(\frac{20 - \pi x^2}{x}\right) + 2x$$

$$\pi x^2 + xy = 20$$

$$xy = 20 - \pi x^2$$

$$= 2\pi x + \frac{40}{x} - 2\pi x + 2x$$

$$y = \frac{20 - \pi x^2}{x}$$

$$= 2x + \frac{40}{x}$$

7

- b) Hence, determine the minimum value of the perimeter and prove that you have indeed found a minimum. (6)

min Perimeter $P' = 0$	
$P = 2x + 40x^{-1}$	
$2 - 40x^{-2} = 0$ ✓	
$2 = \frac{40}{x^2}$	
$2x^2 = 40$	
$\sqrt{x^2} = \sqrt{40}$	6
$x = \sqrt{20}$	
$= 2\sqrt{5}$ ✓	
min $P = 2 \cdot 2\sqrt{5} + \frac{40}{2\sqrt{5}}$ ✓	
$= 4\sqrt{5} + \frac{20}{\sqrt{5}}$	(dec = 17,89)
$= 8\sqrt{5}$	
$P' = 2 - 40x^{-2}$	
$P'' = 80x^{-3}$ ✓	
$= \frac{80}{x^3}$	P'' at $x = 2\sqrt{5}$ is $\frac{80}{(2\sqrt{5})^3}$ ✓
	> 0

∴ it is a minimum

TOTAL: 150



