





ST BENEDICT'S

SUBJECT
GRADE
EXAMINER
NAME
TEACHER

Mathematics
12
MR C BOTHA

PAPER
DATE
MARKS
MODERATOR
DURATION

Prelim Paper 1
July 2019
150
GR 12 TEACHERS
3 hours

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QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1	Algebra	21	
2	Number Patterns and Series and Sequences	13	
3	Functions	16	
4	Finance	8	
5	Calculus	14 (72)	
6	Probability	15	
7	Number patterns	16	
8	Finance	9	
9	Functions	16	
10	Calculus	9	
11	Calculus	13 (78)	
TOTAL		150	

1. This question paper consists of 11 questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
3. Answers only will not necessarily be awarded full marks
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. An information sheet with formulae is included at the end of the question paper.
8. Write neatly and legibly

SECTION A

QUESTION 1

21 MARKS

a. Solve for x:

1) $(x - 2)(4 + x) = 0$

(2)

$x = 2$ ✓

$x = -4$ ✓

2

2) $3x^2 - 2x = 14$ (correct to two decimals)

(4)

$3x^2 - 2x - 14 = 0$ ✓

$$x = \frac{2 \pm \sqrt{4 - 4(3)(-14)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{172}}{6}$$

4

$x = 2,52$ ✓

or

$x = -1,55$ ✓

3) $2^{x+2} + 2^x = 20$

(3)

$2^x(2^2 + 1) = 20$

$2^x(5) = 20$

$2^x = 4$

$x = 2$ ✓

3

4) $(x-1)(x-2) < 6$

(4)

$$x^2 - 3x + 2 < 6$$

$$x^2 - 3x - 4 < 0$$

$$(x-4)(x+1) < 0$$



$$\therefore -1 < x < 4$$

4.

- b. The roots of a quadratic equation are $x = \frac{3 \pm \sqrt{-k-4}}{2}$. For which values of k are the roots real? (2)

$$-k-4 \geq 0$$

$$-k \geq 4$$

$$k \leq -4$$

2

- c. Consider the circle $x^2 + y^2 = 3h$ and the straight line $x + y = h$. For which values of h will the graphs touch each other? (6)

$$y = h - x \quad \checkmark$$

$$x^2 + y^2 = 3h$$

$$x^2 + (h - x)^2 = 3h \quad \checkmark$$

$$x^2 + h^2 - 2xh + x^2 - 3h = 0$$

$$2x^2 - 2xh + h^2 - 3h = 0 \quad \checkmark$$

$$\Delta = b^2 - 4ac$$

$$= (-2h)^2 - 4(2)(h^2 - 3h) \quad \checkmark$$

$$= 4h^2 - 8(h^2 - 3h)$$

$$= 4h^2 - 8h^2 + 24h$$

$$= -4h^2 + 24h$$

$$-4h^2 + 24h = 0 \quad \checkmark$$

$$-4h(h - 6) = 0$$

$$h = 0$$

or

$$h = 6 \quad \checkmark$$

$$N/A$$

6

QUESTION 2

13 MARKS

- a. A geometric sequence has $T_3 = 20$ and $T_4 = 40$.

Determine:

- 1) The common ratio

(1)

$$r = 2$$

- 2) A formula for the general term of this sequence

(3)

$$ar^2 = 20$$

$$a = 5$$

$$T_n = 5(2)^{n-1}$$

- b. The following sequence is given: $\frac{2}{1}; \frac{-1}{5}; \frac{-4}{25}; \dots$

- 1) Write down the FOURTH term of the sequence

(1)

$$\frac{-7}{125}$$

- 2) Determine a formula for the n^{th} term

(3)

$$\text{Numerator: } T_n = -3n + 5$$

$$\text{Denominator: } T_n = 5^{n-1}$$

$$T_n = \frac{-3n+5}{5^{n-1}}$$

3) Determine the 500th term (Leave your answer as a fraction)

(2)

$$T_{500} = \frac{-3(500) + 5}{500 - 1}$$

$$= \frac{-1495}{499}$$

2

4) Which will be the first term of the sequence with the NUMERATOR less than -59?

(3)

$$-3n + 5 < -59$$

$$-3n < -64$$

$$n > \frac{64}{3}$$

$$n > 21\frac{1}{3}$$

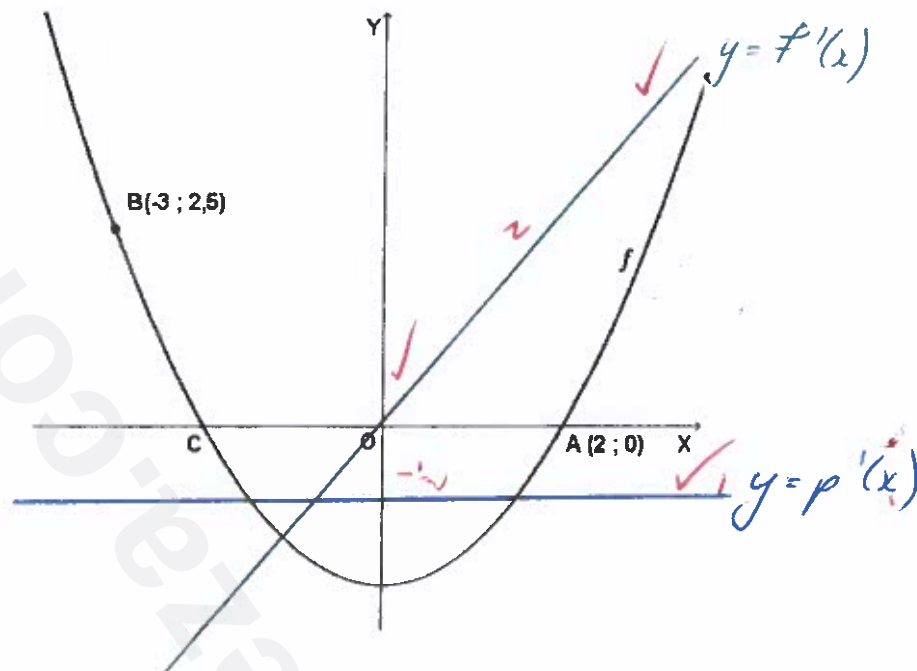
$$\therefore n = 22$$

3

QUESTION 3

16 MARKS

The graph of $f(x) = ax^2 + q$ is sketched below. The points A(2 ; 0) and B(-3 ; 2,5) and C lies on the graph where A and C are the x-intercepts of f .



a. Determine the equation of f

(4)

$$\begin{aligned}
 y &= ax^2 + q \\
 (2, 0) \quad 0 &= 4a + q \\
 (-3, 2.5) \quad 2.5 &= 9a + q \\
 2.5 &= 5a \\
 \frac{1}{2} &= a \\
 0 &= 4\left(\frac{1}{2}\right) + q \\
 -2 &= q \\
 y &= \frac{1}{2}x^2 - 2
 \end{aligned}$$

b. Write down the range of f

(1)

$$y \geq -2$$

- c. Write down the range of $h(x)$ if $h(x) = -f(x) - 2$

(2)

$$y \leq 0$$

✓✓

2

- d. Determine the equation of $p(x)$, the line passing through A and B

(3)

$$m = \frac{-2,5}{5}$$

$$= -\frac{1}{2}$$

✓

$$y - y_1 = -\frac{1}{2}(x - x_1)$$

$$(2;0)$$

$$y = -\frac{1}{2}(x - 2)$$

✓✓

$$= -\frac{1}{2}x + 1$$

3

- e. Determine the equation of the exponential function $g(x) = b^x + q$ that passes through the point A and with range $y > -4$.

(3)

$$g(x) = b^x + q$$

$$y > -4$$

$$g(x) = b^x - 4$$

✓

$$(2;0)$$

$$0 = b^2 - 4$$

✓

$$4 = b^2$$

$$2 = b$$

✓

$$y = 2^x - 4$$

3

- f. On the given diagram on page 8, draw the graphs of $f'(x)$ and $p'(x)$. Clearly mark the graphs and indicate the intercepts with the axis

(3)

See graph

QUESTION 4

8 MARKS

Phillip loves fishing and is planning on developing a campsite to run fishing tours.



- a. In order to set up the campsite, Phillip starts investing R500 at the end of every month for a period of 10 years into an account that offers him an interest rate of 15% per annum, compounded monthly. Determine the total he would accumulate over the 10 years. (4)

$$\begin{array}{c}
 \begin{array}{ccccccc}
 T_0 & T_1 & T_2 & & & T_{119} & T_{120} \\
 & 500 & 500 & & & & 500
 \end{array} \\
 n = 120 \\
 FV = \frac{500 \left[\left(1 + \frac{0,15}{12} \right)^{120} - 1 \right]}{\frac{0,15}{12}} \\
 = R137\,608,53
 \end{array}$$

4.

- b. Ten years later Phillip finds a piece of land on the West Coast of South Africa to set up his fishing campsite. The cost of the piece of land is R1 205 000. He pays a deposit of R135 000. Calculate the monthly payment needed to pay off the loan over a period of 20 years if the rate of borrowing money is 9% pa compounded monthly. (4)

$$\begin{array}{c}
 1205\,000 - 135\,000 = 1\,070\,000,00 \\
 1\,070\,000 = \frac{x \left[1 - \left(1 + \frac{0,09}{12} \right)^{-240} \right]}{\frac{0,09}{12}} \\
 x = R9\,627,02
 \end{array}$$

4.

QUESTION 5

14 MARKS

a. If $f(x) = -2x^2$, determine $f'(x)$ from first principles

(5)

$$\begin{aligned}
 f(x+h) &= -2(x+h)^2 \\
 &= -2(x^2 + 2hx + h^2) \\
 &= -2x^2 - 4hx - 2h^2 \\
 \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} & \checkmark \\
 &= \lim_{h \rightarrow 0} \frac{-2x^2 - 4hx - 2h^2 + 2x^2}{h} \checkmark \\
 &= \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h} \checkmark \\
 &= \lim_{h \rightarrow 0} (-4x - 2h) \checkmark \\
 &= -4x \checkmark
 \end{aligned}$$

b. Determine:

1. $D_x[(3x-2)^2]$

(3)

$$\begin{aligned}
 D_x [9x^2 - 12x + 4] \\
 = 18x - 12
 \end{aligned}$$

2. $\frac{dy}{dx}$ if $y = \frac{2x^2-1}{\sqrt{x}}$

(3)

$$\begin{aligned}
 y &= \frac{2x^2}{x^{\frac{1}{2}}} - \frac{1}{x^{\frac{1}{2}}} \\
 &= 2x^{\frac{3}{2}} - x^{-\frac{1}{2}} \quad \checkmark \\
 \frac{dy}{dx} &= 3x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}} \\
 &= 3x^{\frac{1}{2}} + \frac{1}{2x^{\frac{3}{2}}} \quad \checkmark \\
 &= 3\sqrt{x} + \frac{1}{2\sqrt{x^3}} \quad \checkmark
 \end{aligned}$$

c. Given: $y = \frac{1}{x^2}$

Prove that the gradient of the curve is negative at each point on the curve

where $x > 0$

(3)

$$\begin{aligned}
 y &= \frac{1}{x^2} \\
 &= x^{-2} \quad \checkmark \\
 \frac{dy}{dx} &= -2x^{-3} \\
 &= -\frac{2}{x^3} \quad \checkmark \\
 -\frac{2}{x^3} &< 0 \quad \text{for } x \in \mathbb{R} \quad \checkmark \\
 \text{Gradient is always } < 0
 \end{aligned}$$

SECTION B

QUESTION 6

15 MARKS

- a. Eight boys and seven girls are to be seated randomly in a row. What is the probability that:

1. The row has a girl at each end ?

(3)

$$\frac{7 \times 13! \times 6}{15!} = 0,2$$

2. The row has boys and girls sitting in alternate positions ?

(3)

$$\frac{8! 7!}{15!} = 0,000155$$

3. Two particular boys that have had an argument will not sit next to each other ?

(3)

Next to each other

$$\frac{14! \times 2}{15!} = 0,1333$$

Not next to each other

$$1 - 0,1333 = 0,867$$

- b. Thabo writes an Art and a Music examination. He has a 40% chance of passing the Music examination, 60% of passing the Art examination and a 70 % chance of passing at least one of the exams.

Calculate the probability that Thabo will pass both the Art and the Music examination. (2)

$$P(A \cup M) = P(A) + P(M) - P(A \cap M)$$

$$0,7 = 0,6 + 0,4 - P(A \cap M) \quad \checkmark$$

$$P(A \cap M) = 0,3 \quad \checkmark$$

$$\therefore 30\%$$

- c. A survey was conducted asking 60 people with which hand they write and what their academic achievement were (average higher than 70%). The survey concluded that the "hand used for writing" and "academic achievement" were **independent**.

		Hand used to write with		
		Right	Left	Total
Academic achievement	Above 70%	a	b	20
	Less than 70%	c	d	40
	Total	48	12	60

Calculate the values of a, b and c (4)

$$\frac{a}{60} = \frac{20}{60} \times \frac{48}{60} \quad \checkmark$$

$$3600a = 57600$$

$$a = 16 \quad \checkmark$$

$$\therefore b = 4 \quad \checkmark$$

$$c = 32 \quad \checkmark$$

QUESTION 7

16 MARKS

a. If $S_n = 2n^2 + 3n$, then:1. Calculate T_{15}

(3)

$$S_{15} = 2(15)^2 + 3(15)$$

$$= 495 \quad \checkmark$$

$$S_{14} = 434 \quad \checkmark$$

$$T_{15} = 495 - 434$$

$$= 61 \quad \checkmark$$

3

2. Find T_n in its simplest form

(5)

$$S_1 = 5 = T_1 \quad \checkmark$$

$$S_2 = 14 \quad \therefore T_2 = 14 - 5$$

$$= 9 \quad \checkmark$$

$$S_3 = 27 \quad \therefore T_3 = 27 - 14 \quad \checkmark$$

$$5, 9, 13$$

$$\therefore T_n = 4n + 1 \quad \checkmark \quad \checkmark$$

5

b. Calculate the value of

$$\sum_{n=3}^6 (2n^2 - 1)$$

(2)

$$17 + 31 + 49 + 71 = 168$$

c. In a converging geometric series $S_{\infty} = \frac{40}{3}$ and $T_2 = \frac{5}{2}$, calculate the possible value(s) of the first term in the series. (6)

$$\frac{a}{1-r} = \frac{40}{3} \quad ar = \frac{5}{2} \quad \left[a = \frac{5}{2r} \right]$$

$$a = \frac{40(1-r)}{3}$$

$$\therefore \frac{40(1-r)}{3} \cdot r = \frac{5}{2}$$

$$80r(1-r) = 15$$

$$80r - 80r^2 = 15$$

$$0 = 80r^2 - 80r + 15$$

$$= 16r^2 - 16r + 3$$

$$= (4r - 1)(4r - 3)$$

$$r = \frac{1}{4}$$

$$a = 10$$

$$r = \frac{3}{4}$$

$$a = 3\frac{1}{3}$$

QUESTION 8

9 MARKS

Joe buys a vehicle that costs R450 000. The vehicle is depreciated at 8,1% p.a. on the reducing balance method. After 4 years it will be traded in at this depreciated value and replaced by a new vehicle. A sinking fund is created to cover the replacement of this vehicle. Payments will be made at the end of each month. The first payment will be made at the end of the 13th month and the last payment will be made at the end of the 48th month. The sinking fund earns interest at a rate of 6,2 % p.a, compounded monthly.

- a. Calculate the value of the trade-in vehicle after 4 years (rounded to the nearest R100) (2)

$$A = 450\,000 (1 - 0,081)^4$$

$$= 321\,000$$

- b. Inflation will cause the price of the new vehicle to increase by 6,2 % p.a. Calculate the price of the new vehicle in 4 years time (rounded to the nearest R100) (2)

$$A = 450\,000 (1 + 0,062)^4$$

$$= 572\,400$$

- c. Calculate the monthly payment to the sinking fund. (5)

$$\text{Sinking fund} = 251\,400$$

T_0	T_1	T_2	T_{12}	T_{13}	T_{14}	T_{47}	T_{48}
				x	x	x	x

$$\therefore n = 48 - 13 + 1 = 36$$

$$251\,400 = x \left[\frac{(1 + \frac{0,062}{12})^{36} - 1}{\frac{0,062}{12}} \right]$$

$$6\,361,84$$

$$R\,6\,371,98 = x$$

QUESTION 9

16 MARKS

a. Given: $f(x) = -2(x + 4)^2 + 3$

Determine the turning points of:

1. $f(x) - 1$

(2)

$(-4; 2)$ ✓ ✓ 2

2. $f(x + 1)$

(2)

$(-5; 3)$ ✓ ✓ 2

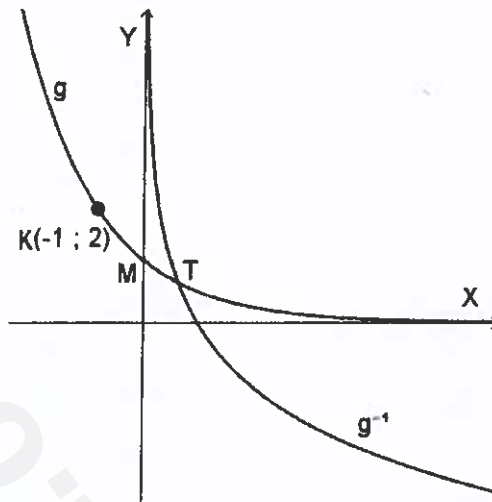
3. $4f(x)$

(2)

$(-4; 12)$ ✓ ✓ 2

- b. Refer to the figure showing a sketch of $y = g(x)$ and $y = g^{-1}(x)$ where $g(x) = b^x$.

T is the point of intersection of the two graphs. The point $K(-1; 2)$ lies on the graph.



1. Show that $b = \frac{1}{2}$

(1)

$$\begin{aligned} y &= b^x \\ (-1; 2) \quad 2 &= b^{-1} \\ \frac{1}{2} &= b \end{aligned}$$

2. Determine the equation of g^{-1} in the form $g^{-1}(x) = \dots$

(2)

$$\begin{aligned} g(x) = y &= \frac{1}{2}^x \\ g^{-1}(x) &= \log_{\frac{1}{2}} x \end{aligned}$$

3. The x -coordinate of T is 0,64 (correct to two decimals). Give the y -coordinate of T (correct to two decimals)

(2)

$$\begin{aligned} y &= 0,64 \quad (\text{lies on } y=x) \\ \text{OR} \\ y &= \left(\frac{1}{2}\right)^{0,64} \\ &= 0,64 \end{aligned}$$

4. Write down the values of x for which $g^{-1}(x) \geq 0$

(3)

$$x - \ln x = 1$$

$$0 < x \leq 1$$

3

5. $h(x)$ is the reflection of $g^{-1}(x)$ about the y-axis.

i. Give the equation of $h(x)$

(1)

$$h(x) = \log_{\frac{1}{2}}(-x)$$

ii. State the domain of $h(x)$

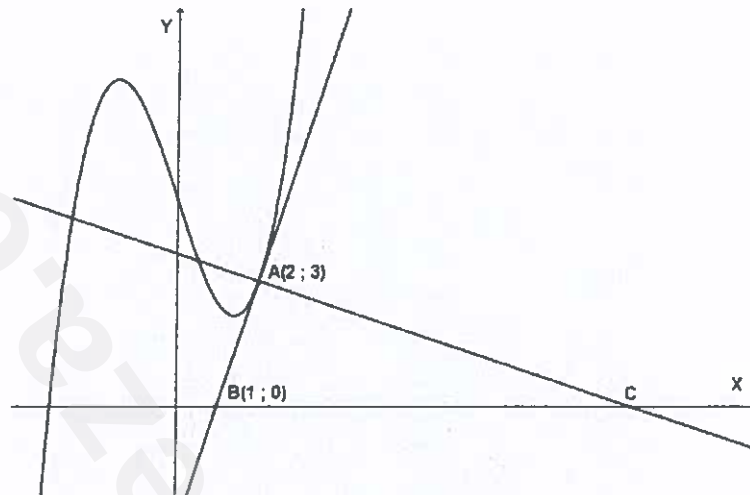
(1)

$$x < 0$$

QUESTION 10

9 MARKS

In the diagram the tangent to the curve $y = f(x) = ax^3 - 3x + b$ at $A(2; 3)$ meets the x -axis at $B(1; 0)$. The line perpendicular to the tangent at A meets the x -axis at C



- a. Prove that value of $a = \frac{1}{2}$ and $b = 5$

(4)

$$f'(x) = 3ax^2 - 3$$

$$\text{gradient} = \frac{3}{1} \quad \checkmark$$

$$\therefore 3ax^2 - 3 = 3$$

$$3ax^2 = 6$$

$$ax^2 = 2 \quad \checkmark$$

$$\text{But } x = 2$$

$$\therefore 4a = 2$$

$$a = \frac{1}{2} \quad \checkmark$$

$$3 = \frac{1}{2}(2)^3 - 3(2) + b$$

$$5 = b \quad \checkmark$$

b. Determine the coordinates of C

(3)

$$\text{Gradient} = -\frac{1}{3}$$

$$y - 3 = -\frac{1}{3}(x - 2)$$

$$y = -\frac{1}{3}x + \frac{2}{3} + 3$$

$$= -\frac{1}{3}x + \frac{11}{3}$$

$$0 = -\frac{1}{3}x + \frac{11}{3}$$

$$\frac{1}{3}x = \frac{11}{3}$$

$$x = 11$$

$$(11, 0)$$

c. Determine the area of ΔABC

(2)

$$BC = 10$$

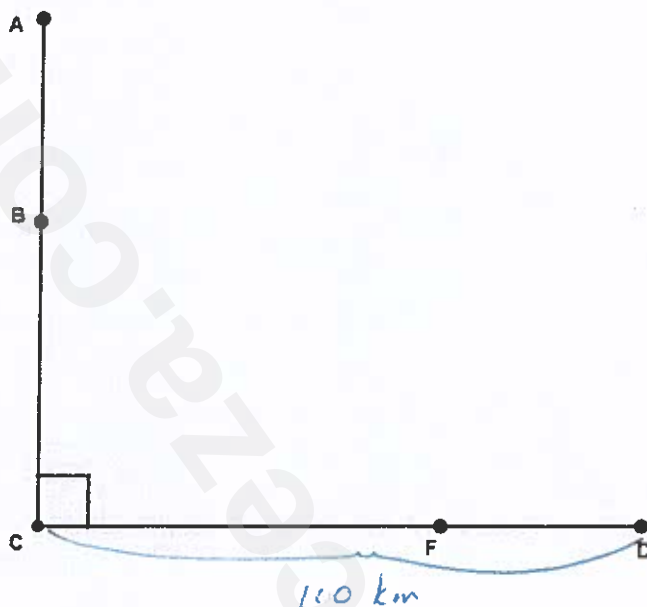
$$\perp h = 3$$

$$\begin{aligned} \therefore \text{Area} &= \frac{1}{2}(3)(10) \\ &= 15 \end{aligned}$$

QUESTION 11

13 MARKS

Two cyclists start to cycle at the same time. One starts at Point C heading due north towards A and the other starts at Point D heading due west to B. The cyclist starting at Point C cycles at 30 km/h and the cyclist starting at Point D cycles at 40 km/h. The distance between C and D is 100 km. After a certain time, t hours, they reach points B and F respectively.



- a. Explain why the distance $BC = 30t$

(1)

$$\begin{aligned} & \text{Speed} \times \text{time} \\ & = 30t \end{aligned}$$

- b. Hence, write down the following distances in terms of t :

DF:	$40t$	✓✓	2
FC:	$100 - 40t$	✓✓	2

(4)

c. Now calculate the length of BF^2 (in terms of t)

(3)

$$\begin{aligned} BF^2 &= (30t)^2 + (100 - 40t)^2 \checkmark \\ &= 900t^2 + (10000 - 8000t + 1600t^2) \checkmark \\ &= 900t^2 + 10000 - 8000t + 1600t^2 \\ &= 2500t^2 - 8000t + 10000 \checkmark \end{aligned} \quad 3$$

d. After how long will the two cyclists be closest to each other

(3)

$$\begin{aligned} \frac{dBF^2}{dt} &= 5000t - 8000 \checkmark \\ 5000t - 8000 &= 0 \checkmark \\ t &= \frac{8}{5} \\ &= 1,6 \text{ hrs} \checkmark \end{aligned} \quad 3$$

e. How far are the two cyclists from each other at this point (determined in (3)) ?

(2)

$$\begin{aligned} BF^2 &= 2500(1,6)^2 - 8000(1,6) + 10000 \checkmark \\ &= 3600 \\ BF &= 60 \text{ km} \checkmark \end{aligned} \quad 2$$

Total: 150