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ST BENEDICT'S

SUBJECT	Mathematics	PAPER	Calculus & Sequence and series
GRADE	12	DATE	17 February 2023
EXAMINER	Mrs K Myburgh	MARKS	40
NAME	Memorandum	MODERATOR	Grade 12 teachers
TEACHER		DURATION	50 minutes

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1	Basics calculus application	10	
2	Determining derivatives	15	
3	Sequences and series (Sum to infinity)	15	
TOTAL		40	

INSTRUCTIONS:

1. This paper consists of 3 questions, 10 pages and an information sheet.
2. Read the questions carefully.
3. Answer all questions in the spaces provided.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. All necessary working details must be shown. Answers only, without the relevant calculations will not be given marks.
6. It is in your interest to write legibly and present your work neatly.

QUESTION 1

10 MARKS

- a) Calculate: $\lim_{x \rightarrow 3} \frac{3x^2 - 4x + 2}{x + 2}$ (1)

$$= \frac{3(3)^2 - 4(3) + 2}{3 + 2}$$

$$= \frac{17}{5} \quad \checkmark$$

- b) Determine the average gradient of the graph $y = 2x^2$ between the point $x = 1$ and $x = 3$ (2)

$$(1; 2) \quad (3; 18) \quad \checkmark$$

$$m = \frac{18 - 2}{3 - 1}$$

$$= 8 \quad \checkmark$$

- c) $P(2; 4)$ is a point on the curve $y = x^2$. Determine the gradient of the function at this point. (2)

$$\frac{dy}{dx} = 2x \quad \checkmark$$

$$m = 2(2)$$

$$= 4 \quad \checkmark$$

- d) The gradient of the curve of $y = x^3 + ax^2$ is equal to -4 at the point $x = 2$. Determine the value of a . (2)

$$\frac{dy}{dx} = 3x^2 + 2ax$$

$$-4 = 3(2)^2 + 2a(2)$$

$$-16 = 4a$$

$$-4 = a$$

- e) What is the value of y on the graph of $y = 3x^2 - 2x + 1$ where the gradient is 4? (3)

$$\frac{dy}{dx} = 6x - 2$$

$$4 = 6x - 2$$

$$1 = x$$

$$(1; 2)$$

QUESTION 2

15 MARKS

a) If $f(x) = 5x^2 + 3x$, determine $f'(x)$ using first principles.

(4)

$$f(x) = 5x^2 + 3x$$

$$f(x+h) = 5(x+h)^2 + 3(x+h)$$

$$= 5x^2 + 10xh + 5h^2 + 3x + 3h$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{5x^2 + 10xh + 5h^2 + 3x + 3h - (5x^2 + 3x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(10x + 5h + 3)}{h}$$

$$f'(x) = 10x + 5(0) + 3$$

$$\underline{f'(x) = 10x + 3}$$

- 2 notation

b) Determine:

$$1) \quad f'(x) \text{ if } f(x) = \frac{x^2}{4} - 3x$$

(2)

$$f(x) = \frac{1}{4} x^2 - 3x$$

$$\underline{f'(x) = \frac{1}{2} x - 3}$$

2) $D_x[(2x^3 - 1)^2]$

(3)

$$D_x [4x^6 - 4x^3 + 1]$$

$$= \underline{24x^5 - 12x^2}$$

3) $\frac{dy}{dt}$ if: $y = \frac{7}{t} - 5\sqrt{t} + \frac{3}{\sqrt[3]{t}}$ (Leave your answer with positive exponential values)

(3)

$$y = 7t^{-1} - 5t^{\frac{1}{2}} + 3t^{-\frac{1}{3}}$$

$$\frac{dy}{dx} = -7t^{-2} - \frac{5}{2}t^{-\frac{1}{2}} - 1t^{-\frac{4}{3}}$$

$$\frac{dy}{dx} = \frac{-7}{t^2} - \frac{5}{2t^{\frac{1}{2}}} - \frac{1}{t^{\frac{4}{3}}}$$

4) $\frac{dy}{dx}$ if $2xy - 3y = 2x^2 - 7x + 6; x \neq \frac{3}{2}$ (3)

$$y(2x-3) = \frac{(x-2)(2x-3)}{(2x-3)}$$

$$y = x - 2$$

$$y = 1$$

QUESTION 3

15 MARKS

a) If $(t+1)$; $(1-t)$ and $(2-5t)$ are the first three terms of an infinite, converging geometric sequence, determine:

1) the value(s) of t and the common ratio.

(5)

$$\frac{1-t}{t+1} = \frac{2-5t}{1-t}$$

$$1 - 2t + t^2 = 2t - 5t^2 + 2 - 5t$$

$$6t^2 + t - 1 = 0$$

$$(3t-1)(2t+1) = 0$$

$$t = \frac{1}{3} \quad t \neq -\frac{1}{2}$$

$$\text{If } t = \frac{1}{3}$$

$$\frac{4}{3} ; \frac{2}{3} ; \frac{1}{3}$$

$$r = \frac{1}{2}$$

$$\text{If } t = -\frac{1}{2}$$

$$\frac{1}{2} ; \frac{3}{2} ; \frac{9}{2}$$

$$r = 3$$

2) the sum to infinity

(2)

$$S_{\infty} = \frac{\frac{4}{3}}{1 - \left(\frac{1}{2}\right)} \\ = \frac{8}{3}$$

b) The following information is given about a geometric series:

$$S_n = -\left(\left(\frac{1}{3}\right)^n - 1\right)$$

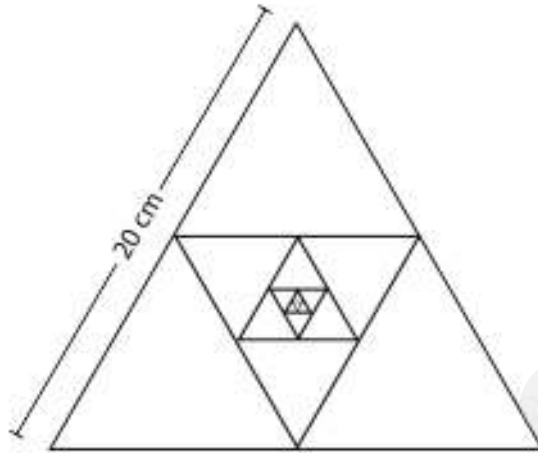
$$\sum_{n=5}^{\infty} T_n = \frac{1}{81}$$

Evaluate:

(3)

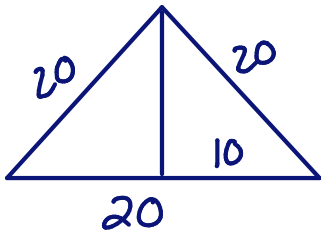
$$\sum_{n=1}^{\infty} T_n \quad S_4 = -\left[\left(\frac{1}{3}\right)^4 - 1\right] \\ = \frac{80}{81} \\ \therefore S_{\infty} = \frac{80}{81} + \frac{1}{81} \\ = 1$$

- c) The figure below shows three equilateral triangles in an infinite sequence of successively smaller equilateral triangles formed by connecting the midpoints of the sides of the proceeding larger triangle. The largest triangle has sides of length 20cm.



- 1) Determine the area of the largest triangle.

(2)



$$\text{Area} = \frac{1}{2} (20)(20) \cdot \sin 60^\circ$$

$$= 173,21$$

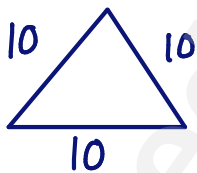
OR

$$\text{Area} = \frac{1}{2} \cdot 20 \cdot (10\sqrt{3})$$

$$= 173,21$$

- 2) Find the total sum of the area for all the triangles in the infinite series.

(3)



$$A_2 = \frac{1}{2} (10)(10) \cdot \sin 60^\circ$$

$$= 25\sqrt{3}$$

$$r = \frac{1}{4}$$

$$S_\infty = \frac{173,21}{1 - (\frac{1}{4})}$$

$$= 230,94 \text{ cm}^2$$

Additional working out space:

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