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ST BENEDICT'S

SUBJECT	**Mathematics**	**PAPER**	**Algebra and Sequences Part B**
GRADE	**12**	**DATE**	1 Feb 2023
EXAMINER	Hayward	**MARKS**	35
NAME		**MODERATOR**	Mrs Povall, Ms Brown, Mrs Myburgh
TEACHER		**DURATION**	40 minutes

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
2	Factor Remainder Theorem	**7**	
3	Nature of Roots	**7**	
4	Sequences and Series	**16**	
5	Problem Solver	**5**	
TOTAL		**35**	

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INSTRUCTIONS:

1. This paper consists of 9 pages. A page for extra working has been provided. Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all questions on the question paper and hand it in at the end of the test.
4. It is in your interest to write legibly and present your work neatly.
5. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. Ensure that your calculator is in **DEGREE** mode.
8. Round off your answers to **ONE** decimal place unless otherwise stated.

Formulae:

$$x = \frac{(-b) \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

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QUESTION 2 **7 MARKS**

Given $f(x) = x^3 + 2x^2 + kx - 6$, find k if:

a) The remainder is 6 when f is divided by $x + 2$. (4)

$$f(-2) = 6$$
$$(-2)^3 + 2(-2)^2 + k(-2) - 6 = 6$$
$$-8 + 8 - 2k - 6 = 6$$
$$k = -6$$

b) $x - 3$ is a factor of f. (3)

$$f(3) = 0$$
$$(3)^3 + 2(3)^2 + k(3) - 6 = 0$$
$$27 + 18 + 3k - 6 = 0$$
$$3k = -39$$
$$k = -13$$

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QUESTION 3 — 7 MARKS

a) Given $p^2x^2 + 6px + 9 = 0$. Show that the roots of the equation are equal for $p \in \mathbb{R}$. (3)

$$\Delta = b^2 - 4ac$$
$$= (6p)^2 - 4(p^2)(9)$$
$$= 36p^2 - 36p^2$$
$$= 0$$
$$\therefore \textit{Roots are Real, Rational and } \boldsymbol{equal}$$

b) For which values of x is $\sqrt{x^2 - 3x}$ real. (4)

$$x^2 - 3x \geq 0$$
$$x(x-3) \geq 0$$
$$C.V: 0 \textit{ and } 3$$
Valid Method
$$x \in (-\infty; 0] \cup [3; \infty)$$

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QUESTION 4 — 16 MARKS

a) Given the arithmetic series: $S_n = \Sigma_{x=1}^{n}(4n-1)$

1) Calculate the sum of the first twenty terms of the sequence. (3)

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{20}{2}[2(3) + (20-1)(4)]$$
$$= 10(6 + 76)$$
$$= 820$$

2) If the nth term of the sequence is 191, determine the value of n. (2)

$$T_n = a + (n-1)d$$
$$191 = 4n - 1$$
$$4n = 192$$
$$n = 48$$

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b) The third term of a geometric sequence is 72 and the sixth term is -243. Determine the first term and the common ratio. (5)

$$T_3 = ar^2 = 72$$

$$T_6 = ar^5 = -243$$

$$r^3 = -\frac{243}{72} = -\frac{27}{8}$$

$$\therefore r = -\frac{3}{2}$$

$$\therefore a\left(-\frac{3}{2}\right)^2 = 72$$

$$a = 32$$

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c) Consider the pattern: $2,\ 7,\ 16,\ 29,\ 46$ ...

1) If the pattern continues in this way, write down the next two terms. (2)

$$67,\ 92$$

2) Determine the nth term of the sequence. (4)

$$2a = 4;\quad 3a + b = 5;\quad a + b + c = 2$$

$$a = 2;\quad b = -1;\quad c = 1$$

$$T_n = 2n^2 - n + 1$$

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QUESTION 5 5 MARKS

The sum of the first three consecutive terms of an arithmetic sequence is 15. If 1, 4 and 19 are respectively added to the three terms, the resulting numbers form three consecutive terms of a geometric sequence. Write down the original arithmetic sequence(s).

A.S: $a;\ \ a+d;\ \ a+2d$

$$(a)+(a+d)+(a+2d)=15$$
$$3a+3d=15$$
$$a=5-d$$

G.S: $a+1;\ \ a+d+4;\ \ a+2d+19$

$$\frac{T_2}{T_1}=\frac{T_3}{T_2}$$
$$\frac{a+d+4}{a+1}=\frac{a+2d+19}{a+d+4}$$
$$\frac{(5-d)+d+4}{(5-d)+1}=\frac{(5-d)+2d+19}{(5-d)+d+4}$$
$$\frac{9}{6-d}=\frac{24+d}{9}$$
$$81=(24+d)(6-d)$$
$$0=144+6d-24d-d^2-81$$
$$0=d^2+18d-63$$
$$0=(d+21)(d-3)$$
$$d=-21\ \ OR\ \ d=3$$
$$a=26\ \ OR\ \ a=2$$

A.S: $26;\ 5;\ -16$ OR $2;\ 5;\ 8$

G.S: $27;\ 9;\ 3$ OR $3;\ 9;\ 27$

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EXTRA WORKING:

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