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ST BENEDICT'S

SUBJECT	Mathematics	PAPER	Sequences and series
GRADE	12	DATE	17/02/2021
EXAMINER	Mrs K Myburgh	MARKS	35
NAME	Memorandum	MODERATOR	Grade 12 teachers
TEACHER		DURATION	1 period

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1	Arithmetic number patterns	13	
2	Geometric number patterns	15	
3	Quadratic number patterns	7	
TOTAL		35	

INSTRUCTIONS:**Downloaded from eduresourceza.com**

1. This paper consists of 3 questions and 8 pages.
2. Read the questions carefully.
3. Answer all questions in the spaces provided.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. Round off your answers to two decimal digits where necessary.
6. All necessary working details must be shown. Answers only, without the relevant calculations will not be given marks. Equations may not be solved solely with a calculator.
7. It is in your interest to write legibly and present your work neatly.

USEFUL FORMULAE:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$T_n = a \times r^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_\infty = \frac{a}{1 - r}$$

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QUESTION 1

13 MARKS

- a) Find the first three terms of an arithmetic sequence if $T_5 = 10$ and $T_{10} = -30$. (4)

$$T_5 = 10$$

$$a + 4d = 10$$

$$T_{10} = -30$$

$$a + 9d = -30 \quad \checkmark m$$

$$\begin{array}{r} a + 4d = 10 \\ - \quad a + 9d = -30 \\ \hline \end{array}$$

$$-5d = 40$$

$$d = -8 \quad \checkmark a$$

$$a + 4(-8) = 10$$

$$a = 42 \quad \checkmark a$$

$$\underline{42; 34; 26} \quad \checkmark ca$$

- b) Consider the following finite series: $5 + 1 + (-3) + \dots + (-255)$

- 1) Determine how many terms are in this finite series. (3)

$$T_n = -4n + 9 \quad \checkmark m$$

$$-255 = -4n + 9$$

$$-264 = -4n$$

$$66 = n \quad \checkmark a$$

2) Find the sum of this finite series.

(2)

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{66} = \frac{66}{2} [2(5) + (66-1)(-4)] \checkmark m$$

$$\underline{S_{66} = -8250 \checkmark a}$$

3) Determine how many terms in the finite series are divisible by 5.

(4)

(5); (1); (-3); (-7); (-11); (-15); (-19); (-23); (-27); (-31); (-35)

$$5; -15; -35 \dots \checkmark m$$

$$\begin{array}{ccc} & \diagdown & \diagup \\ 5 & & -15 \\ & \diagup & \diagdown \\ -20 & & -20 \end{array}$$

$$T_n = -20n + 25 \checkmark m$$

$$-255 = -20n + 25$$

$$-280 = -20n$$

$$\underline{14 = n \checkmark a}$$

QUESTION 2

15 MARKS

a) Consider the following geometric sequence: $8x^2 + 4x^3 + 2x^4 + \dots$ 1) Find the common ratio in terms of x

(1)

$$r = \frac{4x^3}{8x^2}$$

$$= \frac{x}{2} \quad \checkmark a$$

2) Determine the n^{th} term of the series in terms of x .

(1)

$$T_n = a \times r^{n-1}$$

$$T_n = 8x^2 \times \left(\frac{x}{2}\right)^{n-1} \quad \checkmark a$$

3) Determine the values of x for which the series converges.

(2)

$$-1 < r < 1$$

$$-1 < \frac{x}{2} < 1 \quad \checkmark m$$

$$\underline{-2 < x < 2} \quad \checkmark a$$

4) Calculate S_{∞} if: $x = 1$. Downloaded from eduresourceza.com

(2)

$$S_{\infty} = \frac{8}{1 - \frac{1}{2}} \quad \checkmark_m \quad \begin{matrix} a = 8 \\ r = \frac{1}{2} \end{matrix}$$

$$= 2 \checkmark_a \\ = 16$$

5) Calculate S_{∞} if: $x = 32$.

(1)

$$S_{\infty} : \text{Does not exist} \quad \checkmark_a \\ \underline{r > 1} \quad \begin{matrix} a = 8192 \\ r = 16 \end{matrix}$$

b) Calculate, showing all working out:

(4)

$$\sum_{m=0}^{79} \frac{(-1)^{m+1}}{2^m}$$

$$(-1) + \left(\frac{1}{2}\right) + \left(-\frac{1}{4}\right) + \dots$$

$$n = 79 - 0 + 1 \\ = 80$$

$$S_{80} = \frac{(-1) \left(\left(-\frac{1}{2}\right)^{80} - 1 \right)}{-\frac{1}{2} - 1} \quad \checkmark_m \\ = \frac{-2}{3} \checkmark_a \quad ca$$

- c) Determine the greatest number of terms for which the sum of: $8 + 16 + 32 + \dots$ will be less than 504.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

(4)

$$504 = \frac{(8)(2^n - 1)}{2 - 1} \checkmark m$$

$$63 = 2^n - 1$$

$$64 = 2^n$$

$$2^6 = 2^n \checkmark m$$

$$6 = n \checkmark a$$

$$\therefore \underline{5 \text{ terms} \checkmark a}$$

QUESTION 3

7 MARKS

Consider the sequence: 29; 43; 61; 83 ...

- a) Determine the general term for this sequence.

(3)

$$29; 43; 61; 83 \dots$$

$$2a = 4$$

$$a = 2$$

$$14 \quad 18 \quad 22$$

$$3(2) + b = 14$$

$$b = 8$$

$$4 \quad 4$$

$$\therefore \underline{T_n = 2n^2 + 8n + 19}$$

$$2 + 8 + C = 29$$

$$C = 19$$

- b) By completing the square, show that all terms in this sequence will be odd.

(4)

$$T_n = 2 \left[n^2 + 4n + \frac{19}{2} \right]$$

$$T_n = 2 \left[n^2 + 4n + (2)^2 - (2)^2 + \frac{19}{2} \right]$$

$$T_n = 2 \left[(n+2)^2 + \frac{11}{2} \right]$$

$$T_n = 2(n+2)^2 + 11$$

$$\therefore \text{even} + \text{odd} = \text{odd} \checkmark \text{explanation}$$

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