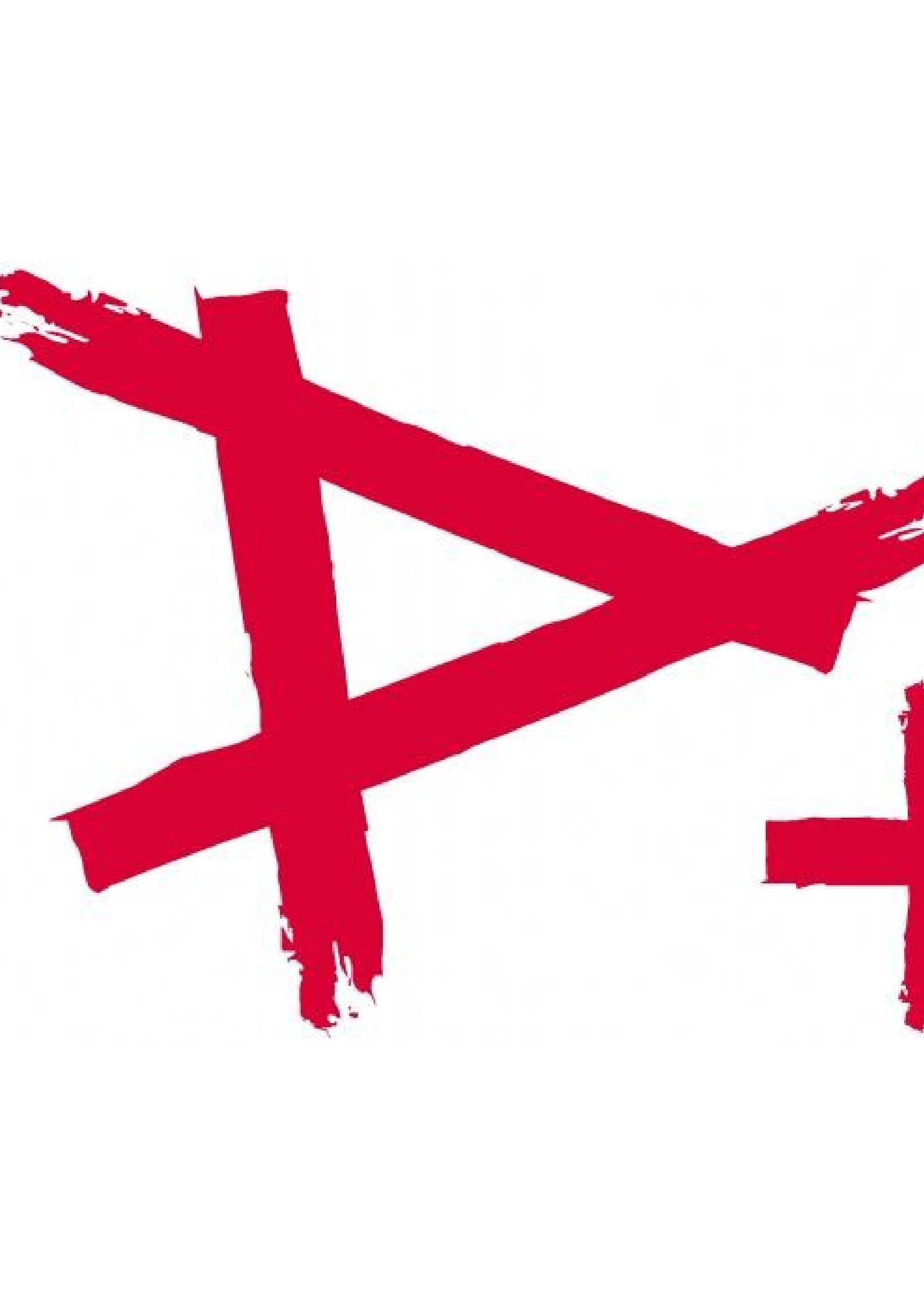






MATHEMATICS – PAPER 1

Time: 3 hours

Exam no. _____

150 marks

NAME: _____

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 26 pages. Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. You may use an approved non-programmable and non-graphical calculator unless otherwise stated. Ensure your calculator is in **DEGREE** mode.
5. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
6. Diagrams are not necessarily drawn to scale.
7. If necessary, round off answers to **ONE** decimal place, unless otherwise stated.
8. It is in your own interest to write legibly and to present your work neatly.
9. Three blank pages (Pages 24 to 26) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.

For official use only.

	Equations	Calculus	Sequences	Functions	Financial Maths	Sequences	Calculus	Probability	Calculus	Functions
Question	1	2	3	4	5	6	7	8	9	10
Mark										
Total	30	12	14	19	15	16	17	11	7	9
Total										
Percentage										
150										
100										

SECTION A

Question 1

(a) Solve for x and show ALL relevant working:

1. $2x^2 - 17x = 0$

$$x(2x - 17) = 0$$

$$\therefore x = 0 \quad \text{or} \quad 2x - 17 = 0$$

$$x = \frac{17}{2} \quad (\text{or } 8, 5)$$

(2)

2. $3x^2 + 10x - 4 = 0$ (correct to 1 decimal place)

$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-4)}}{2(3)}$$

$$x = \frac{-10 \pm \sqrt{148}}{6}$$

$$x = \frac{-10 \pm \sqrt{148}}{6}$$

$$x = 0, 4 \quad \text{or} \quad x = -3, 7$$

(3)

3. 8. $2^x = \left(\frac{1}{4}\right)^{x-2}$

$$2^3 \cdot 2^x = (2^{-2})^{x-2}$$

$$2^{3+x} = 2^{-2x+4}$$

$$\therefore 3 + x = -2x + 4$$

$$3x = 1$$

$$x = \frac{1}{3}$$

(3)

4. $(x-5)^2 < x-5$

$$x^2 - 10x + 25 - x + 5 < 0$$

✓ (x out side)

$$x^2 - 11x + 30 < 0$$

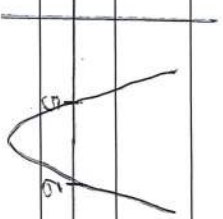
$$(x-6)(x-5) < 0$$

C. Values : $x=5$ or 6

$$+ \frac{9}{5} - \frac{9}{6} +$$

or

$$\frac{1}{6}$$



SOLN : $5 < x < 6$

(4)

5. $\log_3 x = \frac{\log 5}{\log 3}$

$$\log_3 x = \log_3 5$$

$$\text{or } \log_3 x = 1, 4, 6, 4, 9, \dots$$

$$\therefore x = 5$$

$$\therefore 3^1, 3^4, 3^6, \dots = 5$$

(2)

6. $(3^x + 9)(2^x - 27) = 0$

$$3^x + 9 = 0$$

or

$$2^x - 27 = 0$$

$$3^x = -9$$

$$2^x = 27$$

No solution

$$\log_2 27 = x$$

$$\therefore x = 4, 8$$

(4)

7. The roots of a quadratic equation are $\frac{5 \pm \sqrt{17+4m^2}}{2m}$ where $m \neq 0$.

Explain why this equation will have real roots for all $m \in \mathbb{R}$ where $m \neq 0$.

$$\Delta = 17 + 4m^2 \quad \checkmark \text{ (going } \Delta^1 \text{)} \quad m^2 \geq 0$$

$$\therefore 4m^2 \geq 0 \quad \text{and} \quad \therefore 4m^2 + 17 \geq 0$$

$\therefore \Delta$ is always positive $\checkmark$

$\therefore$ Roots are real.

(2)

- (b) Given $f(x) = \sqrt{2x+1}$

1. Give the domain of $f(x)$

$$2x+1 \geq 0 \quad \therefore x \geq -\frac{1}{2} \quad \checkmark$$

(1)

2. If $g(x) = 2x - 1$, solve for $f(x) = g(x)$

$$2x - 1 = \sqrt{2x + 1}$$

$$(2x - 1)^2 = 2x + 1 \quad \checkmark \text{ (m/squaring)}$$

$$4x^2 - 4x + 1 - 2x - 1 = 0$$

$$4x^2 - 6x = 0 \quad \checkmark$$

$$2x(2x - 3) = 0 \quad \checkmark \text{ (Factor)}$$

$$\therefore x = 0 \quad \text{or} \quad x = \frac{3}{2}$$

$$\text{Check: LHS: } -1$$

$$\text{LHS: } 2$$

$$\text{RHS: } 1$$

$$\text{RHS: } \sqrt{2\left(\frac{3}{2}\right) + 1} = 2$$

$$\therefore \text{Soln: } x = \frac{3}{2} \text{ only.} \quad \checkmark$$

(4)

(c) Given: $\log_2 x = y$ and $2^{y+1} + x = 12$, solve simultaneously for x and y .

$$2^y = x \quad \text{and} \quad 12 - 2^{y+1} = x \quad \text{--- (2)}$$

$$\textcircled{1} = \textcircled{2}$$

$$2^y = 12 - 2^{y+1} = 12$$

$$2^y (1 + 2) = 12$$

$$2^y = 4$$

$$2^y = 2^2 \quad \therefore y = 2$$

Subst back into $\textcircled{1}$

$$\therefore 2^2 = x$$

$$x = 4$$

Solution: $(4, 2)$

[30]

Question 2

- (a) 1. Using first principles, find
- $f'(x)$
- if
- $f(x) = x^2 - 3x$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[(x+h)^2 - 3(x+h)] - (x^2 - 3x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 3h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2x + h - 3)}{h}$$

$$= \lim_{h \rightarrow 0} (2x + h - 3)$$

$$= \lim_{h \rightarrow 0} \frac{h(2x + h - 3)}{h}$$

$$= \lim_{h \rightarrow 0} (2x + h - 3) = 2x - 3$$

$$h \rightarrow 0$$

(4)

2. Determine the equation of the tangent to the graph of
- f
- at the point where
- $f'(x) = -1$
- .

$$2x - 3 = -1$$

Egn of tang

$$\therefore x = 1$$

$$\text{So } f(1) = 1^2 - 3(1)$$

$$= -2$$

$$y = -x - 1$$

$$\text{Point } (1, -2)$$

(4)

(b) Determine $\frac{dy}{dx}$ if $y = \sqrt{4x^3} + 2\sqrt{x} - \frac{2}{x}$ (leave your answer with positive exponents)

$$y = 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 2x^{-1} \quad \checkmark^m \text{ (ready to derive)} \quad \checkmark^m \quad \checkmark^m$$

$$\therefore \frac{dy}{dx} = 3x^{\frac{1}{2}} + x^{-\frac{1}{2}} + 2x^{-2} \quad \checkmark^m \quad \checkmark^m$$

$$= 3x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}} + \frac{2}{x^2}$$

(-1) if ans left as -ve expts

(4)

[12]

Question 3

(a) Given: $T_n = 2^{1-n}$

1. Write down the first TWO terms of the sequence.

$$T_1 = 2^{1-1} = 2^0 = 1$$

$$T_2 = 2^{1-2} = 2^{-1} = \frac{1}{2}$$

(2)

2. Determine
- S_4
- of the sequence. (Leave your answer as a fraction)

$$T_3 = \frac{1}{4}$$

$$T_4 = \frac{1}{8}$$

$$\therefore S_4 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

$$= \frac{15}{8}$$

$$\frac{15}{8}$$

(2)

- (b) Determine
- $\sum_{k=-1}^{20} (3k+2)$

$$k = -1 : 3(-1) + 2 = -1$$

$$k = 0 : 3(0) + 2 = 2$$

$$k = 1 : 3(1) + 2 = 5$$

$$\therefore -1 + 2 + 5 + \dots$$

$$a = -1$$

$$d = 3$$

$$S_{22} = \frac{22}{2} [2(-1) + 21(3)]$$

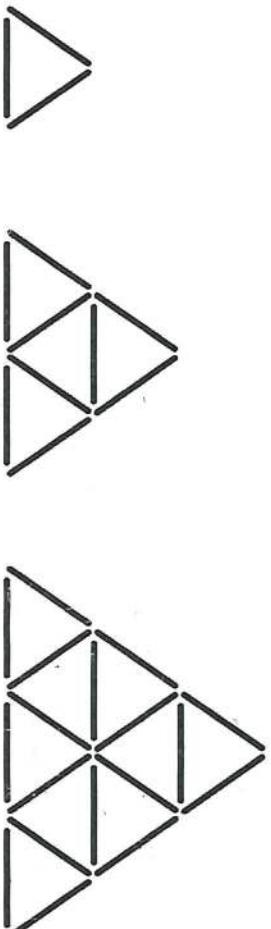
$$= 11(61)$$

$$= 671$$

$$\frac{671}{1}$$

(4)

- (c) The first three patterns formed by the arrangement of toothpicks are shown below:



Pattern 1

3

Pattern 2

9

Pattern 3

18

1. How many toothpicks will the fourth pattern have?

30 ✓¹¹

(1)

2. Determine the n^{th} term of the sequence.

3 ; 9 ; 18 ; 30 ✓¹¹

$$\begin{array}{c} \swarrow \quad \searrow \\ 6 \quad 9 \quad 12 \end{array}$$

$$\begin{array}{c} \swarrow \quad \searrow \\ 3 \quad 3 \end{array}$$

$$2a = 3$$

$$3\left(\frac{3}{2}\right) + b = 6$$

$$\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right) + c = 3$$

$$a = \frac{3}{2} \quad \checkmark$$

$$\therefore b = 6 - \frac{9}{2}$$

$$c = 0 \quad \checkmark$$

$$b = \frac{3}{2} \quad \checkmark$$

$$\therefore T_n = \frac{3}{2}n^2 + \frac{3}{2}n$$

(4)

3. How many toothpicks will be needed to form the 15th pattern?

$$T_{15} = \frac{3}{2}(15)^2 + \frac{3}{2}(15)$$

$$= 360 \quad \checkmark^{11}$$

(1)

[14]

Question 4

- (a) The equations of the graphs of $g(x) = \frac{3x-3}{x-1}$ and $h(x) = -\frac{3}{x-1} + 3$ are given.

1. Show that $g(x)$ is an asymptote of $h(x)$ and state whether it is the vertical or horizontal asymptote.

$$g(x) = \frac{3(x-1)}{x-1} = 3$$

$\therefore g(x) = 3$ is the horizontal asymptote of $h(x)$

(2)

2. Write down the domain and range of $h(x)$.

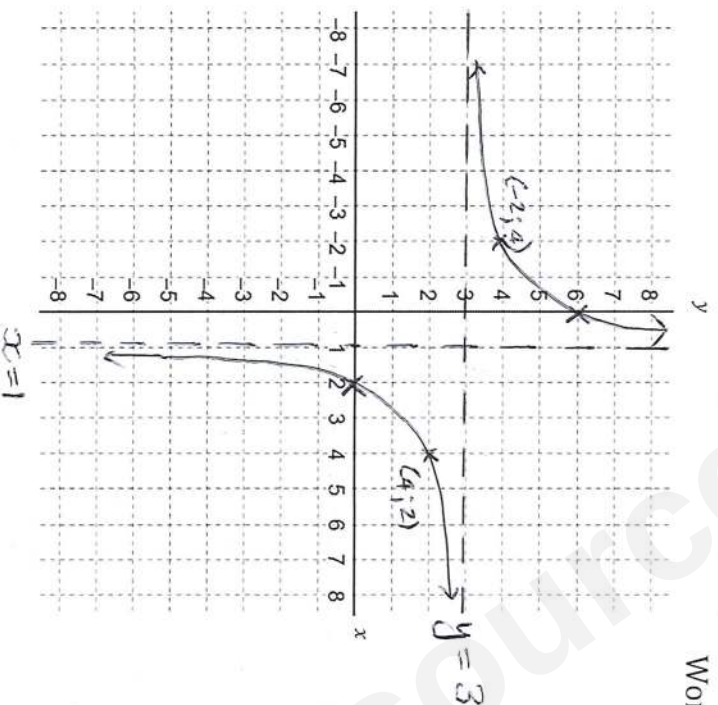
$$\text{Domain: } x \in \mathbb{R}; x \neq 1$$

$$\text{Range: } y \in \mathbb{R}; y \neq 3$$

(2)

3. Sketch the graph of $h(x)$. Show clearly all intercepts with the axes, the asymptotes and two points on each arm of the graph.

Working space



$$\underline{\text{y int: let } x=0}$$

$$\therefore y = \frac{-3}{-1} + 3 = 6$$

(0, 6)

$$\underline{\text{x int: let } y=0}$$

$$0 = \frac{-3}{x-1} + 3$$

$$3 = 3(x-1)$$

$$3xc = 6$$

$$x = 2$$

(2, 0)

$$\underline{\text{Choose } x = -2}$$

$$y = \frac{-3}{-3} + 3 = 4$$

(4)

(-2, 4)

$$\underline{\text{Choose } x = 4}$$

$$y = \frac{-3}{3} + 3 = 2$$

(4, 2)

✓ asymptotes
✓ slope-
✓ intercepts
✓ other points

- (b) The graph shows the function $g(x) = -x + 3$ and $h(x) = b^x$. The two functions intersect at $A(1;2)$.

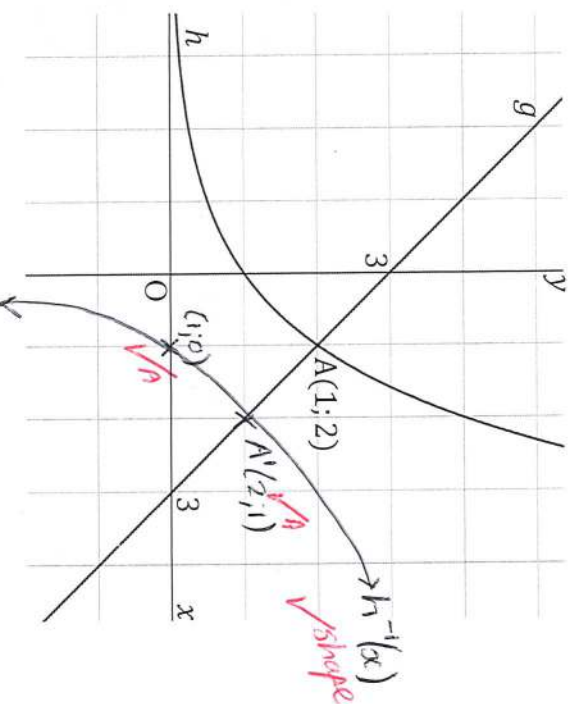
1. Determine the value of b .

Subst $(1;2)$ into $h(x)$ ✓^m

$$\therefore 2 = b^1$$

$$b = 2 \quad \checkmark^A$$

(2)



2. If the point $A(1;2)$ lies on h , determine the new coordinates, A' , when h is reflected about the line $y = x$.

$$A'(2;1) \quad \checkmark^A$$

(1)

3. Write down the inverse function of $h(x)$ in the form $h^{-1}(x) = \dots\dots$

$$\text{Inverse } h(x) : x = 2^y \quad \checkmark^m(\text{switch})$$

$$\therefore y = \log_2 x \quad \therefore h^{-1}(x) = \log_2 x \quad \checkmark^A \text{ (must be in } h^{-1}(x) \text{ form)}$$

(2)

4. On the graph above, sketch h^{-1} labelling any intercepts with the axes as well as point A' .

(3)

5. Use the graphs above to determine the values of x for which $-2^x \cdot x + 6 \cdot 2^{x-1} < 0$.

$$-2^{2^x} \cdot x + 6 \cdot 2^x \cdot 2^{-1} < 0$$

$$\checkmark^m 2^x (-x + 6(\frac{1}{2})) < 0$$

$$2^{2^x} (-x + 3) < 0 \quad \checkmark^A$$

for $x > 3 \quad \checkmark^A$ (Read from x-int of g)

(3)

[19]

Total for Section A: 75

SECTION B

Question 5

- (a) Alice starts saving for retirement and pays R800 into a savings account at the end of each month. After she makes her last payment she has R884 942,47 in her retirement fund. She receives an interest rate of 6,5% per annum compounded monthly.

1. Determine the number of payments that she made.

$$x = R800$$

$$FV = 884\,942,47 = 800 \left[\left(1 + \frac{0,065}{12}\right)^n - 1 \right] \quad \checkmark N$$

$$i = \frac{0,065}{12}$$

$$4\,793,438379 = 800 \left[\left(\frac{2413}{2400}\right)^n - 1 \right]$$

$$n = ?$$

$$6,9917\dots = \left(\frac{2413}{2400}\right)^n$$

$$\therefore \log \left(\frac{2413}{2400}\right)^m = \log \left(\frac{2413}{2400}\right)^n \quad \checkmark m$$

$$n = 360 \text{ payments} \quad \checkmark N$$

(if you use 12n then $n = 30 \text{ years} \times 12 = 360$) (4)

2. Convert the nominal interest rate to an effective interest rate.

$$1 + i_{\text{eff}} = \left(1 + \frac{i_{\text{nom}}}{12}\right)^{12} \quad \checkmark m(\text{sub})$$

$$\therefore 1 + i_{\text{eff}} = \left(1 + \frac{0,065}{12}\right)^{12}$$

$$i_{\text{eff}} = \left(\frac{2413}{2400}\right)^{12} - 1$$

$$i_{\text{eff}} = 0,06697 \quad \checkmark N$$

$$\therefore \text{eff. rate is } 6,7\% \text{ pa} \quad \checkmark$$

(2)

- (b) Thandi took a mortgage loan of R 850 000 to buy a house and was required to pay equal monthly instalments for 30 years. She was charged interest at 8% p.a, compounded monthly.

1. Show that her monthly instalment was R 6237.

$$850\,000 = x \left[1 - \left(1 + \frac{0,08}{12}\right)^{-360} \right] \quad \checkmark m(PV)$$

$$\frac{0,08}{12} \quad \checkmark N$$

$$i = \frac{0,08}{12}$$

$$\therefore x = R6237$$

$$n = 30 \times 12$$

$$x = ?$$

(3)

2. Calculate the outstanding balance on her loan at the end of the first year (Round off your answer to the nearest rand).

$ \begin{aligned} 08 &= FV - FV_{\text{ANNUITY}} \\ &= 850000 \left(1 + \frac{0,08}{12}\right)^{12} - 6237 \left[\left(1 + \frac{0,08}{12}\right)^{12} - 1 \right] \times \frac{0,08}{12} \\ &= 920549,5808 - R\,77650,18859 \\ &\approx R\,842\,899,39 \checkmark^m \\ &= R\,842\,899 \text{ (to nearest rand)} \end{aligned} $	<i>OR using Rule with 29 years to go</i> $ \begin{aligned} &P_r = 6237 \left[1 - \left(1 + \frac{0,08}{12}\right)^{-38} \right] \times \frac{(0,08)}{12} \\ &= R\,842\,899,56 \end{aligned} $
---	--

(3)

3. Hence, or otherwise, calculate how much of the R 74 844 that she paid during the first year, was taken by the finance company as payment towards the interest it charged.

$$\begin{aligned}
 \text{Loan amount reduced by } R\,850\,000 - R\,842\,899 \\
 = R\,7101 \checkmark^m
 \end{aligned}$$

But she has paid $6237 \times 12 = R\,74\,844$ in the 1st year

$\therefore$ Payments made towards interest charged

$$74\,844 - \checkmark^m 7101$$

$$= R\,67\,743 \checkmark^{ca}$$

(3)

[15]

Question 6

- (a) The sum to infinity of a geometric sequence is 4 times the common ratio and the first term is three times the common ratio. Find the second term.

$$S_{\infty} = 4r \quad \text{and} \quad T_1 = a = 3r$$

$$\frac{a}{1-r} = 4r \dots \textcircled{1} \quad a = 3r \dots \textcircled{2} \quad \text{let up, into eqns, to } a \text{ and } r$$

Subst $\textcircled{2}$ into $\textcircled{1}$ *(solve simult)*

$$\therefore \frac{3r}{1-r} = 4r \quad \checkmark$$

$$\therefore 3r = 4r(1-r) \quad \checkmark A$$

$$\therefore 4r^2 - 4r + 3r = 0 \quad \checkmark$$

$$4r^2 - r = 0$$

$$r(4r - 1) = 0$$

$$\cancel{r=0} \quad \text{OR } r = \frac{1}{4} \quad \checkmark A \text{ (for } T_1 \text{ and } T_2) \quad \therefore T_2 = \frac{3}{4} \left(\frac{1}{4} \right) = \frac{3}{16}$$

- (b) Determine the smallest value of n for which $\sum_{k=1}^n \frac{1}{4} (2)^{k-1} > 9$.

$$k=1: \frac{1}{4} (2^0) = \frac{1}{4}$$

$$k=2: \frac{1}{4} (2^1) = \frac{1}{2}$$

$$k=3: \frac{1}{4} (2^2) = 1$$

$$k=4: \frac{1}{4} (2^3) = 2$$

$\therefore$ Geometric series with
 $a = \frac{1}{4}$ $r = 2$ $S_n > 9$

$$\therefore \frac{\frac{1}{4} (2^n - 1)}{2 - 1} > 9$$

$$\therefore 2^n - 1 > 36$$

$$2^n > 37$$

$$2^5 = 32$$

$$2^6 = 64 > 37$$

$\therefore$ 6 terms must be added to give a sum > 9 .

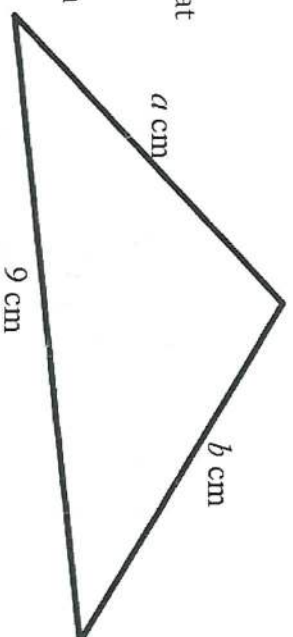
(5)

(c) Refer to the figure (not drawn to scale):

The lengths of the sides of a triangle are such that they form a geometric sequence.

The longest side has a length of 9 cm.

The other two sides are of length a cm and b cm with $a > b$.



1. Show that $a^2 = 9b$

Sequence is : $b; a; 9$ ✓^m

then $\frac{a}{b} = \frac{9}{a}$ ✓^m (ratio)

$$\therefore a^2 = 9b$$

(2)

2. Given that the perimeter of the triangle is 19 cm, determine the length of the shortest side

$$\text{Perimeter : } a + b + 9 = 19 \quad \text{--- (1)} \quad \checkmark^m$$

Subst $b = \frac{a^2}{9}$ into (1)

$$\therefore a + \frac{a^2}{9} + 9 = 19 \quad \checkmark^m \text{ (solve simultaneously)}$$

$$\therefore a + \frac{a^2}{9} = 10$$

$$9a + a^2 = 90 \quad \checkmark^m \text{ eqn}$$

$$a^2 + 9a - 90 = 0 \quad \checkmark^m \text{ (fact)}$$

$$(a + 15)(a - 6) = 0 \quad \checkmark^m \text{ (fact)}$$

$$\therefore a \neq 15 \quad \text{or} \quad a = 6$$

$\therefore$ Shortest side = b

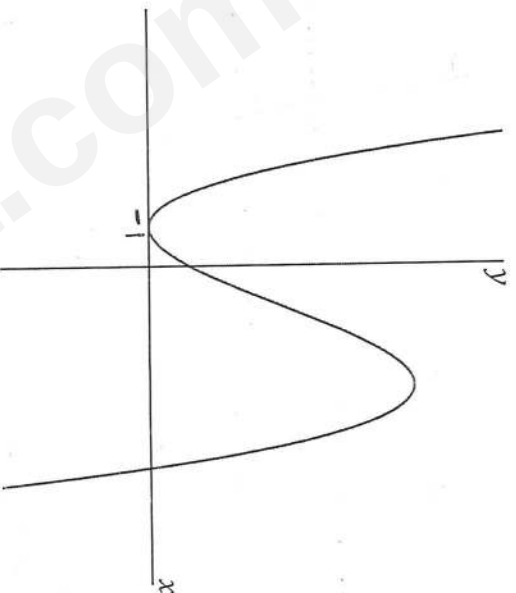
$$\therefore b = \frac{6^2}{9} = 4 \quad \checkmark^m$$

(5)

[16]

Question 7

- (a) Given: $f(x) = -x^3 + 3x^2 + 9x + 5 = -(x+a)^2(x-5)$ and f has a stationary point on the x -axis at $(-1; 0)$.



1. Write down the value of a .

$a = 1$ ✓ⁿ (1)

2. Determine the coordinates of the other stationary point of $f(x)$.

at stat. pt: $f'(x) = 0$ ✓^m

$$\therefore -3x^2 + 6x + 9 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3 \text{ is other stat pt } \therefore f(3) = -3^3 + 3(3)^2 + 9(3) + 5$$

$$= (3; 32) \text{ ✓} = 32 \text{ ✓}^m \text{ (sub)} \text{ (4)}$$

3. Write down the value(s) of x for which $f(x) \cdot f''(x) > 0$

Point of inflection is at $x = 1$

$$x < 1, x \neq 1 \text{ ok } x > 5 \text{ ✓}$$

(3)

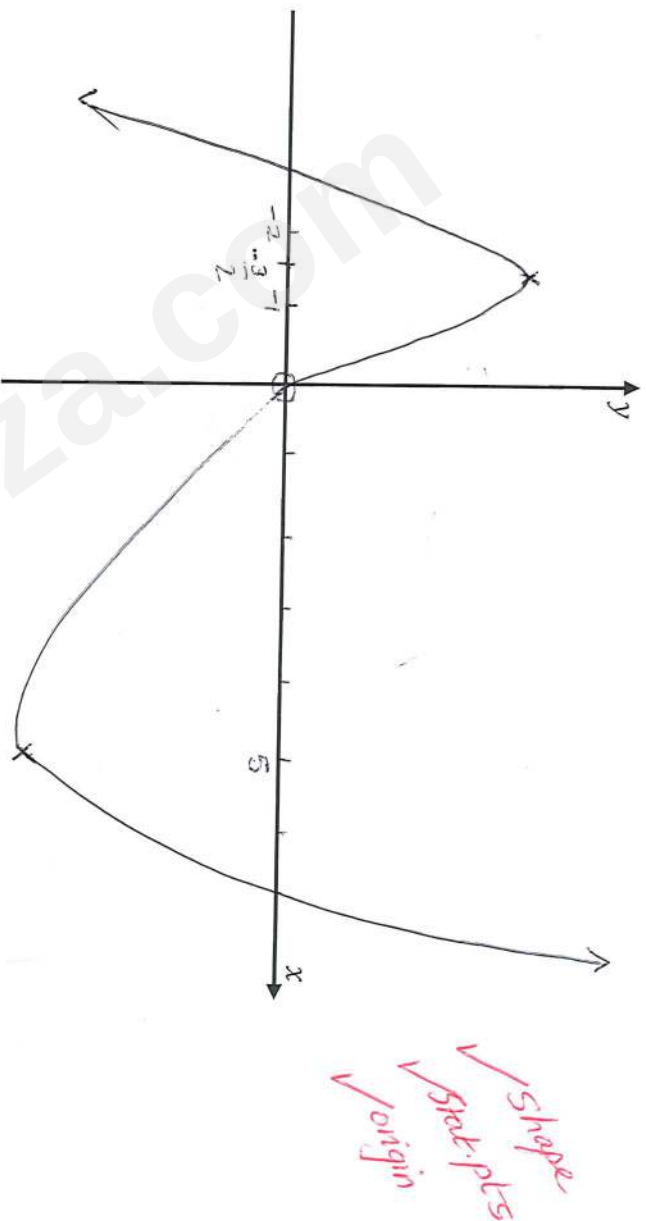
4. For which value(s) of k will $f(x) + k$ have 2 real roots?

$$k = -32 \text{ ✓ or if } k = 0 \text{ ✓}$$

(2)

(b) Given: $f(x) = ax^3 + bx^2 + cx$ with $f'(\frac{-3}{2}) = f'(5) = 0$ and $f(x)$ decreasing for $\frac{-3}{2} < x < 5$

1. Use this information to draw a rough sketch of $y = f(x)$.



2. Determine the value(s) of x for which:

(3)

2.1 $f'(x) > 0$

$x < -3/2$ and $x > 5$

(2)

2.2 $f(x)$ is concave upwards

inflection pt: $x = -\frac{3}{2} + 5 = \frac{7}{2}$

✓m (idea)

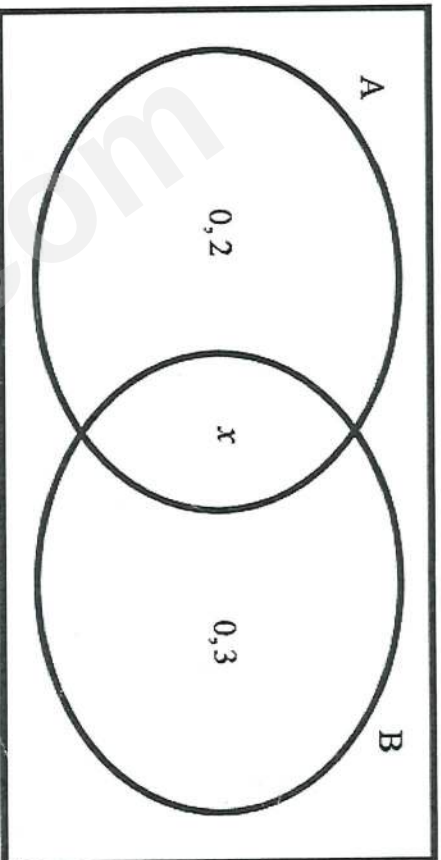
$x > \frac{7}{4}$

(2)

[17]

Question 8

(a) Given the following Venn Diagram:



1. What is the value of
- x
- if
- $P(A \cup B) = 0,9$
- ?

$$x = 0,4 \quad \checkmark_A$$

(1)

2. What is the value of
- x
- if A and B are mutually exclusive?

$$x = 0 \quad \checkmark_A$$

(1)

3. Calculate the value(s) of
- x
- if A and B are independent.

$$\text{independent} \therefore \checkmark_{\text{value}} P(A \cap B) = P(A) \times P(B)$$

$$\therefore (0,2+x) \times (0,3+x) = x \quad \checkmark_A$$

$$0,06 + 0,5x + x^2 = x$$

$$\therefore 6 + 50x + 100x^2 = 100x$$

$$100x^2 - 50x + 6 = 0$$

$$50x^2 - 25x + 3 = 0 \quad \checkmark_{\text{ml solve eqn}}$$

$$(10x - 3)(5x - 1) = 0$$

$$x = \frac{3}{10} \quad \text{OR} \quad x = \frac{1}{5}$$

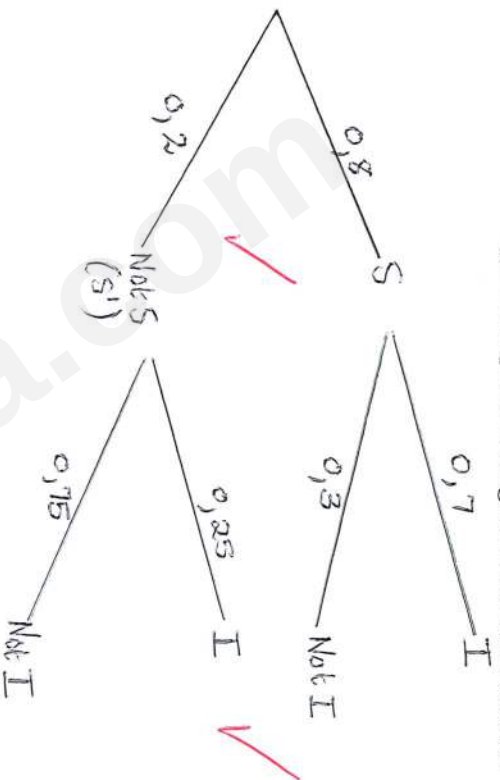
$$\therefore x = 0,3 \quad \text{OR} \quad x = 0,2 \quad \checkmark_A$$

(4)

- (b) The probability that it will be a sunny day is 0,8.
On sunny days, the probability that Lawrence will buy an ice cream is 0,7.
Given that it is **not** a sunny day, the probability that he buys an ice cream is 0,25.



1. Draw a tree diagram representing the scenario above.



(2)

2. Determine the probability that on any given day, Lawrence will buy an ice cream.
(Give your answer correct to two decimal places.)

$$\begin{aligned}
 P(\text{Buy Icecream}) &= P(S \cap I) \text{ or } P(S' \cap I) \\
 &= (0,8 \times 0,7) + (0,2 \times 0,25) \\
 &= 0,56 + 0,05 \\
 &= 0,61
 \end{aligned}$$

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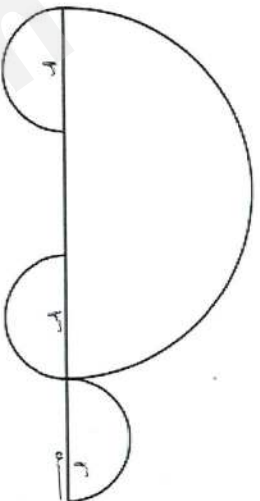
(3)

[11]



Question 9

A company manufactures metal tortoises to decorate one of their products. The tortoise is made up of a large semi-circle with a radius R , and the three smaller semicircles with a radius r . The values of R and r may vary, but $R + r = 300$ mm.



(a) Show that the area of the figure (in mm^2) is given by:

$$A = 2\pi R^2 - 900\pi R + 135\,000\pi$$

$$\begin{aligned} \text{Area} &= \text{large semicircle} + 3 \text{ small semicircles} & R + r &= 300 \\ &= \frac{1}{2}(\pi R^2) + 3\left(\frac{1}{2}\pi r^2\right) & \therefore r &= 300 - R \end{aligned}$$

$$= \frac{1}{2}\pi R^2 + \frac{3}{2}\pi r^2 \dots \textcircled{2}$$

Sub ① into ②

$$\begin{aligned} \therefore A &= \frac{1}{2}\pi R^2 + \frac{3}{2}\pi(300 - R)^2 \\ &= \frac{1}{2}\pi R^2 + \frac{3}{2}\pi(90000 - 600R + R^2) \\ &= \frac{1}{2}\pi R^2 + 135000\pi - 900\pi R + \frac{3}{2}\pi R^2 \\ &= 2\pi R^2 - 900\pi R + 135000\pi \end{aligned}$$

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(b) Determine the value of R if A , the area of the tortoise, is at a minimum.

$$\text{for min area, } \frac{dA}{dR} = 0 \quad \checkmark \text{ (idea)}$$

$$\therefore 4\pi R - 900\pi = 0$$

$$4R - 900 = 0$$

$$R = \frac{900}{4}$$

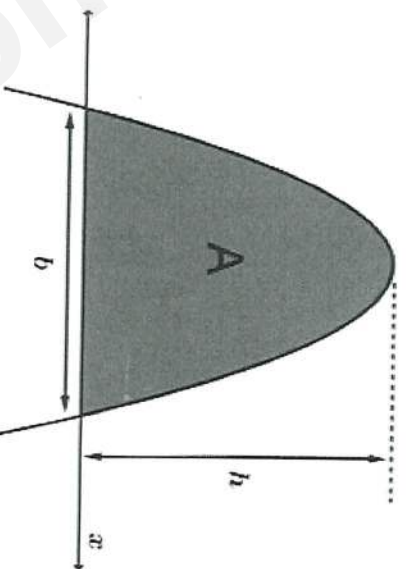
$$R = 225 \text{ mm} \quad \checkmark \text{ (A)}$$

(3)

[7]

Question 10

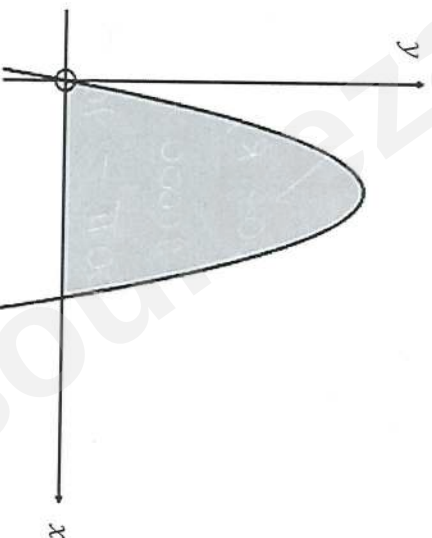
The diagram shows the area bound by a parabola and a horizontal line.



Let b be the horizontal base of the area and h the height ie: h is the vertical distance from the turning point to the base. The area A is given by the formula $A = \frac{2}{3}bh$.

The equation of the curve below is $y = 4x - x^2$. Determine the area of the shaded region for each of the following:

a)



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$$y = 4x - x^2$$

$$\text{x-ints: } 0 = 4x - x^2$$

$$\text{Tpt } \Rightarrow x = -\frac{4}{2(-1)} = 2$$

$$0 = x(4 - x)$$

$$\therefore y = 4(2) - (2)^2$$

$$\therefore x = 0 \text{ OR } x = 4$$

$$y = 4$$

$$\therefore \text{base} = 4u$$

$$\text{Tpt } (2, 4) \therefore \text{height } (h) = 4u$$

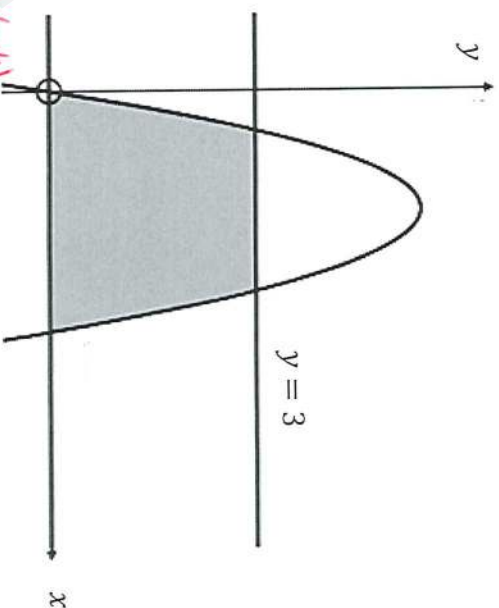
$$\therefore \text{Area shaded} = \frac{2}{3}(4)(4)$$

$$= \frac{32}{3}u^2 \quad \text{(or } 10\frac{2}{3}u^2)$$

(4)

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(b)



$$\text{at } y = 3 \quad \therefore 3 = 4x - x^2 \quad (\text{m(sub)})$$

$$\therefore x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \quad \text{or} \quad x = 1 \quad \checkmark^A$$

$$\therefore \text{base unshaded} = 2u^2 \quad \checkmark^A$$

$$\text{and height} = 4 - 3 = 1u \quad \checkmark^A$$

$$\therefore \text{Area unshaded}$$

$$= \frac{2}{3}(2)(1)$$

$$= \frac{4}{3}u^2$$

$$\therefore \text{Area shaded} = \text{whole area} - \text{unshaded area} \quad \checkmark^M$$

$$= \frac{32}{3} - \frac{4}{3}$$

$$= \frac{28}{3}u^2 \quad \checkmark^A \quad (\text{or } 9\frac{2}{3}u^2)$$

(5)

[9]

Total for Section B: 75

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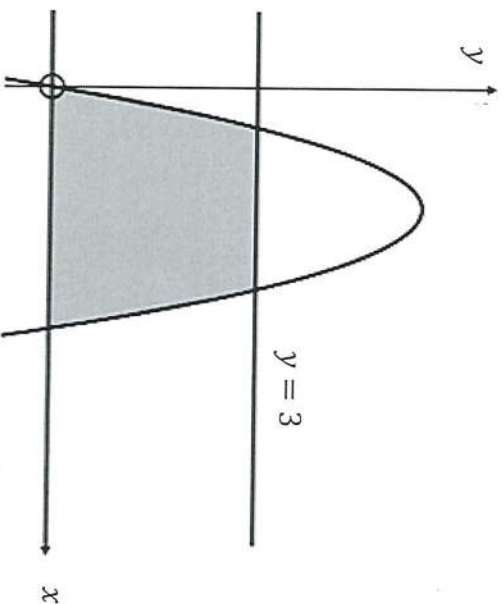
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(b)



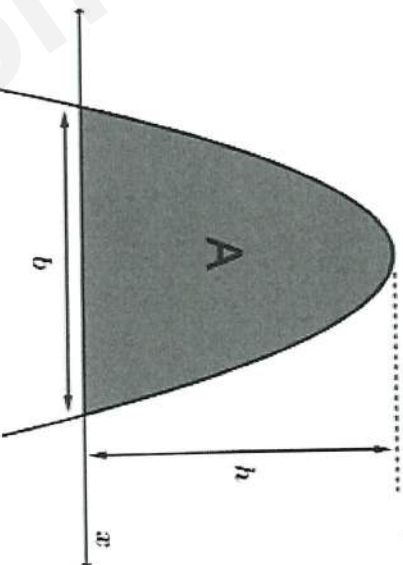
(5)

[9]

Total for Section B: 75

Question 10

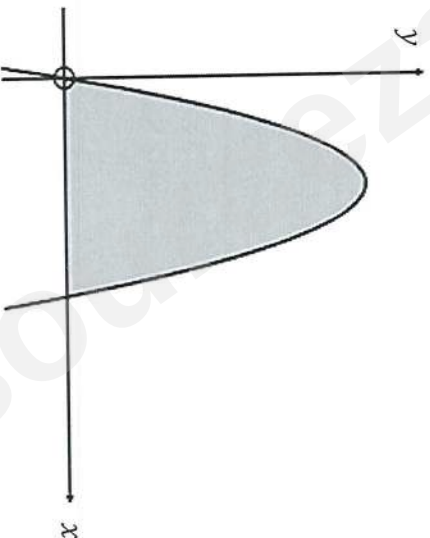
The diagram shows the area bound by a parabola and a horizontal line.



Let b be the horizontal base of the area and h the height ie: h is the vertical distance from the turning point to the base. The area A is given by the formula $A = \frac{2}{3}bh$.

The equation of the curves below is $y = 4x - x^2$. Determine the area of the **shaded region** for each of the following:

(a)

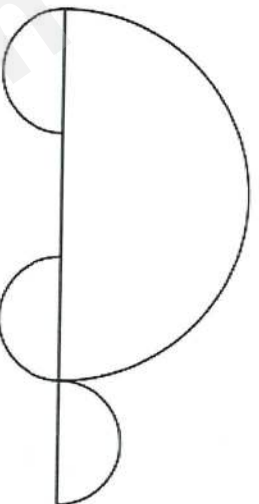


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Question 9

A company manufactures metal tortoises to decorate one of their products. The tortoise is made up of a large semi-circle with a radius R , and the three smaller semicircles with a radius r . The values of R and r may vary, but $R + r = 300 \text{ mm}$.



- (a) Show that the area of the figure (in mm^2) is given by:

$$A = 2\pi R^2 - 900\pi R + 135\,000\pi$$

(4)

- (b) Determine the value of R if A , the area of the tortoise, is at a minimum.

(3)

[7]



(b) The probability that it will be a sunny day is 0,8.

On sunny days, the probability that Lawrence will buy an ice cream is 0,7.

Given that it is **not** a sunny day, the probability that he buys an ice cream is 0,25.

1. Draw a tree diagram representing the scenario above.

[2]

2. Determine the probability that on any given day, Lawrence will buy an ice cream.
(Give your answer correct to two decimal places.)

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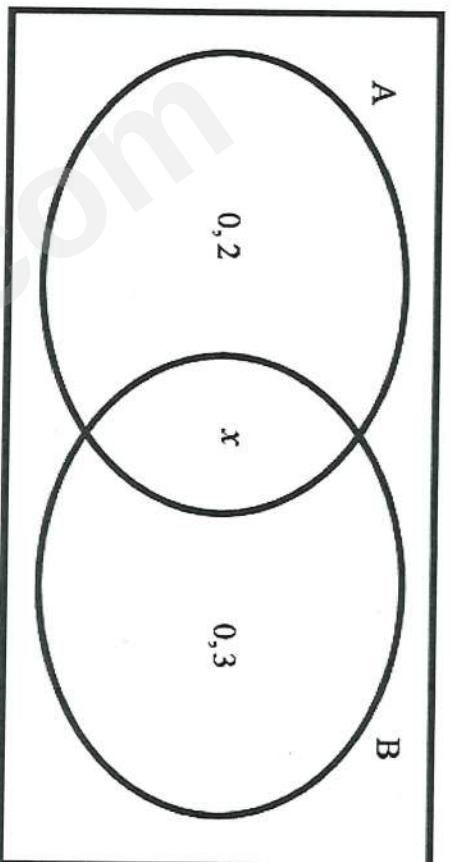
[3]

[11]



Question 8

(a) Given the following Venn Diagram:



1. What is the value of x if $P(A \cup B) = 0,9$?

(1)

2. What is the value of x if A and B are mutually exclusive?

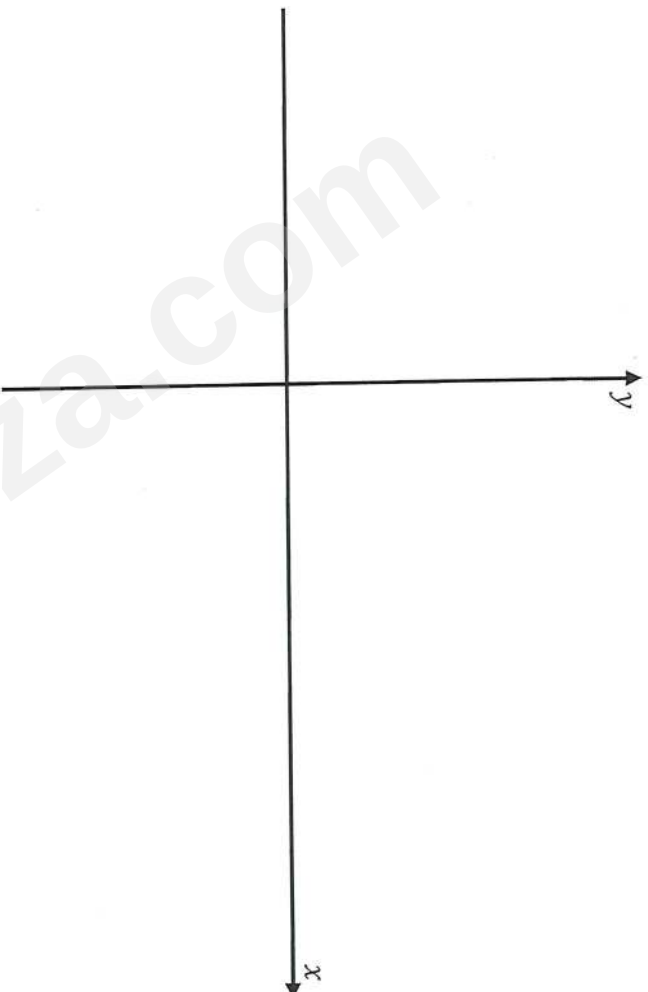
(1)

3. Calculate the value(s) of x if A and B are independent.

(4)

(b) Given: $f(x) = ax^3 + bx^2 + cx$ with $f'(\frac{-3}{2}) = f'(5) = 0$ and $f(x)$ decreasing for $\frac{-3}{2} < x < 5$

1. Use this information to draw a rough sketch of $y = f(x)$.



(3)

2. Determine the value(s) of x for which:

2.1 $f'(x) > 0$

(2)

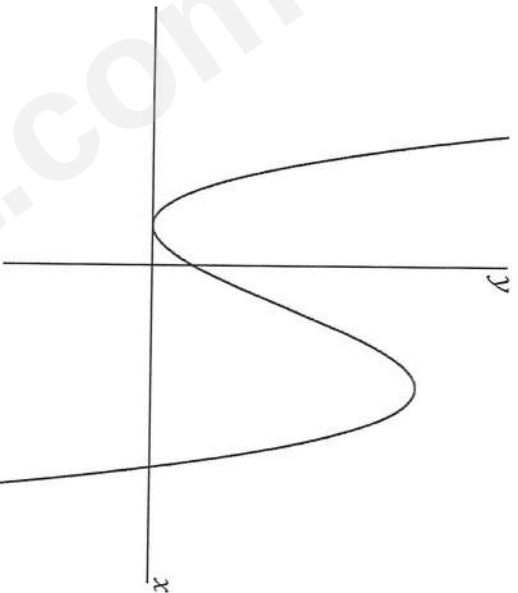
2.2 $f(x)$ is concave upwards

(2)

[17]

Question 7

- (a) Given: $f(x) = -x^3 + 3x^2 + 9x + 5 = -(x + a)^2(x - 5)$ and f has a stationary point on the x -axis at $(-1; 0)$.



1. Write down the value of a .

_____ (1)

2. Determine the coordinates of the other stationary point of $f(x)$.

_____ (4)

3. Write down the value(s) of x for which $f(x) \cdot f''(x) > 0$

_____ (3)

4. For which value(s) of k will $f(x) + k$ have 2 real roots?

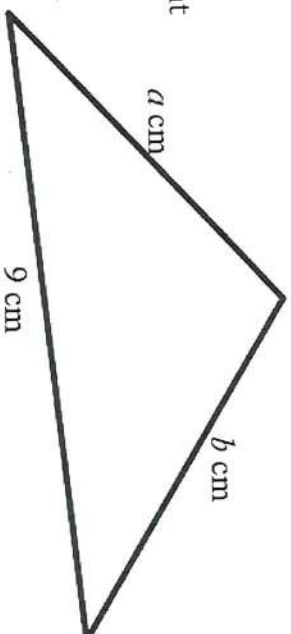
_____ (2)

(c) Refer to the figure (not drawn to scale):

The lengths of the sides of a triangle are such that they form a geometric sequence.

The longest side has a length of 9 cm.

The other two sides are of length a cm and b cm with $a > b$.



1. Show that $a^2 = 9b$

(2)

2. Given that the perimeter of the triangle is 19 cm, determine the length of the shortest side.

(5)

[16]

- (b) Determine the smallest value of n for which $\sum_{k=1}^n \frac{1}{4} (2)^{k-1} > 9$.

(5)

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2. Calculate the outstanding balance on her loan at the end of the first year (Round off your answer to the nearest rand).

(3)

3. Hence, or otherwise, calculate how much of the R 74 844 that she paid during the first year, was taken by the finance company as payment towards the interest it charged.

(3)

[15]

SECTION B**Question 5**

(a) Alice starts saving for retirement and pays R800 into a savings account at the end of each month. After she makes her last payment she has R884 942,47 in her retirement fund. She receives an interest rate of 6,5% per annum compounded monthly.

1. Determine the number of payments that she made.

(4)

2. Convert the nominal interest rate to an effective interest rate.

(2)

(b) Thandi took a mortgage loan of R 850 000 to buy a house and was required to pay equal monthly instalments for 30 years. She was charged interest at 8% p.a, compounded monthly.

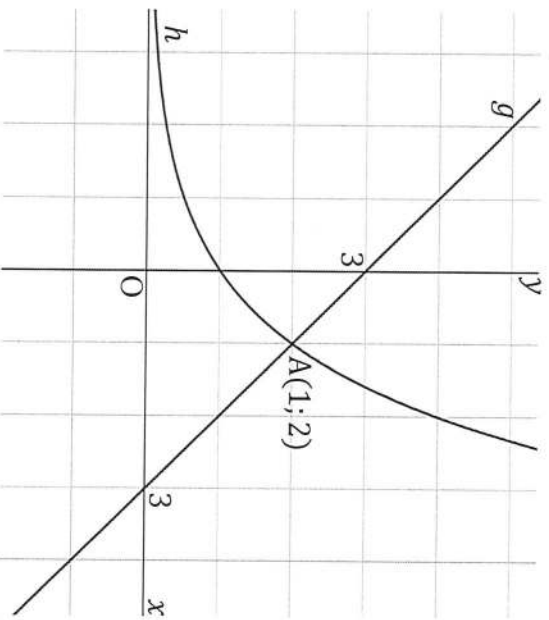
1. Show that her monthly instalment was R 6237.

(3)

- (b) The graph shows the function $g(x) = -x + 3$ and $h(x) = b^x$. The two functions intersect at $A(1; 2)$.

1. Determine the value of b .

_____ (2)



2. If the point $A(1; 2)$ lies on h , determine the new coordinates, A' , when h is reflected about the line $y = x$.

_____ (1)

3. Write down the inverse function of $h(x)$ in the form $h^{-1}(x) = \dots\dots\dots$

_____ (2)

4. On the graph above, sketch h^{-1} labelling any intercepts with the axes as well as point A' .

(3)

5. Use the graphs above to determine the values of x for which $-2^x \cdot x + 6 \cdot 2^{x-1} < 0$.

_____ (3)

[19]

Total for Section A: 75

Question 4

(a) The equations of the graphs of $g(x) = \frac{3x-3}{x-1}$ and $h(x) = -\frac{3}{x-1} + 3$ are given.

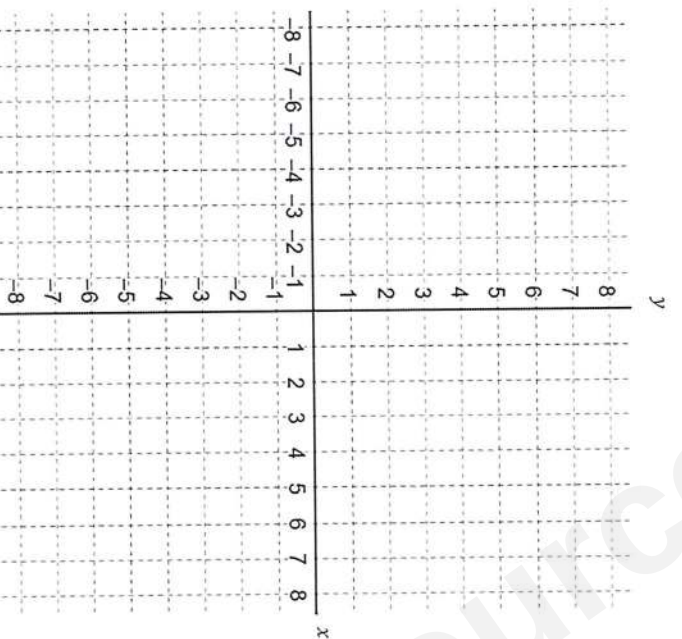
1. Show that $g(x)$ is an asymptote of $h(x)$ and state whether it is the vertical or horizontal asymptote.

(2)

2. Write down the domain and range of $h(x)$.

(2)

3. Sketch the graph of $h(x)$. Show clearly all intercepts with the axes, the asymptotes and two points on each arm of the graph.



(4)

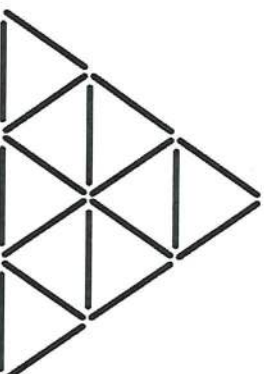
- (c) The first three patterns formed by the arrangement of toothpicks are shown below:



Pattern 1



Pattern 2



Pattern 3

1. How many toothpicks will the fourth pattern have?

_____ (1)

2. Determine the n^{th} term of the sequence.

_____ (4)

3. How many toothpicks will be needed to form the 15th pattern?

_____ (1)

[14]

Question 3

(a) Given: $T_n = 2^{1-n}$

1. Write down the first TWO terms of the sequence.

_____ (2)

2. Determine S_4 of the sequence. (Leave your answer as a fraction)

_____ (2)

- (b) Determine $\sum_{k=-1}^{20} (3k + 2)$

_____ (4)

(b) Determine $\frac{dy}{dx}$ if $y = \sqrt{4x^3} + 2\sqrt{x} - \frac{2}{x}$. (leave your answer with positive exponents)

(4)

[12]

100

20

49

- ed $\mathbf{T} = \mathbf{T}_1$.

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(4)

7. The roots of a quadratic equation are $\frac{5 \pm \sqrt{17 + 4m^2}}{2m}$ where $m \neq 0$.

Explain why this equation will have real roots for all $m \in \mathbb{R}$ where $m \neq 0$.

(2)

- (b) Given $f(x) = \sqrt{2x + 1}$

1. Give the domain of $f(x)$

(1)

2. If $g(x) = 2x - 1$, solve for $f(x) = g(x)$

(4)

4. $(x - 5)^2 < x - 5$

(4)

5. $\log_3 x = \frac{\log 5}{\log 3}$

(2)

6. $(3^x + 9)(2^x - 27) = 0$

(4)

Question 1

1. $2x^2 - 17x = 0$

2. $3x^2 + 10x - 4 = 0$ (correct to 1 decimal place) [2]

2. $3x^2 + 10x - 4 = 0$ (correct to 1 decimal place)

3. $8.2^x = \left(\frac{1}{4}\right)^{x-2}$

3. $8 \cdot 2^x = \left(\frac{1}{4}\right)^{x-2}$



ST ANNE'S DIOCESAN COLLEGE
TRIALS EXAMINATION – 2022
GRADE 12

MATHEMATICS – PAPER 1

Time: 3 hours Exam no. _____

150 marks

NAME: _____

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 26 pages. Please check that your question paper is complete.
2. Read the questions carefully.

Answer all the questions.

You may use an approved non-programmable and non-graphical calculator unless otherwise stated. Ensure your calculator is in **DEGREE** mode.

Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**

Diagrams are not necessarily drawn to scale.

If necessary, round off answers to **ONE** decimal place, unless otherwise stated.

It is in your own interest to write legibly and to present your work neatly.

Three blank pages (Pages 24 to 26) are included at the end of the paper.

If you run out of space for a question, use these pages.

Clearly indicate the question number of your answer should you use this extra space.

For official use only.

	Equations	Calculus	Sequences	Functions	Financial Maths	Sequences	Calculus	Probability	Calculus	Functions
Question	1	2	3	4	5	6	7	8	9	10
Mark										
Total	30	12	14	19	15	16	17	11	7	9
Total										
Percentage										
150										
100										