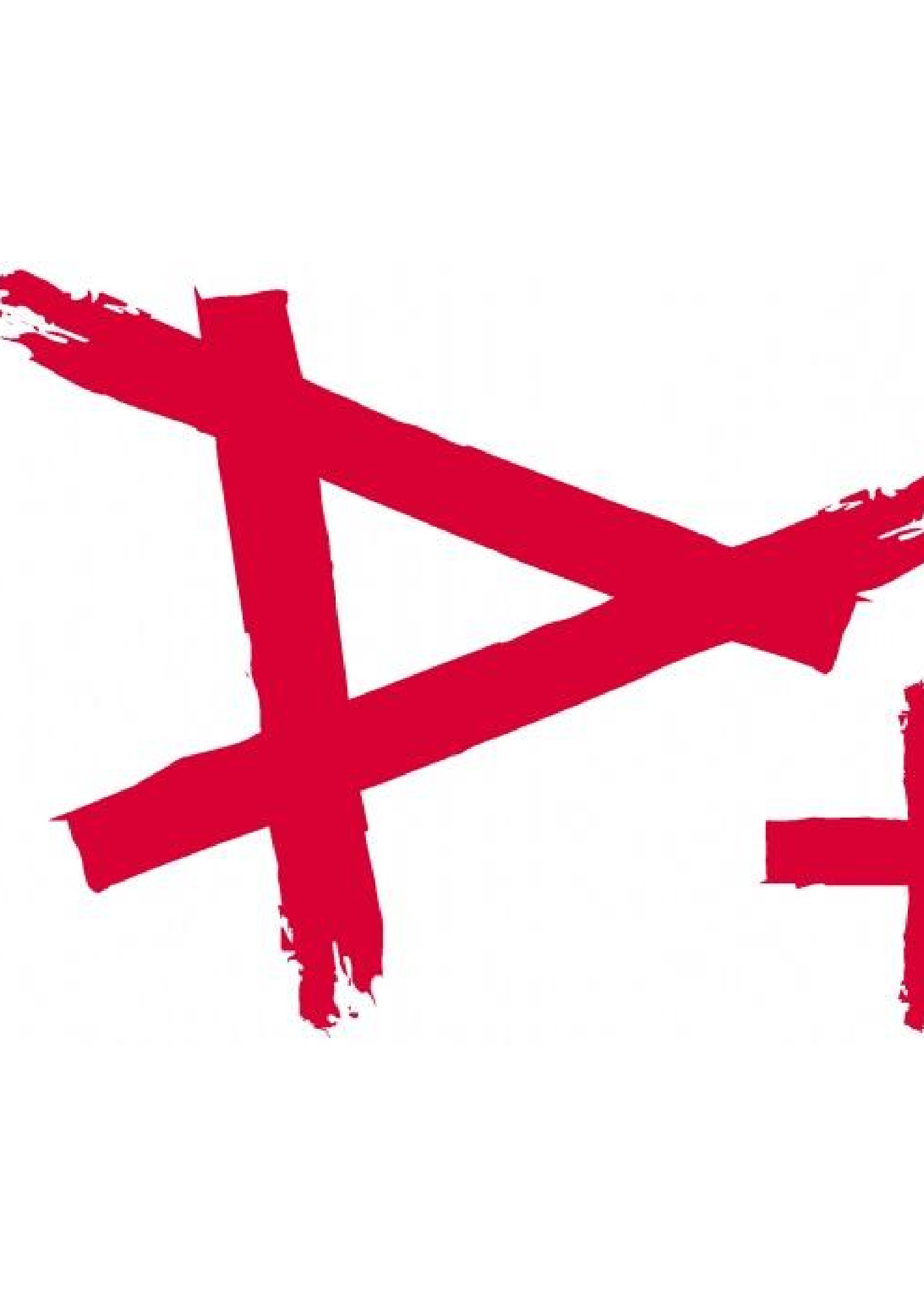






Grade 12

Core Mathematics: Paper I

August 2021

Time: 3 hours

Marks: 150

Please read the following instructions carefully:

1. This question paper consists of 16 pages and an Information sheet. Please check that your question paper is complete.
2. **Answer all the questions on the question paper.**
3. You may use an approved non-programmable and non-graphical calculator.
4. All necessary working details must be clearly shown.
5. Round off answers to **1 decimal place** where necessary, unless otherwise stated.
6. Ensure that your calculator is in **DEGREE** mode.
7. Diagrams may not be drawn to scale.
8. **A mark is not necessarily allocated to the correct answer but rather the working. Show all working.**

SECTION A					SECTION B				
	Equations	Financial Maths	Number patterns	Functions	Probability	Functions	Calculus	Calculus	Calculus
Question	1	2	3	4	5	6	7	8	9
Mark									
Total	27	14	24	14	11	24	11	18	7
Total									
Percentage									
150									
100									

IEB exam number: _____

Name: _____

MEMORANDUM

Teacher: _____

SECTION A
Question 1

1.1 Solve for x and show ALL relevant working:

1.1.1 $(2 - 3x)(x - 4) = 0$

$$2 - 3x = 0$$

$$\text{or } x - 4 = 0$$

✓m solving linear factors

(2)

$$-3x = -2$$

$$x = 4$$

$$x = \frac{2}{3}$$

✓a for both answers

1.1.2 $-2x^2 + 4x = -5$

(correct to TWO decimal places)

(3)

$$-2x^2 + 4x + 5 = 0$$

✓m standard format

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(-2)(5)}}{2(-2)}$$

✓m quadratic formula

$$x = \frac{-4 \pm \sqrt{56}}{-4}$$

$$x = -0,87$$

$$\text{or } x = 2,87$$

✓a

1.1.3 $3^x - 3^{x-1} = 162$

(4)

$$3^x - 3^x \cdot 3^{-1} = 162$$

$$3^x (1 - 3^{-1}) = 162$$

✓m common factor

$$3^x \left(1 - \frac{1}{3}\right) = 162$$

$$3^x = 243$$

✓a

$$3^x = 3^5$$

✓m exponential form to equate

$$x = 5$$

✓a

1.1.4 $\sqrt{3-x} = \frac{x^2-x-6}{\sqrt{3-x}}$

(5)

$$3-x = x^2-x-6$$

✓m multiply by LCD

$$0 = x^2 - 9$$

✓a

$$0 = (x-3)(x+3)$$

✓m factorise

$$\therefore x \neq 3 \quad \text{or } x = -3$$

✓a two possible answers

$$\therefore x = -3 \text{ only}$$

✓a

1.2 If $\log_3 m = k$, determine in terms of k :

1.2.1 $\log_3 3m$

$$= \log_3 3 + \log_3 m \quad \checkmark \text{m separate product}$$
$$= 1 + k \quad \checkmark a$$

(2)

1.2.2 $\log_3 \sqrt[3]{m}$ $\checkmark \text{m exponent rule}$

$$= \log_3 m^{\frac{1}{3}} \quad \checkmark \text{m exponent rule}$$
$$= \frac{1}{3} \log_3 m$$
$$= \frac{1}{3} k \quad \checkmark a$$

(2)

1.3 Solve for x if $\log x^3 - \log x = 4$

$\checkmark \text{m coefficient rule}$

(3)

$$3 \log x - \log x = 4$$

$$2 \log x = 4$$

$$\log x = 2 \quad \checkmark \text{m convert to exponential notation}$$

$$x = 10^2$$

$$x = 100 \quad \checkmark a$$

1.4 The roots of a quadratic equation are given by the following:

$$x = \frac{6 \pm \sqrt{p(5-p)}}{4}$$

1.4.1 Describe the nature of roots if $p = 4$

$$\Delta = b^2 - 4ac = 4(5-4) \quad \checkmark \text{m substitution}$$

$$= 4 \quad \checkmark a \quad \therefore \text{Roots are real, rational, unequal}$$

$\checkmark a$ for all 3 descriptions

(3)

1.4.2 For which values of p will the roots be non-real?

(3)

roots are non-real if $\Delta < 0$

$$p(5-p) < 0$$

$$\text{Crit. Val: } p=0 \quad \text{or} \quad p=5 \quad \checkmark \text{m numberline or parabola}$$

$\checkmark \text{m critical values}$

$$\frac{0}{-0 + 0 -}$$

$$\therefore p < 0 \quad \text{or} \quad p > 5 \quad \checkmark a$$

[27]

Question 2

2.1 A car is bought for R125 000 and its value depreciates at a rate of 17% p.a.

Calculate the book value of the car in 6 years' time using the reducing-balance method. (2)

$$A = 125000(1 - 0,17)^6 \quad \checkmark m$$

$$A = R40\,867,55 \quad \checkmark a$$

2.2 Calculate the effective interest rate per annum if an investment earns interest at a rate of 9,5% p.a. compounded monthly. (3)

$$1 + i_{\text{eff}} = \left(1 + \frac{0,095}{12}\right)^{12} \quad \checkmark m \quad \checkmark a$$

$$1 + i_{\text{eff}} = 1,099247584$$

$$i_{\text{eff}} = 0,099247584$$

$$\text{rate} = 9,9\% \text{ p.a.} \quad \checkmark a$$

2.3 An entrepreneur made an initial deposit of R40 000 into an investment account earning interest at 8% p.a. compounded monthly. Two years later, she made another deposit of R60 000 into the account and the interest rate changed to 10% p.a. compounded quarterly. Calculate the value of her investment account at the end of 5 years. (5)



For compounding and interest $\checkmark a$ $\frac{0,1}{4} m$ multiplying $\checkmark m$ adding

$$A = 40000\left(1 + \frac{0,08}{12}\right)^{24} \left(1 + \frac{0,1}{4}\right)^{12} + 60000\left(1 + \frac{0,1}{4}\right)^{12}$$

$$A = R143\,789,48 \quad \checkmark a$$

(not consistent accuracy) $\checkmark a$ for this bracket showing compounding and years

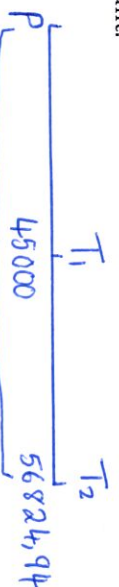
2.4 A company took out a loan that was paid back in two instalments.

At the end of the first year, an amount of R45 000 was repaid.

At the end of the second year, a final amount of R56 824,94 was repaid.

The loan earned interest of 8% p.a. compounded monthly.

Calculate the loan amount. (4)



$\frac{0,08}{12} m$ addition $\checkmark m$ negative exponents

$$P = 45000\left(1 + \frac{0,08}{12}\right)^{-12} + 56824,94\left(1 + \frac{0,08}{12}\right)^{-24}$$

$$P = R90\,000 \quad \checkmark a$$

(not a consistent accuracy.)

[14]

Question 3

3.1 The sequence 2 ; 6 ; 10 ; 14 ; ... ; 130 is an arithmetic sequence.

3.1.1 Determine the n^{th} term of the sequence. (2)

$$a = 2 \quad d = 4$$

$$T_n = 2 + (n-1)(4) \quad \checkmark m$$

$$T_n = 2 + 4n - 4$$

$$T_n = 4n - 2 \quad \checkmark a$$

3.1.2 How many terms are in the sequence? (2)

$$130 \checkmark m \text{ substitute for } T_{130}$$

$$4n = 132$$

$$n = 33 \quad \checkmark a$$

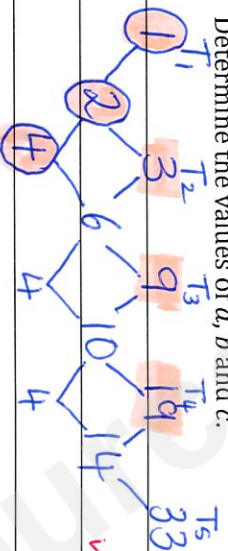
$$\therefore 33 \text{ terms in sequence } (T_{33} = 130)$$

3.1.3 It is further given that the above sequence forms the row of the first

differences for a quadratic sequence $T_n = an^2 + bn + c$ where $T_5 = 33$.

Determine the values of a , b and c .

(5)



work backwards to determine quadratic sequence

m subst. into formula to solve.

$$2a = 4$$

$$3a + b = 2$$

$$a + b + c = 1 \quad \checkmark a$$

$$a = 2 \quad \checkmark a$$

$$6 + b = 2$$

$$2 - 4 + c = 1$$

$$b = -4$$

$$c = 3$$

$$T_n = 2n^2 - 4n + 3 \quad \checkmark ca \text{ (must be quadratic)}$$

3.2 Given:

$$S_n = \frac{3(1-2^n)}{-8}$$

3.2.1 Calculate S_6 and leave your answer as a fraction.

$$S_6 = \frac{3(1-2^6)}{-8} \quad \checkmark \text{ in substitute for } n$$

$$S_6 = \frac{189}{8} \quad \checkmark^a \quad (2,3,6)$$

3.2.2 Calculate T_7

$$T_7 = S_7 - S_6 \quad \checkmark \text{ in subtract sums}$$

$$T_7 = \frac{3(1-2^7)}{-8} = \frac{189}{8} \quad \checkmark^a$$

$$T_7 = 24 \quad \checkmark^a$$

3.3 Given:

$$\sum_{k=1}^n (3k+4) = m \quad \text{and} \quad \sum_{k=1}^{22} (3k+4) = 847$$

Determine, in terms of m , the value of:

$$\sum_{k=23}^n (3k+4)$$

$$= m - 847 \quad \checkmark^a \quad \text{in subtracting totals}$$

3.4 Given:

$$\sum_{p=1}^n 64 \cdot 2^{1-p}$$

3.4.1 Determine the first 3 terms of the series:

$$T_1: \text{let } p=1: 64 \cdot 2^{1-1} = 64 \quad \checkmark \text{ substituting for } p$$

$$T_2: \text{let } p=2: 64 \cdot 2^{1-2} = 32$$

$$T_3: \text{let } p=3: 64 \cdot 2^{1-3} = 16$$

$$\therefore 64 + 32 + 16 + \dots \quad \checkmark a$$

3.4.2 If $n \in \mathbb{N}$, calculate the smallest value of n for which:

$$\sum_{p=1}^n 64 \cdot 2^{1-p} > 127$$

$$a = 64; r = \frac{1}{2}; S_n > 127$$

$\checkmark$ m geom sum formula

$$\frac{64(1-(\frac{1}{2})^n)}{1-\frac{1}{2}} > 127$$

$$1-(\frac{1}{2})^n > \frac{127}{64}$$

$$-(\frac{1}{2})^n < -\frac{1}{64}$$

$$(\frac{1}{2})^n < (\frac{1}{2})^7 \quad \checkmark a \quad (\frac{1}{2})^8 < (\frac{1}{2})^7$$

$$\therefore n = 8 \quad \checkmark ca$$

3.4.3 Calculate:

$$\sum_{p=1}^{\infty} 64 \cdot 2^{1-p}$$

$$a = 64; r = \frac{1}{2}; n = \infty$$

$$S_{\infty} = \frac{64}{1-\frac{1}{2}} \quad \checkmark m \quad \checkmark a \quad (\text{infinite sum formula})$$

$$S_{\infty} = 128 \quad \checkmark ca$$

[24]

Question 4

Given: $f(x) = -\frac{4}{x-2} + 1$

4.1 Write down the equations of the asymptotes of f .

$x = 2$ ✓^a (vertical asymptote)

$y = 1$ ✓^a (horizontal asymptote)

4.2 Calculate:

4.2.1 the y-intercept of f

let $x=0$: ✓^m $y = \frac{-4}{0-2} + 1$

$y = 2 + 1$

$y = 3$ ✓^a

(0,3)

4.2.2 the x-intercept of f

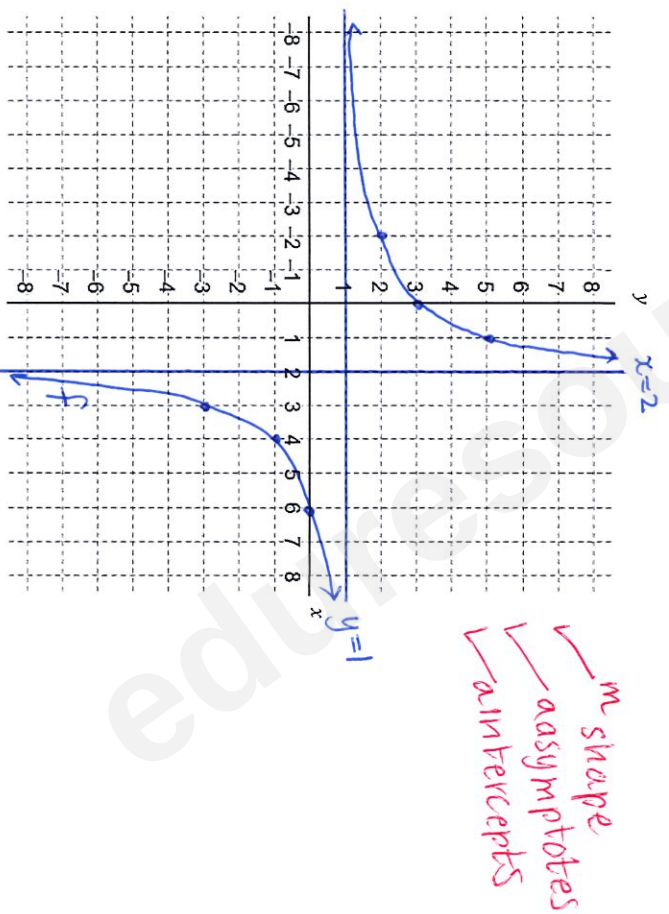
let $y=0$: ✓^m $0 = \frac{-4}{x-2} + 1$

$-1 = \frac{-4}{x-2}$

$2-x = -4$

$x = 6$ ✓^a (6,0)

4.3 Sketch the graph of f showing clearly the asymptotes and the intercepts of the axes.



- 4.4 Determine the equation of the line of symmetry of f that has a negative gradient. (3)
Leave your answer in the form $y = \dots$

point of intersection of asymptotes (2,1)

$$y - y_1 = m(x - x_1) \quad \text{subst. into straight line equation}$$

$$y - 1 = -1(x - 2) \quad \checkmark^m \quad \checkmark^a$$

$$y = -x + 2 + 1$$

$$y = -x + 3 \quad \checkmark^a$$

- 4.5 Give the equation of the graph g if it is the translation of f two units right and 3 units down. (2)

$$g(x) = \frac{-4}{x-4} - 2 \quad \checkmark^a$$

[14]
Total of section A [77]

SECTION B

Question 5

5.1 For any two events, A and B, it is given that $P(A) = 0,48$ and $P(B) = 0,26$.

Determine, to 2 decimal places:

5.1.1 $P(A \text{ and } B)$ if A and B are independent events.

$$P(A \cap B) = P(A) \times P(B) \quad \checkmark \text{m product}$$

$$= 0,48 \times 0,26$$

$$= 0,12 \quad \checkmark a$$

5.1.2 $P(A \cup B)$ if $P(A \cap B) = 0,12$

$$P(A \cup B) = 0,48 + 0,26 - 0,12 \quad \checkmark \text{m identity}$$

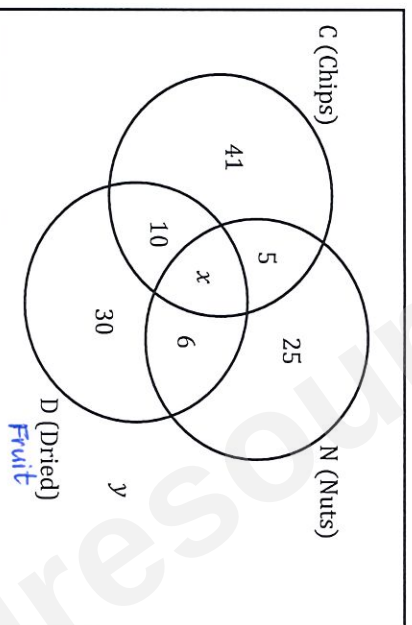
$$P(A \cup B) = 0,62 \quad \checkmark a$$

5.1.3 $P(A \text{ or } B)$ if A and B are mutually exclusive events.

$$P(A \cup B) = P(A) + P(B) \quad \checkmark \text{m sum}$$

$$= 0,48 + 0,26 = 0,74 \quad \checkmark a$$

5.2 A survey was conducted among 130 Grade 12 learners to establish which snack they prefer to eat while they watch television. The results were summarised in the Venn diagram below. However, some information is missing.



5.2.1 If 29 learners prefer at least two types of snacks, calculate the value of x and y.

$$5 + 10 + 6 + 2 = 29 \quad \checkmark m$$

$$y = 130 - 125$$

$$x = 8 \quad \checkmark a$$

$$y = 5 \quad \checkmark ca$$

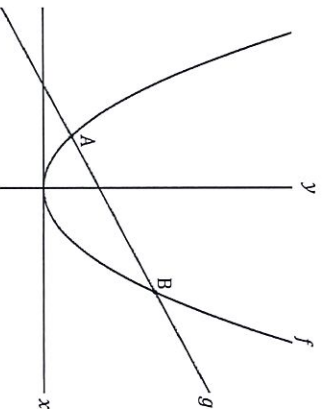
5.2.2 Determine the probability that a learner who does not eat nuts will have at least one other snack while watching television.

$$= \frac{41 + 10 + 30}{130} = \frac{81}{130} = 0,6$$

Also accept $\frac{81}{130}$ when read as conditional probability (AP)

Question 6

6.1 The functions $f(x) = \frac{1}{4}x^2$ and $g(x) = \frac{1}{2}x + 2$ are drawn in the diagram below.



6.1.1

Determine the coordinates of A and B, the points of intersection of graphs f and g . (4)

$$\frac{1}{4}x^2 = \frac{1}{2}x + 2 \quad \checkmark m \text{ equalise: solve simultaneously.}$$

$$x^2 = 2x + 8$$

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

$$x = 4 \quad \text{or} \quad x = -2 \quad \checkmark a$$

$$\therefore y = \frac{1}{2}(4) + 2 \quad y = \frac{1}{2}(-2) + 2 \quad \checkmark m \text{ subst } x\text{-values to solve for } y.$$

$$y = 4$$

$$y = 1$$

$$B(4; 4) \quad A(-2; 1) \quad \checkmark a \text{ both coordinates}$$

6.1.2

For what value/s of x is:

(i) $f(x) > g(x)$

$$x < -2 \quad \checkmark a \quad \text{or} \quad x > 4 \quad \checkmark a$$

(2)

(ii) $f(x) \cdot g(x) \geq 0$

$$x\text{-int: } 0 = \frac{1}{2}x + 2 \quad \checkmark m \text{ find } x\text{-int of } g$$

$$x = -4 \quad \checkmark a$$

$$x \geq -4 \quad \checkmark a$$

(3)

6.1.3

Determine the equation of a tangent to $f(x)$ where $f'(x) = -3$

(5)

$$f'(x) = \frac{1}{2}x \quad \checkmark m \text{ derivate}$$

$$-3 = \frac{1}{2}x$$

$$x = -6 \quad \checkmark a \text{ (x-coordinate)}$$

Equation of tangent:

$$f(-6) = \frac{1}{4}(-6)^2$$

$$= 9 \quad \checkmark ca \text{ (y-coordinate)}$$

$$y - 9 = -3(x + 6) \quad \checkmark m \text{ subst. into straight line}$$

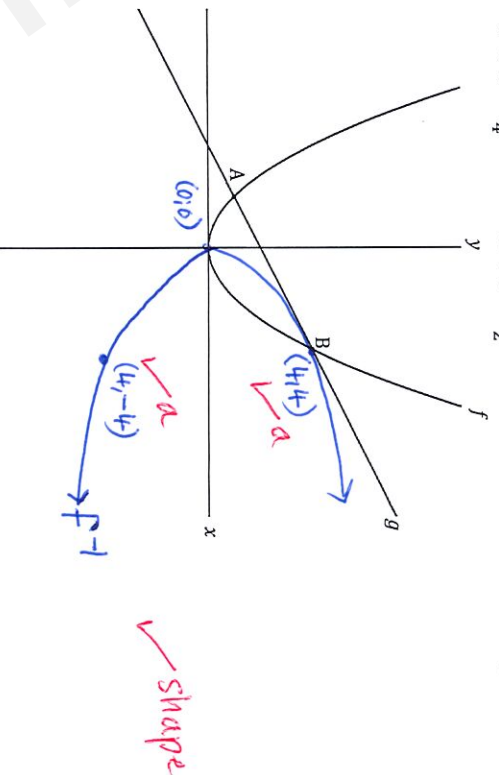
$$y - 9 = -3x - 18$$

$$(-6; 9)$$

$$y = -3x - 9 \quad \checkmark ca$$

$$\therefore m = -3 \text{ at } (-6; 9)$$

6.2 The same functions $f(x) = \frac{1}{4}x^2$ and $g(x) = \frac{1}{2}x + 2$ are drawn in the diagram below.



6.2.1 Determine the equation of $f^{-1}(x)$. (3)

$$f: y = \frac{1}{4}x^2$$

$$f^{-1}: x = \frac{1}{4}y^2 \quad \checkmark \text{m swap } x \text{ and } y$$

$$4x = y^2$$

$$y^2 = 4x$$

$$y = \pm 2\sqrt{x} \quad \therefore f^{-1}(x) = \pm 2\sqrt{x} \quad \checkmark \text{a} \quad \checkmark \text{a}$$

6.2.2 Draw a rough sketch of $f^{-1}(x)$ on the same set of axes above. Show the intercept/s with the axes and 2 other coordinates. (3)

6.2.3 Show that $g(x)$ is a tangent to $f^{-1}(x)$. (4)

$$\frac{1}{2}x + 2 = \pm 2\sqrt{x} \quad \checkmark \text{m equate}$$

$$\left(\frac{1}{2}x + 2\right)^2 = 4x \quad \checkmark \text{m square both sides}$$

$$\frac{1}{4}x^2 + 2x + 4 = 4x$$

$$x^2 + 8x + 16 = 16x$$

$$x^2 - 8x + 16 = 0 \quad \checkmark \text{a}$$

$$(x-4)(x-4) = 0$$

$$\therefore x = 4 \text{ only} \quad \checkmark \text{ca (must be one solution)}$$

$$\therefore \text{only one pt. of intersection}$$

$$\therefore g \text{ is a tangent to } f \text{ at } x = 4$$

[24]

Question 7

7.1 From first principles, determine $f'(x)$ if $f(x) = x^2 - x$

(5)

$$(x; x^2 - x) \quad (x+h; (x+h)^2 - (x+h))$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 - (x+h) - (x^2 - x)}{x+h-x} \quad \checkmark^a \quad \checkmark^m \text{ difference of } y \text{ and } x$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h} \quad \checkmark^a$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2 - h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(2x + h - 1)}{h} \quad \checkmark^m \text{ common factor, } h$$

$$f'(x) = 2x - 1 \quad \checkmark^a \quad (\text{deduct a mark for incorrect notation})$$

7.2 Determine the following leaving your answers with positive, rational exponents:

6.2.1 $D_x \left[\frac{x^3 + 3x + 1}{x} \right] \quad (3)$

$$D_x [x^2 + 3 + x^{-1}] \quad \checkmark^m \text{ split numerators}$$

$$= 2x - x^{-2} \quad \checkmark^a$$

$$= 2x - \frac{1}{x^2} \quad \checkmark^a$$

max of 1 mark deducted for incorrect notation between 7.2.1 and 7.2.2

7.2.2 $\frac{dy}{dp}$ if $y = (3k - 2p)^2 \quad (3)$

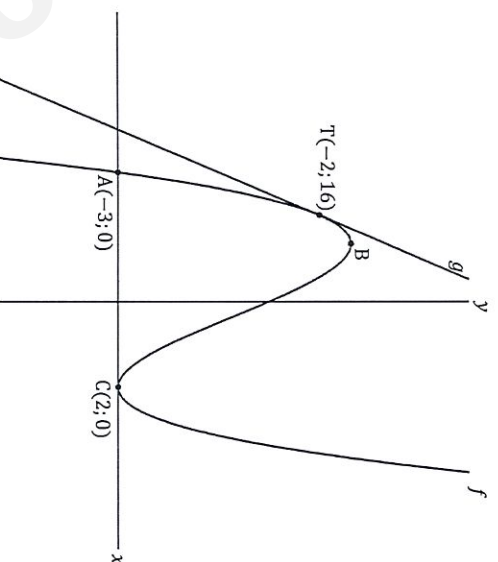
$$y = 9k^2 - 12kp + 4p^2 \quad \checkmark^m \text{ multiply out brackets}$$

$$\frac{dy}{dp} = -12k + 8p \quad \checkmark^a$$

[11]

Question 8

The function $f(x) = x^3 + bx^2 + cx + d$ and its tangent, $g(x) = mx + 32$ are drawn in the sketch below. The x-intercepts are at $A(-3; 0)$ and $C(2; 0)$, the turning points are at B and $C(2; 0)$. The tangent touches the curve at $T(-2; 16)$



8.1 Show that the equation of $f(x) = x^3 - x^2 - 8x + 12$ (3)

$$f(x) = a(x-2)^2(x+3) \quad \text{m intercepts}$$

$$16 = a(-2-2)^2(-2+3) \quad \text{In question, it is given that } a=1$$

$$16 = a(16) \quad \text{Because the value of 'a' is already}$$

$$a=1 \quad \text{given in question, it is also accepted if said:}$$

$$f(x) = (x-2)^2(x+3) \quad y=(x+3)(x-2)^2 \text{ and multiplied out.}$$

$$= (x^2 - 4x + 4)(x+3)$$

$$= x^3 - 4x^2 + 4x + 3x^2 - 12x + 12$$

$$= x^3 - x^2 - 8x + 12$$

8.2 Calculate the coordinates of B. (5)

$$f'(x) = 3x^2 - 2x - 8 \quad \text{m derivative}$$

$$0 = 3x^2 - 2x - 8 \quad \text{m}$$

$$0 = (3x+4)(x-2)$$

$$x = -\frac{4}{3} \quad \text{or} \quad x = 2$$

$$f(-\frac{4}{3}) = (-\frac{4}{3})^3 - (-\frac{4}{3})^2 - 8(-\frac{4}{3}) + 12 \quad \text{m subst into f(x)}$$

$$= \frac{500}{27}$$

$$= 18,5$$

$$B(-\frac{4}{3}; 18,5) \quad \text{ca}$$

8.3 For what value/s of k will $x^3 - x^2 - 8x + 12 = k$ have one real solution? (2)

when $k < 0$ or $k > 18,5$

8.4 For what value/s of x is $f''(x) < 0$? (3)

$$f''(x) = 6x - 2 \quad \text{m second derivative}$$

$$6x - 2 < 0$$

$$6x < 2$$

$$x < \frac{1}{3} \quad \text{ca}$$

8.5.1 Show that $m = 8$ for the equation of the tangent, $g(x) = mx + 32$ (2)

$$\begin{aligned} m &= f'(-2) = 3(-2)^2 - 2(-2) - 8 \quad \text{m subst. into derivative} \\ &= 12 + 4 - 8 \\ &= 8 \end{aligned}$$

$$\therefore m = 8 \text{ of tangent } g(x) \text{ at } x = -2$$

8.5.2 For what x -value will another tangent be parallel to the tangent at $T(-2; 16)$? (3)

$$m = f'(x) = 8$$

m Subst. For gradient in derivative

$$8 = 3x^2 - 2x - 8 \quad \text{ca}$$

$$0 = 3x^2 - 2x - 16$$

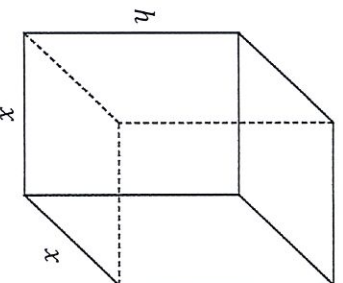
$$0 = (3x - 8)(x + 2)$$

$$\therefore x = \frac{8}{3} \quad \text{or} \quad x = -2$$

$$\therefore x = \frac{8}{3} \quad \text{ca}$$

Question 9

A piece of cardboard with a surface area of 1350 cm^2 is folded to form a closed rectangular box with a square base. The side of the square is $x \text{ cm}$ and the height of the box is $h \text{ cm}$.



9.1. Show that $h = \frac{675 - x^2}{2x}$

$$SA = 2x^2 + 4xh$$

$$1350 = 2x^2 + 4xh \quad \checkmark \text{ surface area formula}$$

$$1350 - 2x^2 = 4xh$$

$$\frac{1350 - 2x^2}{4x} = h \quad \checkmark a$$

$$\frac{675 - x^2}{2x} = h$$

Determine the value of x for which the volume will be a maximum.

(4)

$$\text{Volume} = x^2h$$

$$V(x) = \frac{x^2 \cdot (675 - x^2)}{2x} \quad \checkmark \text{ Volume in terms of } x$$

$$V(x) = \frac{675}{2}x - \frac{1}{2}x^3$$

$$V'(x) = \frac{675}{2} - \frac{3}{2}x^2 \quad \checkmark a$$

$$\checkmark \text{ m } 0 = \frac{675}{2} - \frac{3}{2}x^2$$

$$0 = 675 - 3x^2$$

$$3x^2 = 675$$

$$x^2 = 225$$

$$x = 15 \quad \checkmark a$$