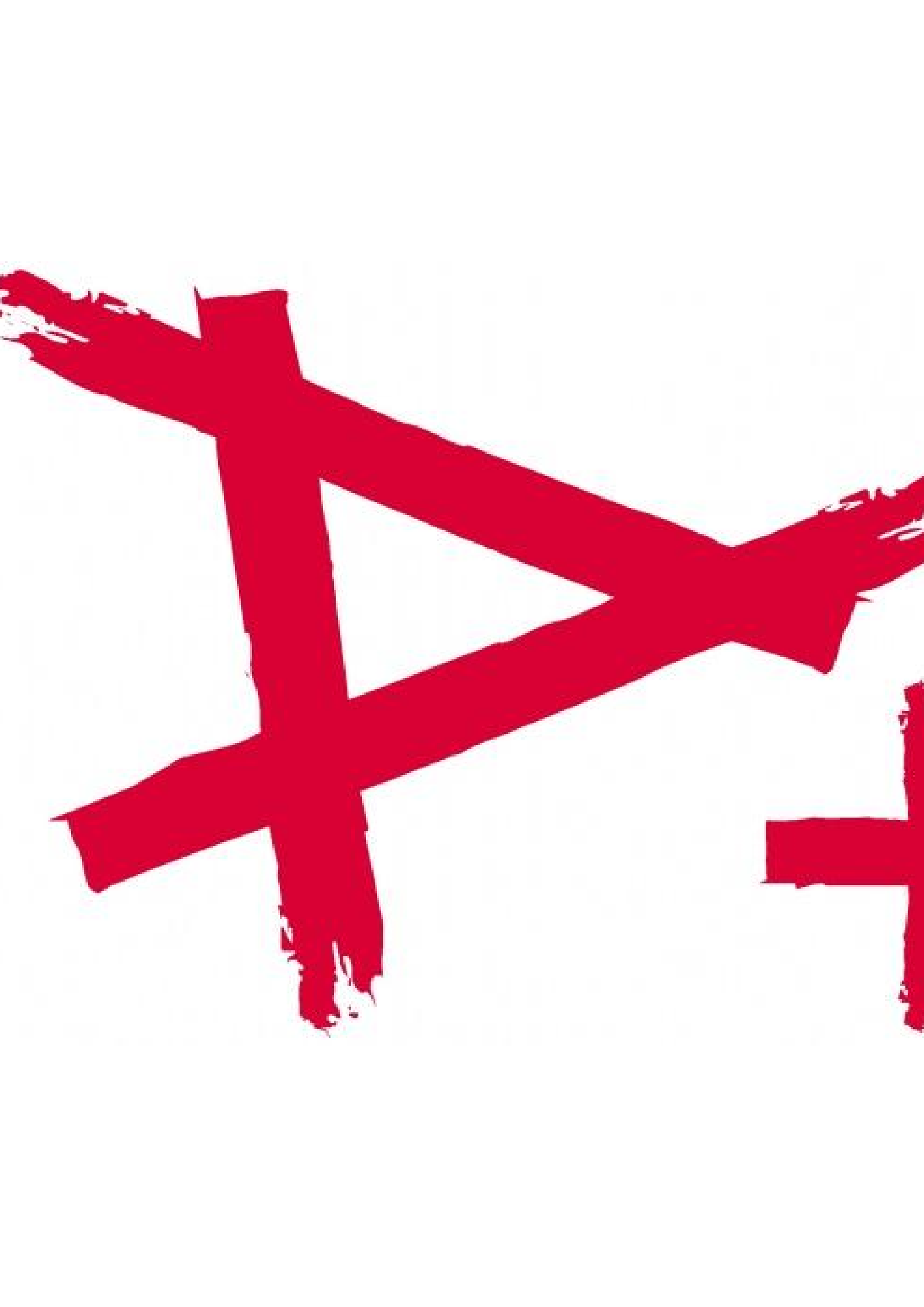




SECTION A

QUESTION 1

(a) Solve for x . Answers only will not be awarded full marks.

(1) $-x(x+1) = 0$

(2)

$$-x = 0 \quad \text{or} \quad x+1 = 0$$

$$\therefore x = 0 \quad x = -1$$

(2) i. $4x^2 + 7x - 2 = 0$, by using the quadratic formula.

(3)

$$a = 4 \quad b = 7 \quad c = -2$$

$$x = \frac{-7 \pm \sqrt{81}}{8}$$

$$= \frac{-7 \pm 9}{8}$$

$$= \frac{1}{4} \quad \text{or} \quad -2$$

ii. Hence, or otherwise, solve $4 \cdot 2^{2x} + 7 \cdot 2^x = 2$.

(3)

$$\therefore 4 \cdot 2^{2x} + 7 \cdot 2^x - 2 = 0$$

from (i) $2^x = \frac{1}{4}$ or $2^x = -2$

$$2^x = 2^{-2}$$

$$\therefore x = -2$$

(3) $\sqrt{5-x} - 1 = x$

(5)

$$\sqrt{5-x} = x+1$$

$$5-x = (x+1)^2$$

$$5-x = x^2 + 2x + 1$$

$$0 = x^2 + 3x - 4$$

$$= (x+4)(x-1)$$

$$\therefore x = -4 \quad \text{or} \quad x = 1$$

CHECK: $x = 1$ is the only soln

✓ a (can be ca if checked answers and concluded no soln as a result)

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(4) $6x - 2x(x-4) \geq 0$

(4)

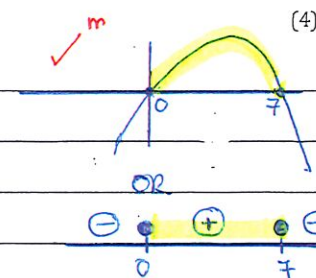
$$6x - 2x^2 + 8x \geq 0$$

$$14x - 2x^2 \geq 0$$

$$2x(7-x) \geq 0$$

Critical Values: $x = 0$ or $x = 7$

Soln: $0 \leq x \leq 7$

(b) Solve for x and y simultaneously if:

$$x^2 + xy - 2y^2 = 0 \quad \text{and} \quad x - y = 3$$

(5)

$$x^2 + xy - 2y^2 = 0 \quad \dots (1)$$

$$x = 3 + y \quad \dots (2)$$

$$(1) \rightarrow (2): (3+y)^2 + (3+y)y - 2y^2 = 0$$

$$9 + 6y + y^2 + 3y + y^2 - 2y^2 = 0$$

$$9y + 9 = 0$$

$$9y = -9$$

$$y = -1 \quad \dots (3)$$

$$(3) \rightarrow (2):$$

$$x = 3 + (-1)$$

$$= 2$$

$$\therefore \text{Soln } (2; -1)$$

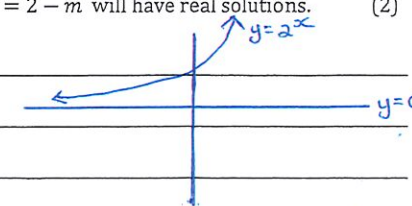
(c) Determine the values of m , for which the equation $2^x = 2 - m$ will have real solutions.

(2)

$$2 - m > 0$$

$$2 > m$$

$$\text{ie } m < 2$$



[24]

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QUESTION 2

- (a) Thobile is a young farmer. She has just bought her first tractor for R500 000. The tractor depreciates at a rate of 8% per annum on a reducing balance. Thobile would like to replace the tractor in 4 years' time. The cost of a new tractor appreciates at 9% per annum due to inflation.

(1) Determine the book value of the tractor after 4 years.

(2)

$$A = 500\,000 (1 - 0,08)^4 \quad \checkmark^m$$

$$= R\,358\,196.48 \quad \checkmark^a$$

(2) Determine the cost of a new tractor in 4 years' time.

(2)

$$A = 500\,000 (1 + 0,09)^4 \quad \checkmark^m$$

$$= R\,705\,790.81 \quad \checkmark^a$$

(3) Thobile plans to trade in the old tractor after 4 years and buy a new one. How much money will she need in a sinking fund in 4 years' time, in order to cover the cost of a new tractor?

(2)

$$705\,790.81 - 358\,196.48 \quad \checkmark^m$$

$$= R\,347\,594.33 \quad \checkmark^a$$

(4) Thobile decides to save R400 000 in this sinking fund and immediately deposits an amount of Rx at the beginning of the month. She then pays Rx at the end of the first month and continues to make the same monthly deposits of Rx at the end of each month for 4 years. If the sinking fund earns interest at a rate of 9% p.a. compounded monthly over a 4 year period, determine the value of x.

(3)

$$400\,000 = x \left[\left(1 + \frac{0,09}{12}\right)^{(12 \times 4) + 1} - 1 \right] \quad \checkmark^{\text{extra paym}} \quad \checkmark^{Fv}$$

$$x = R\,6\,785.17 \quad \checkmark^a$$

- (b) Zandile buys a plot of land on her friend's farm for R1 275 000. She wishes to build a B&B on the property. Zandile takes out a loan for the full amount at an interest rate of 10,25% per annum compounded monthly over a period of 20 years. The equal monthly instalments on this loan are R12 515,95.

(1) Calculate the outstanding balance on the loan after 7 years.

(3)

$$OB = 1\,275\,000 \left(1 + \frac{0,1025}{12}\right)^{7 \times 12} - \frac{12\,515,95 \left[\left(1 + \frac{0,1025}{12}\right)^{7 \times 12} - 1\right]}{\frac{0,1025}{12}} \quad \checkmark^m$$

$$= 2\,604\,912,784 - 1\,528\,389,96$$

$$= R\,1\,076\,522.82 \quad \checkmark^a$$

$$\text{OR } OB = \frac{12\,515,95 \left[1 - \left(1 + \frac{0,1025}{12}\right)^{-(13 \times 12)}\right]}{\frac{0,1025}{12}} = R\,1\,076\,522,16 \quad \checkmark^a$$

payments still to be made

(2) Immediately after 7 years Zandile misses the next 5 consecutive payments, due to financial difficulty. Thereafter she continues making monthly payments into the loan account until the end of the 20 year period. * 13 years left $\therefore$ no. paym = $(13 \times 12) - 5 = 151$

Determine the new monthly payments Zandile will now have to make.

(4)

$$1\,076\,522.82 \left(1 + \frac{0,1025}{12}\right)^5 = x \left[1 - \left(1 + \frac{0,1025}{12}\right)^{-151}\right] \quad \checkmark^a \quad \checkmark^m$$

$$1\,123\,291.485 = x (84.66259 \dots)$$

$$x = R\,13\,267,86 \quad \checkmark^a$$

$$\text{OR } R\,13\,267,85 \quad \checkmark^a$$

[16]

QUESTION 3

(a) Given: $f(x) = \frac{1}{x}$

(1) Determine the average gradient in the interval $[1; 3]$.

(3)

$$f(3) = \frac{1}{3} \rightarrow (3; \frac{1}{3}) \quad \checkmark^m$$

$$f(1) = 1 \rightarrow (1, 1) \quad \checkmark^m$$

$$\therefore \text{Ave Grad} = \frac{\frac{1}{3} - 1}{3 - 1} \quad \checkmark^m$$

$$= -\frac{2}{3} \times \frac{1}{2}$$

$$= -\frac{1}{3} \quad \checkmark^a$$

(2) Determine $f'(x)$ from first principles.

(5)

$$f(x) = \frac{1}{x} ; f(x+h) = \frac{1}{x+h}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \quad \checkmark^m$$

$$= \lim_{h \rightarrow 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h} \quad \checkmark^{LCD}$$

$$= \lim_{h \rightarrow 0} \frac{x - x - h}{x(x+h)} \div \frac{h}{1} \quad \checkmark^{notation}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{x(x+h)} \times \frac{1}{h} \quad \checkmark^{cancel\ h's}$$

$$= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)}$$

$$= -\frac{1}{x^2} \quad \checkmark^a$$

(b) Determine the following (where necessary leave answers in positive exponential form):

(1) $\frac{dy}{dx}$ if $y = (1 - 3x)^2$

(3)

$$y = 1 - 6x + 9x^2 \quad \checkmark^m \quad \checkmark^a$$

$$\therefore \frac{dy}{dx} = -6 + 18x \quad \checkmark^{ca}$$

(2) $D_x \left[\frac{x-8}{x^2} \right]$

(3)

$$= D_x \left[\frac{x}{x^2} - \frac{8}{x^2} \right] \quad \checkmark^m$$

$$= D_x [x^{-1} - 8x^{-2}]$$

$$= D_x [x^{-1} - 8x^{-2}] \quad \checkmark^a$$

$$= -x^{-2} + 16x^{-3}$$

$$= \frac{-1}{x^2} + \frac{16}{x^3} \quad \checkmark^{ca}$$

[14]

QUESTION 4

(a) Given the arithmetic sequence: 8; 10; 12; 14; ...

(1) Determine the general term of the sequence.

$$\begin{aligned} \text{AP: } T_n &= 8 + (n-1)(2) \\ &= 8 + 2n - 2 \\ &= 6 + 2n \end{aligned}$$

(2) What term has a value of 166?

$$166 = 6 + 2n$$

$$160 = 2n$$

$$\therefore n = 80 \quad \therefore T_{80} = 166$$

(3) Calculate the sum of the first 80 terms.

$$S_{80} = \frac{80}{2} [2(8) + 79(2)]$$

$$= 40 [174]$$

$$= 6960$$

(4) It is further given that the above sequence forms the row of the first differences for a quadratic sequence, $T_n = an^2 + bn + c$, where $T_5 = 47$.

Determine the values of a , b and c .

$$\begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \\ 3 & 11 & 21 & 33 & 47 \\ \swarrow & \nwarrow & \swarrow & \nwarrow & \swarrow \\ & 8 & 10 & 12 & 14 \\ & \searrow & \swarrow & \searrow & \swarrow \\ & & 2 & 2 & 2 \end{array}$$

$$2a = 2$$

$$\therefore a = 1$$

$$3a + b = 8$$

$$3 + b = 8$$

$$\therefore b = 5$$

$$a + b + c = 3$$

$$1 + 5 + c = 3$$

$$c = -3$$

$$\therefore T_n = n^2 + 5n - 3$$

(b) Given: $\sum_{k=1}^n (2k - 5) = x$ and $\sum_{k=1}^{15} (2k - 5) = 165$

Determine, in terms of x , the value of $\sum_{k=16}^n (2k - 5)$.

$$\sum_{k=16}^n (2k - 5) = x - 165$$

QUESTION 5

Given the graph $f(x) = -\frac{4}{x-2} - 1$.

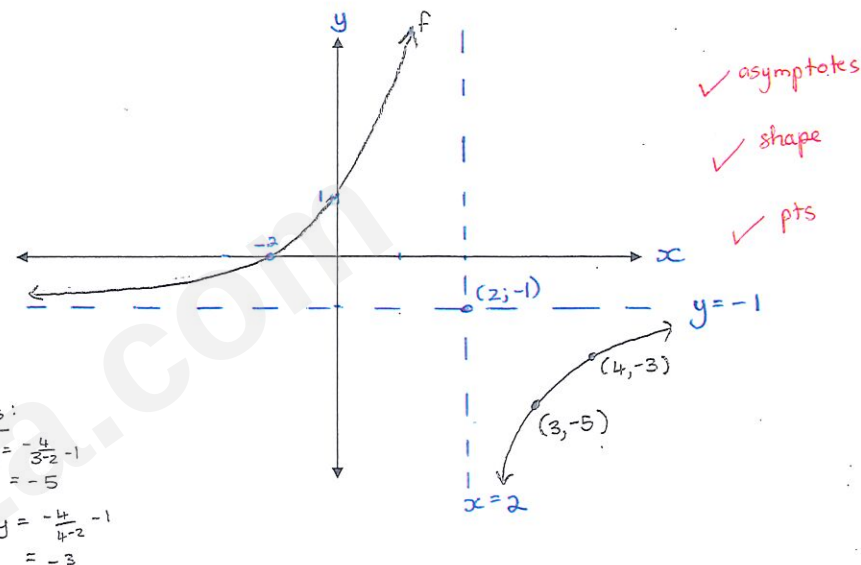
(a) Write down the equations of the asymptotes. (2)

$x = 2$ ✓
 $y = -1$ ✓

(b) Calculate the intercepts with the axes. (3)

x-ints: let $y = 0$
 $0 = -\frac{4}{x-2} - 1$
 $1 = -\frac{4}{x-2}$
 $x-2 = -4$
 $x = -2$ $(-2, 0)$ ✓
 y-int: let $x = 0$
 $y = -\frac{4}{0-2} - 1$
 $= 2 - 1$
 $= 1$ $(0, 1)$ ✓

(c) Sketch the graph of f on the axes below. Label clearly the axes, intercepts and the asymptotes and show at least two coordinates on each arm. (3)



(d) One of the axes of symmetry of f is an increasing function. Determine its equation. (3)

$y = x + c$ $(m=1; \text{incr})$ $(2, -1)$ ✓
 $-1 = 2 + c$ ✓
 $-3 = c$ $\therefore y = x - 3$ ✓ (must have $m=1$)

(e) Circle your answer: If $f(x) = -\frac{4}{x-2} - 1$ is reflected about the y-axis to form h , then: (2)

A. $h(x) = \frac{4}{x-2} - 1$ B. $h(x) = \frac{4}{x-2} + 1$ C. $h(x) = \frac{4}{x+2} + 1$ D. $h(x) = \frac{4}{x+2} - 1$ ✓✓

$y = -\frac{4}{(-x)-2} - 1$
 $= -\frac{4}{-x-2} - 1$
 $= +\frac{4}{x+2} - 1$

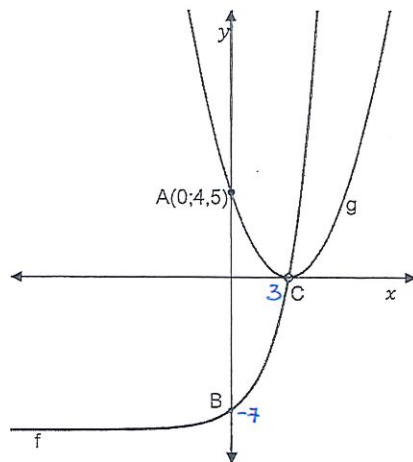
SECTION B

QUESTION 6

The graphs of $f(x) = 2^x - 8$ and $g(x) = a(x+p)^2 + q$ are drawn below.

The point A $(0; 4\frac{1}{2})$ and B are the y-intercepts of $g(x)$ and $f(x)$ respectively.

The graphs intersect at C, which is the turning point of $g(x)$, as well as the common x-intercept of $f(x)$ and $g(x)$.



(a) Write down the equation of the asymptote of f .

(1)

$$y = -8$$

(b) Determine the coordinates of B and C.

(3)

$$B: y = 2^0 - 8 = -7$$

$$\therefore B(0; -7)$$

$$C: 0 = 2^x - 8$$

$$8 = 2^x$$

$$2^3 = 2^x$$

$$\therefore x = 3$$

$$C(3; 0)$$

(c) Determine the value of a , p and q if $g(x) = a(x+p)^2 + q$.

(4)

$$\begin{aligned} \text{T.P. } (3; 0) &\therefore y = a(x-3)^2 + 0 \\ (0; 4\frac{1}{2}) &4\frac{1}{2} = a(0-3)^2 \\ 4\frac{1}{2} &= 9a \\ \frac{1}{2} &= a \end{aligned}$$

$$\therefore a = \frac{1}{2}; p = -3; q = 0$$

(d) (1) Describe the transformation that has taken place for $g(x)$ to become $h(x) = \frac{1}{2}x^2$.

(1)

$$g(x) = \frac{1}{2}(x-3)^2 \rightarrow h(x) = \frac{1}{2}x^2$$

Translation of 3 units to the left.

(2) The domain of $h(x)$ is restricted so that h^{-1} is the function $y = -\sqrt{2x}$. Write down the restriction on h .

(1)

$$x \leq 0$$

(e) Determine k , if $k(x) = f(3x) + 8$.

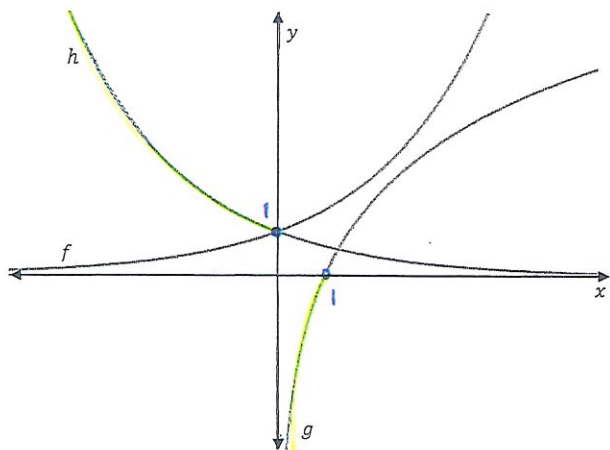
(2)

$$\begin{aligned} k(x) &= 2^{3x} - 8 + 8 \\ &= 2^{3x} \end{aligned}$$

[12]

QUESTION 7

The diagram below shows the graphs of g , h and $f(x) = \left(\frac{3}{2}\right)^x$.



(a) Write down the equation of g if $g(x) = f^{-1}(x)$.

(2)

$$f^{-1}: x = \left(\frac{3}{2}\right)^y \quad \checkmark^m$$

$$\therefore y = \log_{3/2} x$$

$$\therefore g(x) = \log_{3/2} x \quad \checkmark^a$$

(b) Write down the equation of h if $h(x) = b^x$ is the reflection of f in the y -axis and $0 < b < 1$. (2)

$$(x, y) \rightarrow (-x, y) \therefore y = \left(\frac{3}{2}\right)^{-x} \quad \checkmark^m$$

$$y = \left(\frac{2}{3}\right)^x \quad \checkmark^a$$

$$\hookrightarrow 0 < \frac{2}{3} < 1$$

(c) Determine the value(s) of x for which:

(1) $h(x) \geq f(x)$

(1)

$$x \leq 0 \quad \checkmark^1$$

(2) $\log_{3/2} x \leq 0$

(2)

$$\text{ie. } g(x) \leq 0$$

$$\checkmark^m \quad x\text{-int: } 0 = \log_{3/2} x$$

$$\left(\frac{3}{2}\right)^0 = x$$

$$\therefore x \leq 1 \quad \checkmark^a$$

$$1 = x$$

(d) Without drawing the graph determine the domain of $y = \log_{3/2}(x+1)$

(2)

$$x+1 > 0 \quad \checkmark$$

$$x > -1 \quad \checkmark^a$$

[9]

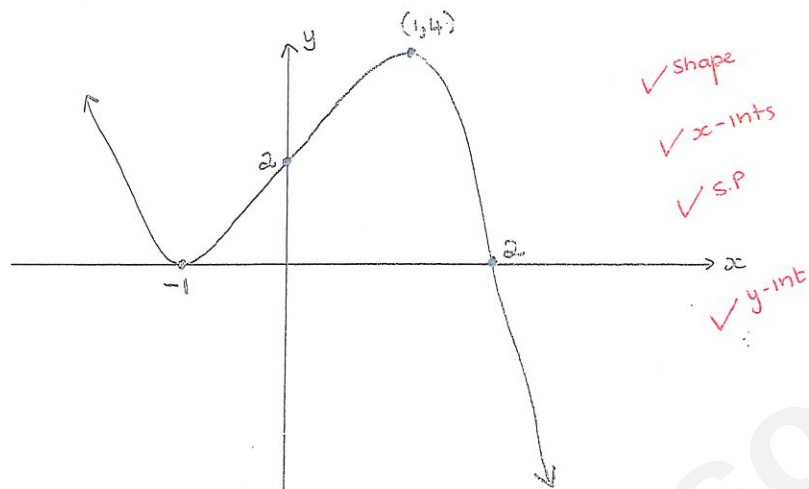
QUESTION 8

A cubic function, $f(x) = ax^3 + bx^2 + cx + d$, has the following properties:

$$a < 0 \quad f(-1) = f(2) = 0 \quad f'(-1) = f'(1) = 0 \quad f(1) = 4 \text{ and } f(0) = 2$$

(a) Use this information to draw a neat sketch graph in the space below. Label intercepts and stationary points.

Shape:  $\checkmark$ x -ints: $(-1, 0) \checkmark$ $(2, 0) \checkmark$ S.P.: $(-1, 0) \checkmark$ $(1, 4) \checkmark$ y -int: $(0, 2) \checkmark$



(b) For which values of x is f decreasing?

$$x < -1 \checkmark \text{ca} \quad \text{or} \quad x > 1 \checkmark \text{ca}$$

(c) Use the intercepts to show that the defining equation of the graph is $f(x) = -x^3 + 3x + 2$. (You do not need to show that a given point lies on f).

$$\begin{aligned} y &= a(x-2)(x+1)^2 \quad \checkmark \text{brackets using } x\text{-ints} \\ 2 &= a(0-2)(0+1)^2 \quad \checkmark \text{sub } (0, 2) \\ 2 &= -2a \\ \therefore a &= -1 \quad \checkmark a \\ y &= -(x-2)(x+1)^2 \\ &= -(x-2)(x^2+2x+1) \\ &= -(x^3+2x^2+x-2x^2-4x-2) \\ &= -(x^3-3x-2) \\ \therefore f(x) &= -x^3+3x+2 \end{aligned}$$

(d) Determine:

1. The coordinates of the point of inflection.

pt of inflection lies mid-way between stationary Pts.

$$x = \frac{-1+1}{2} = 0 \checkmark m \quad \text{or} \quad f(x) = -x^3+3x+2$$

$$\therefore \text{P.O.I.} = (0, 2) \quad f'(x) = -3x^2+3$$

$$f''(x) = -6x = 0 \quad x = 0$$

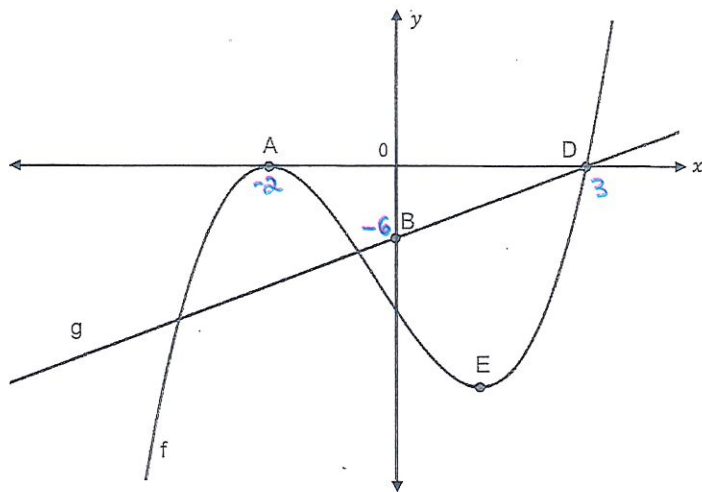
2. The values of x for which the graph of $f(x)$ is concave down.

$$x > 0 \checkmark \text{ca}$$

QUESTION 9

Sketched below is the graph of $f(x) = x^3 + x^2 - 8x - 12$ and $g(x) = 2x + c$.

A and D are the x -intercepts of f and g . OB is 6 units. A and E are stationary points of f .



- (a) Show that $(x - 3)$ is a factor of $f(x)$.

(2)

$$f(3) = (3)^3 + (3)^2 - 8(3) - 12 = 0$$

$$\therefore (x - 3) \text{ is a factor}$$

- (b) Determine the length AD. Show ALL necessary working.

(3)

$$f(x) = x^3 + x^2 - 8x - 12$$

$$= (x - 3)(x^2 + px + 4)$$

$$= (x - 3)(x^2 + 4x - 4)$$

$$= (x - 3)(x + 2)(x + 2)$$

$$\therefore x = 3 \text{ or } x = -2$$

$$A(-2, 0) \quad D(3, 0)$$

$$\therefore AD = 3 - (-2)$$

$$= 5 \text{ units}$$

- (c) Write down the equation of $g(x)$.

(1)

$$g(x) = 2x - 6$$

- (d) Determine the equation of the tangent to $f(x)$ that has the point of tangency at the y -intercept of f .

(4)

$$y\text{-int: } (0, -12)$$

$$f'(x) = 3x^2 + 2x - 8$$

$$f'(0) = -8 = m$$

$$\therefore y = -8x - 12$$

- (e) Use the graph to determine the value(s) of m such that $f(x) = m$ has only one real, positive root.

(2)

$$m > 0$$

- (f) For which value(s) of x is $f'(x) \cdot g(x) < 0$ where $x < 0$?

(2)

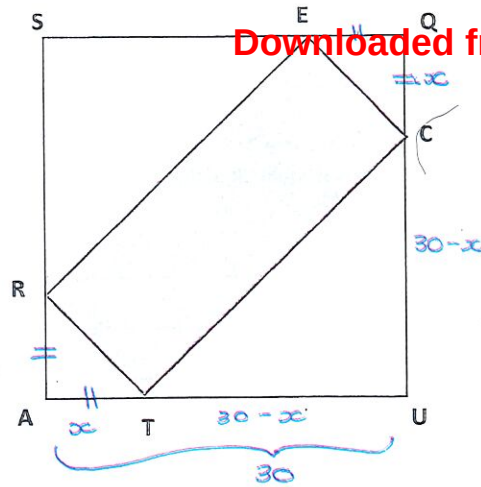
$\hookrightarrow$ grad $\hookrightarrow$ y -values

$$x < -2$$

[14]

QUESTION 10

SQUA is a square with sides of 30 cm each. RECT is a rectangle that fits exactly inside the square, such that its vertices touch the sides of the square and



(a) Prove that the area of RECT is $A(x) = -2x^2 + 60x$.

(4)

$$EC^2 = x^2 + x^2 \quad (\text{Thm of Pythag})$$

$$= 2x^2$$

✓m (Pythag)

$$\therefore EC = x\sqrt{2}$$

✓a (30-x)

$$TC^2 = (30-x)^2 + (30-x)^2 = 2(30-x)^2 \quad (\text{Thm of Pythag})$$

$$\therefore TC = \sqrt{2}(30-x)$$

$$\therefore \text{Area} = l \times b = \sqrt{2} \cdot (30-x) \times x \cdot \sqrt{2}$$

✓m

$$= 2x(30-x)$$

$$\therefore A(x) = 60x - 2x^2$$

(b) Determine the dimensions of RECT for which the area of RECT is a maximum.

(4)

$$A'(x) = -4x + 60 = 0$$

$$60 = 4x$$

$$\therefore x = 15$$

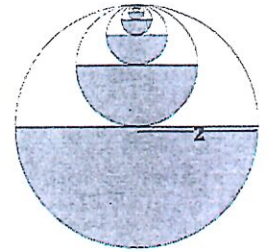
$$\therefore \text{breadth} = 15\sqrt{2}$$

$$\text{length} = \sqrt{2}(30-15) = 15\sqrt{2}$$

[8]

QUESTION 11

(a) The diagram represents a sketch of circles. Half of the area of each circle is shaded as shown. The radius of the largest circle is 2 units. The diameter of each smaller circle is equal to the radius of the previous circle. Determine the sum of the shaded areas of all the circles if this process goes on indefinitely. Give your answer in terms of π and simplify. (4)



$$\text{Sum of Areas} = \frac{1}{2}\pi(2)^2 + \frac{1}{2}\pi(1)^2 + \frac{1}{2}\pi\left(\frac{1}{2}\right)^2 + \dots$$

$$= 2\pi + \frac{1}{2}\pi + \frac{1}{8}\pi + \dots$$

$$\therefore a = 2\pi \quad ; \quad r = \frac{1}{4}$$

✓a (both 'a' and 'r')

$$\therefore S_{\infty} = \frac{2\pi}{1 - \frac{1}{4}}$$

$$= 2\pi \div \frac{3}{4}$$

$$= 2\pi \times \frac{4}{3}$$

$$= \frac{8\pi}{3} \text{ units}^2$$

(b) If $S_n = -n^2 + 2n$ for an arithmetic series, consisting of 12 terms, determine:

(1) The first term of the series.

(2)

$$T_1 = S_1 = -1^2 + 2(1)$$

$$= 1$$

(2) The sum of the last 4 terms.

(3)

$$S_{12} - S_8 = (-12^2 + 2(12)) - (-8^2 + 2(8))$$

$$= -120 + 48$$

$$= -72$$

- (c) The ratio of the sum of the first eight terms to the sum of the first four terms of a geometric series is 97:81. If the first term is 9, determine the next two terms of the sequence, if it is given that all terms are positive.

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$$S_8 = \frac{9(r^8 - 1)}{r - 1}$$

$$S_4 = \frac{9(r^4 - 1)}{r - 1}$$

$$S_8 : S_4 = 97 : 81 \quad \text{or} \quad \frac{S_8}{S_4} = \frac{97}{81}$$

$$\therefore \frac{9(r^8 - 1)}{r - 1} \div \frac{9(r^4 - 1)}{r - 1} = \frac{97}{81}$$

$$\checkmark \text{ cancel } \frac{9(r^8 - 1)}{r - 1} \times \frac{r - 1}{9(r^4 - 1)} = \frac{97}{81}$$

$$\frac{r^8 - 1}{r^4 - 1} = \frac{97}{81}$$

$$\checkmark + \frac{(r^4 - 1)(r^4 + 1)}{r^4 - 1} = \frac{97}{81}$$

$$r^4 + 1 = \frac{97}{81}$$

$$r^4 = \frac{16}{81} \checkmark a$$

$$r = \pm \frac{2}{3}$$

but terms are positive

$$\therefore r = \frac{2}{3} \checkmark ca$$

$$\therefore 9 \xrightarrow{\times \frac{2}{3}} 6 \xrightarrow{\times \frac{2}{3}} 4 \dots$$

$$T_2 = 6$$

$$T_3 = 4$$

} $\checkmark ca$

[15]

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