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St Anne's Diocesan College
CORE MATHEMATICS: PAPER I
TRIAL EXAMINATION

Grade 12
Time: 3 hours

August 2020
150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 22 pages and a formulae sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. Round off your answers to **one decimal digit** where necessary.
6. All the necessary working details must be clearly shown.
7. It is in your best interest to write legibly and to present your work neatly.
8. **Write down your Exam Number and /Name:**

Exam Number:

Name:

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Question	1	2	3	4	5	6	7	8	9	10	11	Total
Topic	Algebra	Finance	Calculus	Sequences	Functions	Functions	Functions	Calculus	Calculus	Calculus Application	Series	%
Max Mark	24	16	14	12	13	12	9	13	14	8	15	150
Your Mark												

SECTION A**QUESTION 1**

(a) Solve for x . **Answers only will not be awarded full marks.**

(1) $-x(x + 1) = 0$

(2)

(2) i. $4x^2 + 7x - 2 = 0$, by using the quadratic formula.

(3)

ii. Hence, or otherwise, solve $4 \cdot 2^{2x} + 7 \cdot 2^x = 2$.

(3)

(3) $\sqrt{5 - x} - 1 = x$

(5)

(4) $6x - 2x(x - 4) \geq 0$

(4)

(b) Solve for x and y simultaneously if:

$$x^2 + xy - 2y^2 = 0 \text{ and } x - y = 3$$

(5)

(c) Determine the values of m , for which the equation $2^x = 2 - m$ will have real solutions.

(2)

[24]

QUESTION 2

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- (a) Thobile is a young farmer. She has just bought her first tractor for R500 000. The value of the tractor depreciates at a rate of 8% per annum on a reducing balance. Thobile would like to replace the tractor in 4 years' time. The cost of a new tractor appreciates at 9% per annum due to inflation.

(1) Determine the book value of the tractor after 4 years. (2)

(2) Determine the cost of a new tractor in 4 years' time. (2)

(3) Thobile plans to trade in the old tractor after 4 years and buy a new one. How much money will she need in a sinking fund in 4 years' time, in order to cover the cost of a new tractor? (2)

(4) Thobile decides to save R400 000 in this sinking fund and immediately deposits an amount of Rx at the beginning of the month. She then pays Rx at the end of the first month and continues to make the same monthly deposits of Rx at the end of each month for 4 years. If the sinking fund earns interest at a rate of 9% p.a. compounded monthly over a 4 year period, determine the value of x . (3)

- (b) Zandile buys a plot of land on her friend's farm for R1 275 000. She wishes to build a B&B on the property. Zandile takes out a loan for the full amount at an interest rate of 10,25% per annum compounded monthly over a period of 20 years.
The equal monthly instalments on this loan are R12 515,95.

(1) Calculate the outstanding balance on the loan after 7 years.

(3)

- (2) Immediately after 7 years Zandile misses the next 5 consecutive payments, due to financial difficulty. Thereafter she continues making monthly payments into the loan account until the end of the 20 year period.

Determine the new monthly payments Zandile will now have to make.

(4)

[16]

QUESTION 3

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(a) Given : $f(x) = \frac{1}{x}$

(1) Determine the average gradient in the interval $[1; 3]$.

(3)

(2) Determine $f'(x)$ from first principles.

(5)

(b) Determine the following (where necessary leave answers in positive exponential form):

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(1) $\frac{dy}{dx}$ if $y = (1 - 3x)^2$ (3)

(2) $D_x \left[\frac{x-8}{x^2} \right]$ (3)

[14]

QUESTION 4

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(a) Given the arithmetic sequence: 8 ; 10 ; 12 ; 14 ;

(1) Determine the general term of the sequence.

(2)

(2) What term has a value of 166?

(2)

(3) Calculate the sum of the first 80 terms.

(2)

- (4) It is further given that the above sequence forms the row of the first differences for a quadratic sequence, $T_n = an^2 + bn + c$, where $T_5 = 47$.

Determine the values of a , b and c .

(4)

- (b) Given: $\sum_{k=1}^n (2k - 5) = x$ and $\sum_{k=1}^{15} (2k - 5) = 165$

Determine, in terms of x , the value of $\sum_{k=16}^n (2k - 5)$.

(2)

[12]

QUESTION 5

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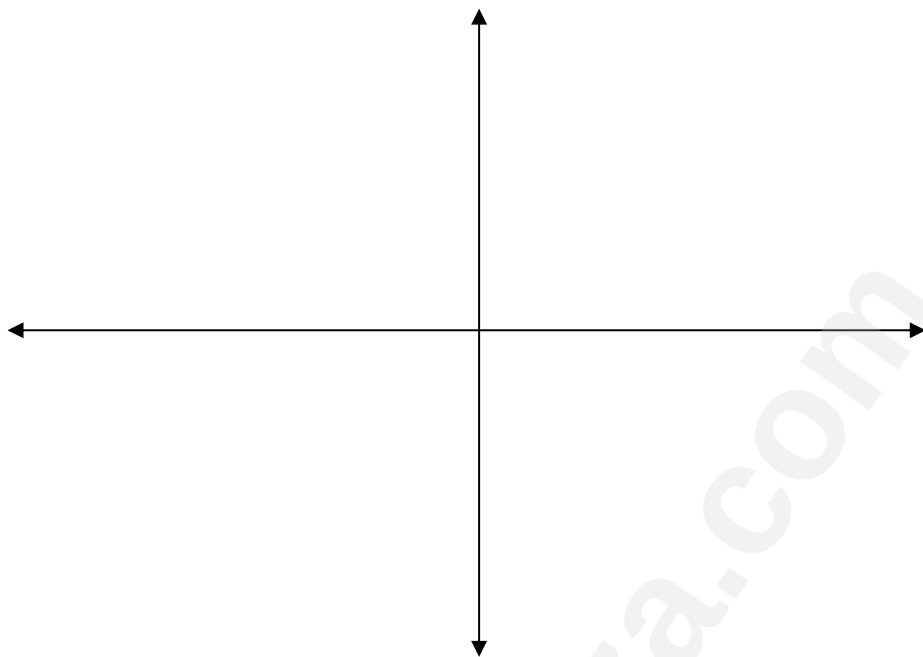
Given the graph $f(x) = -\frac{4}{x-2} - 1$.

(a) Write down the equations of the asymptotes. (2)

(b) Calculate the intercepts with the axes. (3)

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- (c) Sketch the graph of f on the axes below. Label clearly the axes, intercepts and the asymptotes and show at least two coordinates on each arm. (3)



- (d) One of the axes of symmetry of f is an increasing function. Determine its equation. (3)

- (e) Circle your answer: If $f(x) = -\frac{4}{x-2} - 1$ is reflected about the y -axis to form h , then: (2)

A. $h(x) = \frac{4}{x-2} - 1$ B. $h(x) = \frac{4}{x-2} + 1$ C. $h(x) = \frac{4}{x+2} + 1$ D. $h(x) = \frac{4}{x+2} - 1$

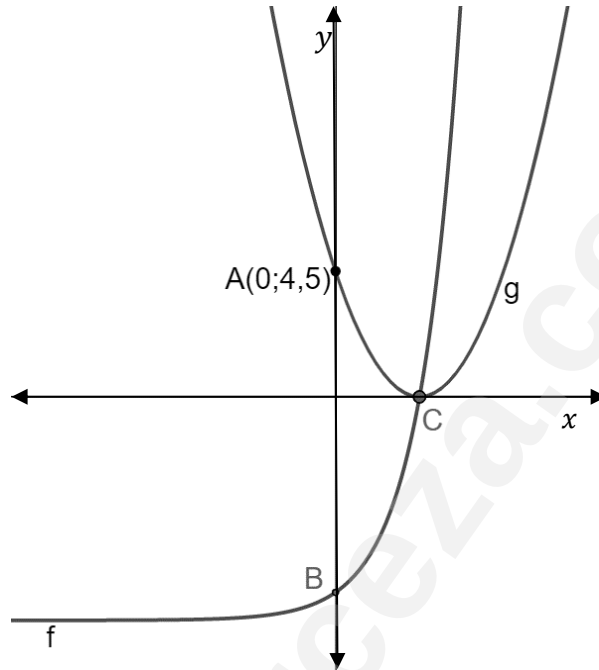
[13]

QUESTION 6

The graphs of $f(x) = 2^x - 8$ and $g(x) = a(x + p)^2 + q$ are drawn below.

The point $A(0; 4\frac{1}{2})$ and B are the y -intercepts of $g(x)$ and $f(x)$ respectively.

The graphs intersect at C, which is the turning point of $g(x)$, as well as the common x -intercept of $f(x)$ and $g(x)$.



- (a) Write down the equation of the asymptote of f . (1)

- (b) Determine the coordinates of B and C. (3)

(c) Determine the value of a , p and q if $g(x) = a(x + p)^2 + q$. (4)

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(d) (1) Describe the transformation that has taken place for $g(x)$ to become $h(x) = \frac{1}{2}x^2$. (1)

(2) The domain of $h(x)$ is restricted so that h^{-1} is the function $y = -\sqrt{2x}$.
Write down the restriction on h . (1)

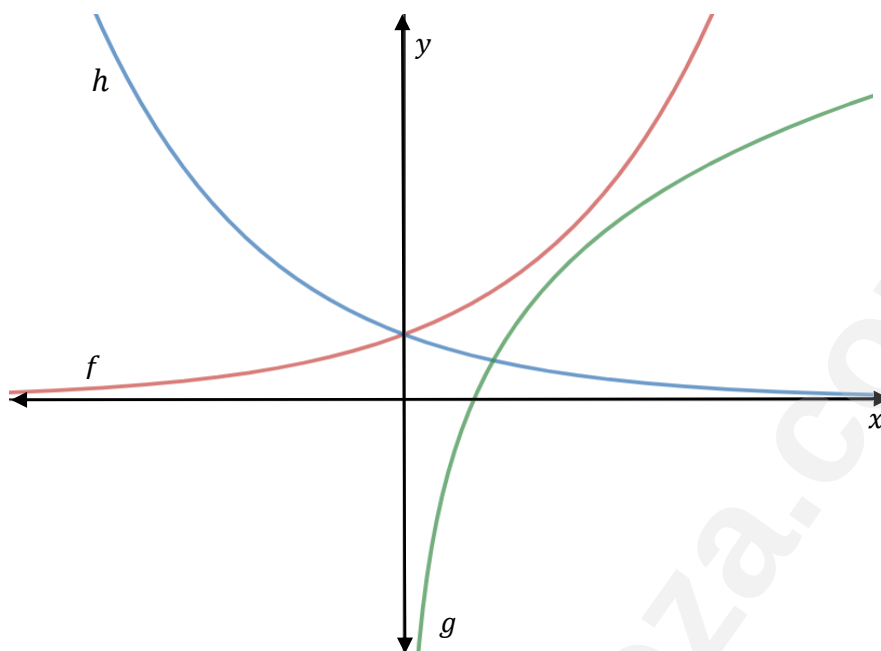
(e) Determine k , if $k(x) = f(3x) + 8$. (2)

[12]

QUESTION 7

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The diagram below shows the graphs of g , h and $f(x) = \left(\frac{3}{2}\right)^x$.



- (a) Write down the equation of g if $g(x) = f^{-1}(x)$. (2)

- (b) Write down the equation of h if $h(x) = b^x$ is the reflection of f in the y -axis and $0 < b < 1$. (2)

(c) Determine the value(s) of x for which:

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(1) $h(x) \geq f(x)$

(1)

(2) $\log_{\frac{3}{2}} x \leq 0$

(2)

(d) Without drawing the graph determine the domain of $y = \log_{\frac{3}{2}}(x + 1)$

. (2)

[9]

QUESTION 8

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A cubic function, $f(x) = ax^3 + bx^2 + cx + d$, has the following properties:

$a < 0$	$f(-1) = f(2) = 0$	$f'(-1) = f'(1) = 0$	$f(1) = 4$ and $f(0) = 2$
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- (a) Use this information to draw a neat sketch graph in the space below. Label intercepts and stationary points.

(4)

- (b) For which values of x is f decreasing?

(2)

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- (c) Use the intercepts to show that the defining equation of the graph is $f(x) = -x^3 + 3x + 2$.
(It is not sufficient to show that a given point lies on f).

(4)

(d) Determine:

1. The coordinates of the point of inflection.

(2)

2. The values of x for which the graph of $f(x)$ is concave down.

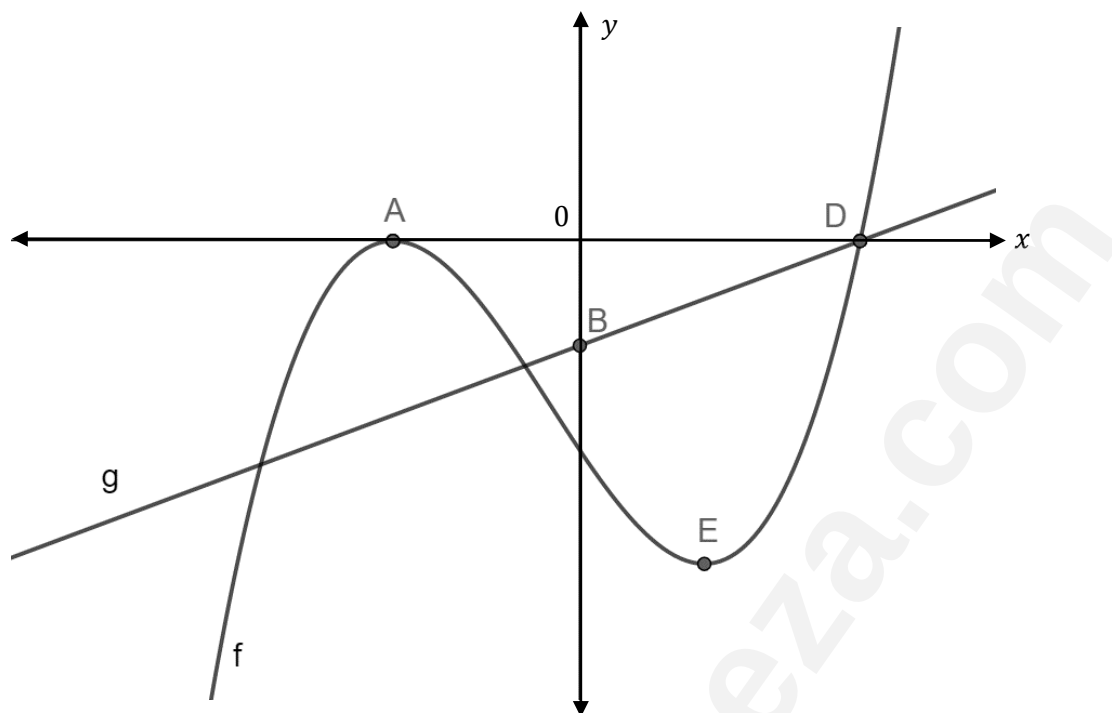
(1)

[13]

QUESTION 9

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Sketched below is the graph of $f(x) = x^3 + x^2 - 8x - 12$ and $g(x) = 2x + c$.
A and D are the x -intercepts of f and g . OB is 6 units. A and E are stationary points of f .



- (a) Show that $(x - 3)$ is a factor of $f(x)$. (2)

- (b) Determine the length AD. Show ALL necessary working. (3)

(c) Write down the equation of $g(x)$.

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(1)

(d) Determine the equation of the tangent to $f(x)$ that has the point of tangency at the y -intercept of f .

(4)

(e) Use the graph to determine the value(s) of m such that $f(x) = m$ has only one real, positive root.

(2)

(f) For which value(s) of x is $f'(x) \cdot g(x) < 0$ where $x < 0$?

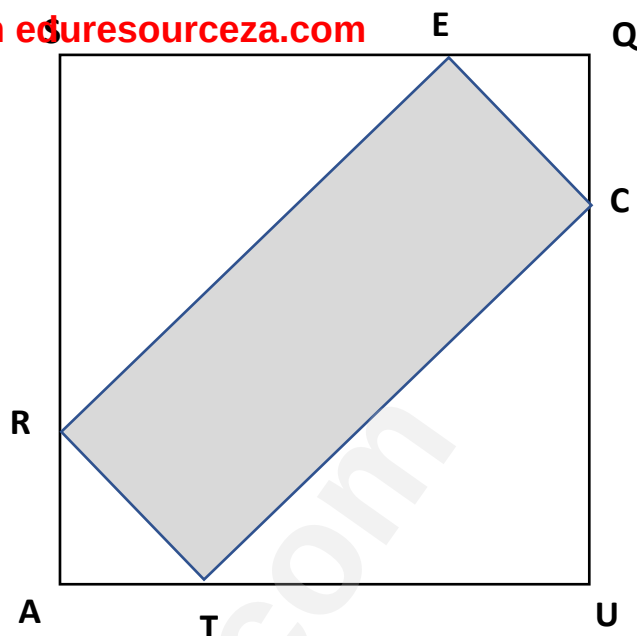
(2)

[14]

QUESTION 10

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SQUA is a **square** with sides of 30 cm each.
RECT is a rectangle that fits exactly inside the square, such that its vertices touch the sides of the square and
 $EQ = QC = AT = AR = x$ cm.



- (a) Prove that the area of RECT is $A(x) = -2x^2 + 60x$. (4)

- (b) Determine the dimensions of RECT for which the area of RECT is a maximum. (4)

[8]

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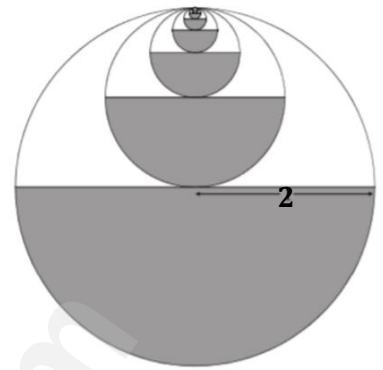
QUESTION 11

- (a) The diagram represents a sketch of circles.
Half of the area of each circle is shaded as shown.
The radius of the largest circle is 2 units.
The diameter of each smaller circle is equal to the radius of the previous circle.

Determine the sum of the shaded areas of all the circles if this process goes on indefinitely.

Give your answer in terms of π and simplify.

(Area of a semicircle = $\frac{1}{2}\pi r^2$)

[illegible]

- (b) If $S_n = -n^2 + 2n$ for an arithmetic series, consisting of 12 terms, determine:

- (1) The first term of the series.

(2)

- (2) The sum of the last 4 terms.

(3)

- (c) The ratio of the **sum of the first eight terms** to the **sum of the first four terms** of a **geometric series** is 97 : 81. If the first term is 9, determine the next two terms of the sequence, if it is given that all terms are positive. (6)

(6)

[15]