



eduresourceza.com



Grade 12
Time: 3 hours

Core Mathematics: Paper II

September 2019
Marks: 150

Please read the following instructions carefully:

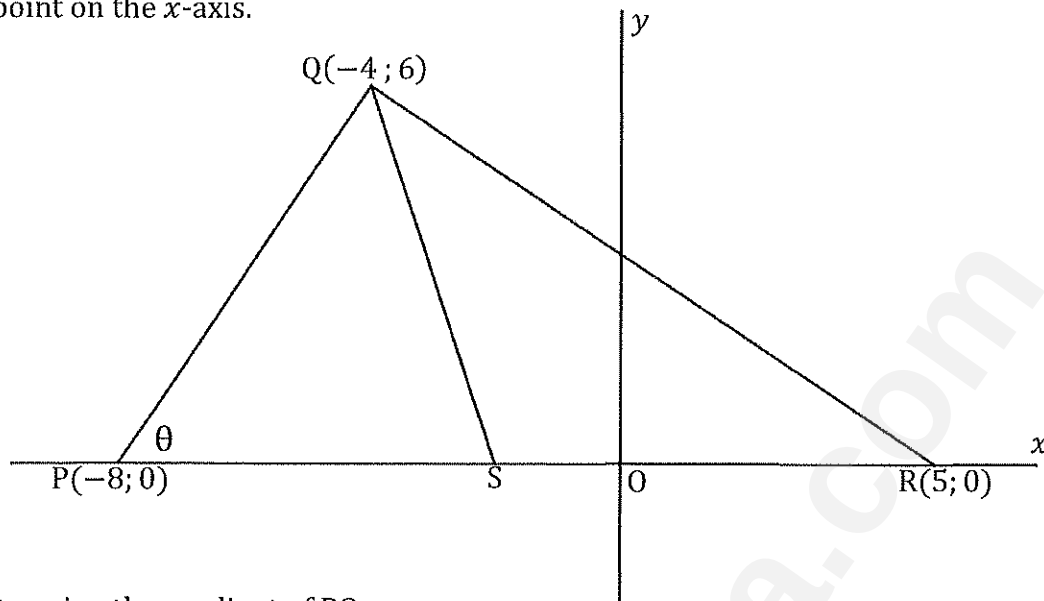
1. This question paper consists of 23 printed pages and an Information sheet. Please check that your question paper is complete.
2. Write your examination number in the space provided on this question paper.
3. **Answer all the questions on the question paper and hand this in at the end of the examination.**
4. All necessary working details must be clearly shown. Give reasons unless otherwise stated.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off answers to **1 decimal digit** where necessary, unless otherwise stated.
7. Ensure that your calculator is in **DEGREE** mode.
8. Diagrams are not drawn to scale.
9. It is in your own interest to present your work neatly.
10. The last page can be used for additional working, if necessary. If this space is used, make sure that you indicate clearly which question is being answered.

EXAMINATION NUMBER:

	SECTION A						SECTION B							
Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Mark														
Total	9	13	19	7	14	15	9	11	19	9	5	10	6	4
Total											Percentage			
	150												100	

QUESTION 1

In the sketch below, the vertices of ΔPQR are $P(-8; 0)$, $Q(-4; 6)$ and $R(5; 0)$.
S is a point on the x-axis.



(a) Determine the gradient of PQ.

(1)

$$m_{PQ} = \frac{6-0}{-4+8} = \frac{6}{4} = \frac{3}{2} \checkmark$$

(b) Prove that $\widehat{PQR} = 90^\circ$.

(2)

$$m_{QR} = \frac{6-0}{-4-5} = \frac{6}{-9} = -\frac{2}{3} \checkmark$$

$$\therefore m_{PQ} \times m_{QR} = \frac{3}{2} \times -\frac{2}{3} = -1 \checkmark$$

$$\therefore PQ \perp QR \text{ ie: } \widehat{PQR} = 90^\circ$$

(c) Calculate the size of θ , rounded to one decimal place.

(2)

$$\tan \theta = \frac{3}{2} \checkmark$$

$$\therefore \theta = 56,3^\circ \checkmark$$

(d) Calculate the gradient of QS if it is given that QS bisects $\widehat{PQR}$.

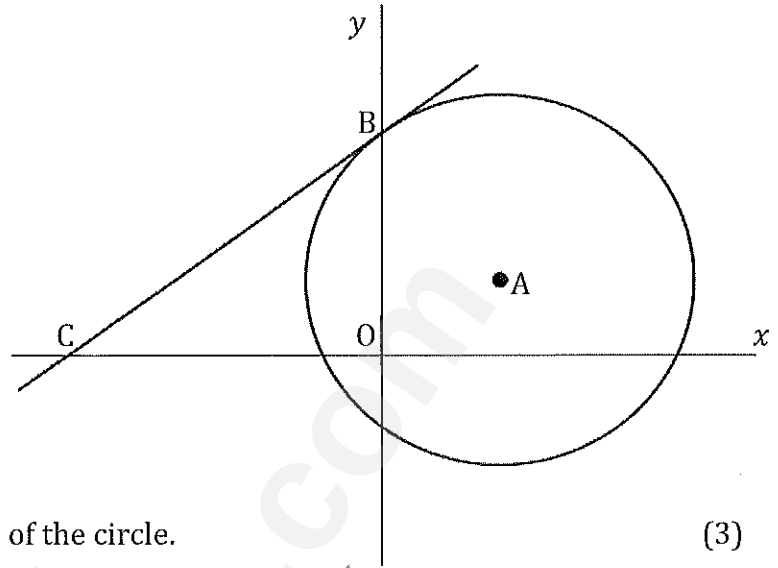
(4)

$$\hat{Q}_1 = \hat{Q}_2 = 45^\circ \checkmark (\text{given}) \therefore \widehat{QSR} = 101,3^\circ \checkmark (\text{ext } \angle \text{ of } \Delta PQS)$$

$$\therefore m \text{ of } QS = \tan 101,3^\circ \checkmark$$

$$= -5 \checkmark$$

In the diagram below, a circle with the equation $x^2 - 6x + y^2 - 4y = 12$, cuts the y -axis at B. Line BC is a tangent to the circle. C lies on the x -axis.



(a) Determine the co-ordinates of A, the centre of the circle.

(3)

$$x^2 - 6x + 9 + y^2 - 4y + 4 = 12 + 9 + 4 \quad \checkmark m$$

$$(x-3)^2 + (y-2)^2 = 25 \quad \checkmark A$$

$$\text{Centre} : (3; 2) \quad \checkmark CA$$

(b) Show that B is the point (0; 6).

(2)

$$B: \text{y-int. of circle} \Rightarrow (0-3)^2 + (y-2)^2 = 25 \quad \checkmark m(\text{sub } x=0)$$

$$\text{let } x=0$$

$$(y-2)^2 = 16$$

$$y-2 = \pm 4 \quad \checkmark \text{solve eqn.}$$

$$\therefore y = 2 \pm 4$$

$$\text{at } B: y = 6 \quad \therefore \underline{B(0; 6)}$$

(c) Determine the equation of tangent BC.

(3)

$$m_{AB} = \frac{6-2}{0-3} = -\frac{4}{3} \quad \checkmark A$$

$$\therefore m_{BC} = \frac{3}{4} \quad \checkmark m \quad (\text{rad} \perp \text{tang})$$

$$\therefore \text{eqn. of tangent BC: } y = \frac{3}{4}x + 6 \quad \checkmark CA$$

(5)

Let $D(x, y)$

$$\therefore \text{Midpt AC} = \left(\frac{3-8}{2}, \frac{2+0}{2} \right)$$

$$= \left(-\frac{5}{2}, 1\right)$$

✓ in (idea)

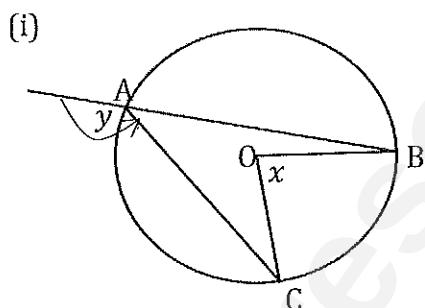
$$\text{So, } \frac{x+0}{2} = -\frac{5}{2} \text{ and } \frac{y+6}{2} = 1$$

(diags par m bisect)

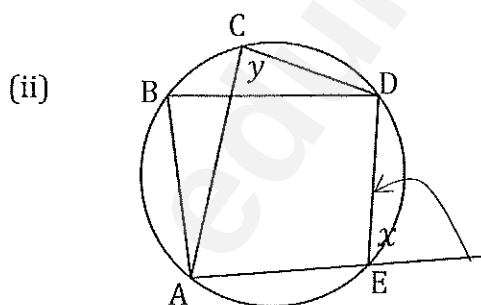
$\therefore x = -5$ and $y = -4$

$$\therefore D(-5; -4) \checkmark$$

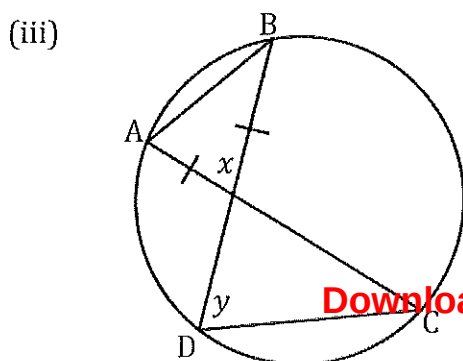
(a) In each of the following diagrams, O is the circle centre and $x = 80^\circ$. Write down the value of y . (3)



$$y = 140^\circ \checkmark A$$



$$y = 80^\circ \checkmark$$

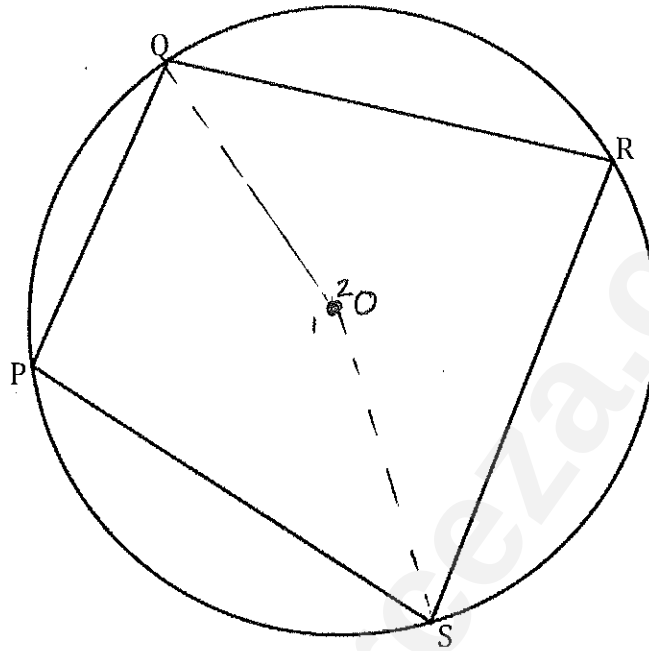


$y = 50^\circ$ ✓ A

(b) Use the diagram below to prove the theorem that states:

"The opposite angles of a cyclic quadrilateral are supplementary"

(4)



Required to prove: $\hat{P} + \hat{R} = 180^\circ$

Construction: Join O to Q and O to S. ✓

Proof: $\hat{O}_1 = 2\hat{R}$ } (L at centre thm)
 $\hat{O}_2 = 2\hat{P}$ } (L at centre thm)

$$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{R} + 2\hat{P}$$

$$\text{But } \hat{O}_1 + \hat{O}_2 = 360^\circ \quad (\text{L's round a pt})$$

$$\therefore 2(\hat{R} + \hat{P}) = 360^\circ$$

$$\hat{R} + \hat{P} = 180^\circ$$

(c) Refer to the diagram:

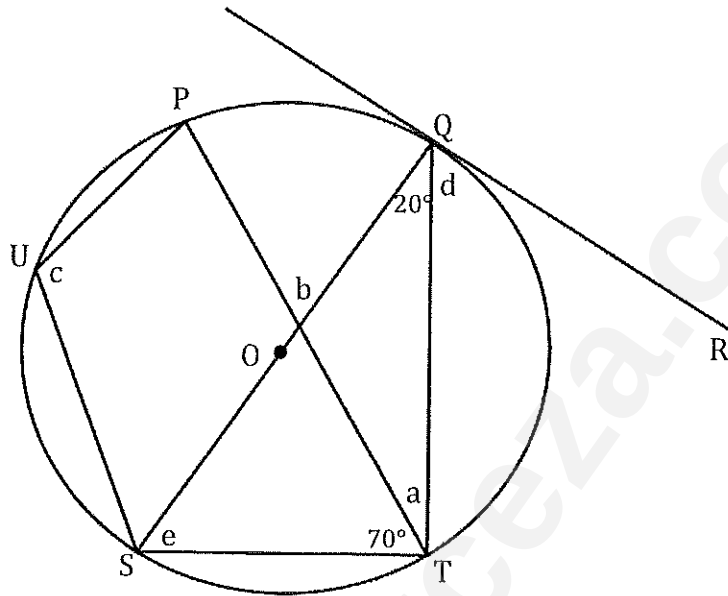
RQ is a tangent to circle QTSUP with centre O.

SOQ and PT are straight lines.

$\widehat{PTS} = 70^\circ$ and $\widehat{SQT} = 20^\circ$.

Find, with reasons, the size of the letters marked (a) to (e).

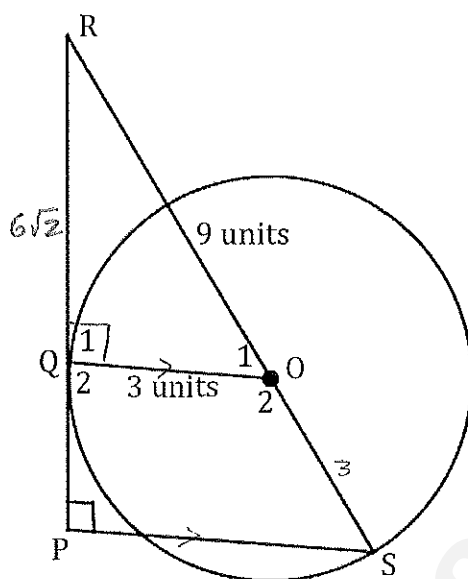
Fill your answers in the table below (solve alphabetically).



Angle	Answer	Reason
a	20°	$\angle$ in semicircle ✓
b	40°	ext. $\angle$ of Δ ✓
c	110°	opp. $\angle$'s of cyclic quad PTSU ✓
d	70°	rad $\perp$ tang ✓
e	70°	tan-chord thm. ✓

(5)

- (d) In the diagram below, O is the centre of the circle. Tangent RQ is produced to P such that $\widehat{RPS} = 90^\circ$. SO is produced to meet the tangent at R. $OR = 9$ units and $OQ = 3$ units.



- (i) Prove that $QO \parallel PS$.

(2)

$$\hat{Q}_1 = 90^\circ \quad (\text{rad} \perp \text{tang})$$

$$\hat{P} = 90^\circ \quad (\text{given})$$

$$\therefore QO \parallel PS \quad (\text{corr. l's eq.})$$

- (ii) Calculate, giving reasons, the lengths of the following leaving answers in simplest surd form where necessary.

1. RQ

(2)

$$\Delta RQO: RQ^2 = 9^2 - 3^2 \quad (\text{Pythag})$$

$$RQ^2 = 72$$

$$RQ = 6\sqrt{2} \text{ u}$$

2. QP

(3)

$$\text{In } \Delta RPS: \frac{QP}{QR} = \frac{OS}{OR} \quad (\text{prop. int. thm, } OQ \parallel PS)$$

$$\therefore \frac{QP}{6\sqrt{2}} = \frac{3}{9}$$

$$QP = 2\sqrt{2} \text{ u}$$

Answer the following question without the use of a calculator.

(a) If $\sin \alpha = 6 \cos \alpha$, determine the value of:

(i) $\tan \alpha$

(1)

$$\frac{\sin \alpha}{\cos \alpha} = 6$$

$$\therefore \tan \alpha = 6$$

(ii) $\frac{\sin(\alpha - 45^\circ)}{\cos \alpha}$

(4)

$$= \frac{\sin \alpha \cdot \cos 45^\circ - \cos \alpha \cdot \sin 45^\circ}{\cos \alpha}$$

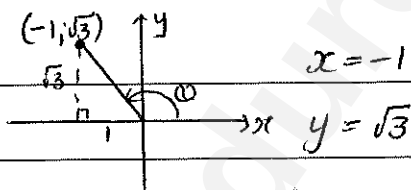
$$= \frac{\frac{1}{\sqrt{2}} \sin \alpha - \frac{1}{\sqrt{2}} \cos \alpha}{\cos \alpha}$$

$$= \frac{\frac{1}{\sqrt{2}} (6 \cos \alpha - \cos \alpha)}{\cos \alpha}$$

$$= \frac{\frac{1}{\sqrt{2}} (5 \cos \alpha)}{\cos \alpha} = \frac{5}{\sqrt{2}} \text{ OR } \frac{5\sqrt{2}}{2}$$

(b) If $\tan \theta = -\sqrt{3}$ and $0^\circ \leq \theta \leq 180^\circ$, determine the value of: $\cos(\theta + 30^\circ)$.

(4)



$$r^2 = (\sqrt{3})^2 + (-1)^2 \text{ (Pythag)}$$

$$r = 2$$

$$\cos(\theta + 30^\circ)$$

$$= \cos \theta \cdot \cos 30^\circ - \sin \theta \cdot \sin 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$= -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$= -\frac{\sqrt{3}}{2}$$

(c) If $p = \sin 25^\circ$, express each of the following in terms of p :

(i) $\cos 50^\circ$ (3)

$$= \cos 2(25^\circ) \quad \text{✓ m(double angle)}$$

$$= 1 - 2 \sin^2 25^\circ \quad \text{✓ m}$$

$$= 1 - 2p^2 \quad \text{✓ m}$$

(ii) $\sin 205^\circ$ (2)

$$= \sin (180^\circ + 25^\circ) \quad \text{✓ m}$$

$$= -\sin 25^\circ$$

$$= -p \quad \text{✓ m}$$

[14]

QUESTION 6

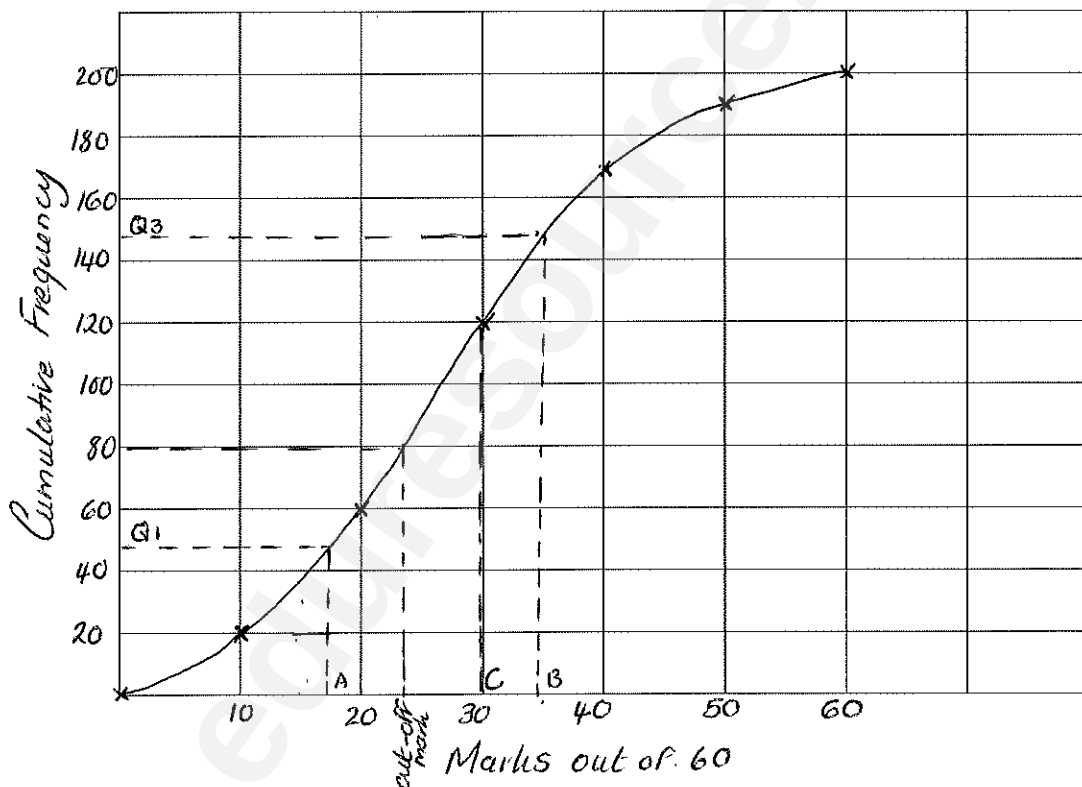
The following frequency table shows the distribution of marks of 200 students writing the NBT exam out of 60.

NBT Mark	Frequency	Cumulative Frequency
$0 \leq x \leq 10$	20	20
$10 < x \leq 20$	40	60
$20 < x \leq 30$	60	120
$30 < x \leq 40$	50	170
$40 < x \leq 50$	20	190
$50 < x \leq 60$	10	200

(a) Complete the cumulative frequency table in the space provided. *Points to plot: (10; 20) (20; 60) (30; 120) (40; 170) (50; 190) (60; 200)* (1)

(b) Draw the cumulative frequency ogive on the grid below. (3)

Cumulative frequency graph showing NBT results.



(c) Use your graph to estimate the interquartile range. *✓ (on graph)*

$$IQR = Q_3 - Q_1 = 35 - 18$$

$$= 17 \quad \checkmark \text{ (read at A and B)}$$

- (d) The top 40% of the students won't need to rewrite the test.

Use the graph to determine the cut-off mark.

(2)

$$\frac{60}{100} \times 200 = 120 \therefore 120^{\text{th}} \text{ position}$$

$\therefore$ cut-off mark is 30 out of 60 (Read at C).

- (e) Calculate the estimated mean of the data.

(2)

$$\bar{X} = \frac{(5 \times 20) + (15 \times 40) + (25 \times 60) + (35 \times 50) + (45 \times 20) + (55 \times 10)}{200}$$

$$= 27 \checkmark$$

- (f) It is given that the standard deviation is 12,9.

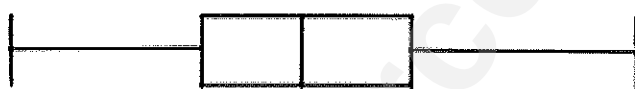
The marks for the NBT test are too low and are all raised by 5 marks.

Write down the new estimated mean and new standard deviation for the new set of marks. (2)

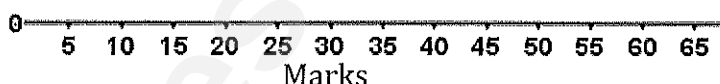
$$\text{New } \bar{X} = 32 \checkmark \text{ and std.dev} = 12,9 \checkmark$$

- (g) Below are two box and whisker diagrams representing the marks of 200 students each from two different Universities, for the same NBT test out of 60.

University A



University B



- (i) What percentage of University B's results were above 55 out of 60?

(1)

$$25\% \checkmark$$

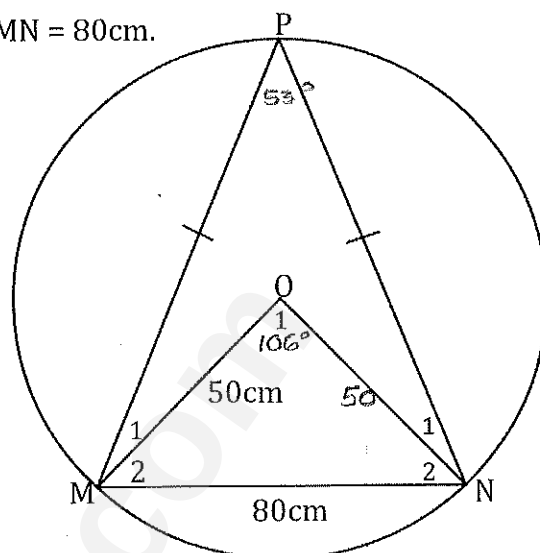
- (ii) University A claim that their overall results are better than those of University B.

Is this true? Refer to the diagrams to justify your answer.

(2)

No $\checkmark$ Uni A and B both have max. marks of 60 but 50% of Uni A's students got ± 27 and above whereas Uni B's had 50% achieving 50 and above marks. Uni A also had 25% of candidates getting between 0 and 17 in comparison to the lowest mark for Uni B being 15 and 25% of students getting between 15 and 25.

O is the centre of the circle and P, M and N lie on the circle. $MN = 80\text{cm}$.
The radius of the circle is 50 cm . $PM = PN$.



Determine:

(a) The size of $\hat{O}_1$, rounded to the nearest degree.

In $\triangle MON$: $OM = ON$ (radii)

$$80^2 = 50^2 + 50^2 - 2(50)(50)\cos \hat{O}_1$$

$$\therefore \cos \hat{O}_1 = \frac{-7}{25}$$

$$\therefore \hat{O}_1 = 180^\circ - 74^\circ$$

$$= 106^\circ$$

(3)

(b) The length of PM, rounded to one decimal digit.

$PM = PN$ (given) and $\hat{P} = 53^\circ$ (Lat centre thm)

$\therefore \hat{M} = \hat{N} = 63,5^\circ$ (isos $\triangle$ and L sum of $\triangle$)

$$\text{In } \triangle PMN: \frac{PM}{\sin 63,5^\circ} = \frac{80}{\sin 53^\circ}$$

$$PM = 89,6\text{ cm}$$

(4)

(c) The area of $\triangle PNM$.

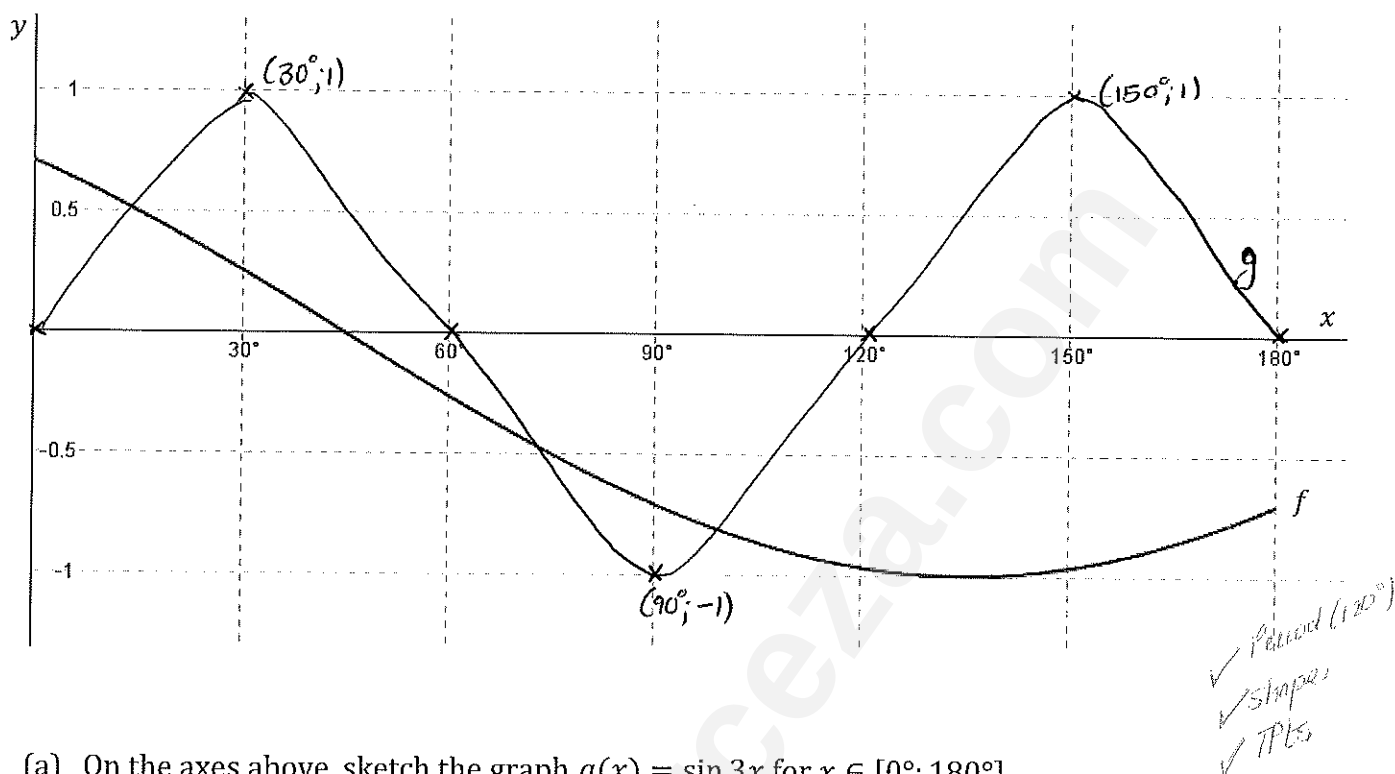
$$\text{Area } \triangle PNM = \frac{1}{2} (89,6)(89,6) \sin 53^\circ$$

$$= 3205,8\text{ cm}^2$$

(2)

QUESTION 8

The diagram below is a sketch of $f(x) = \cos(x + 45^\circ)$ for $x \in [0^\circ; 180^\circ]$.



- (a) On the axes above, sketch the graph $g(x) = \sin 3x$ for $x \in [0^\circ; 180^\circ]$. Label all turning points.

(3)

- (b) Determine the value(s) of x for which $g(x) = f(x)$ for $x \in [0^\circ; 180^\circ]$.

(6)

$$\cos(x + 45^\circ) = \sin 3x$$

$$\text{OR } \sin(90^\circ - (x + 45^\circ)) = \sin 3x$$

$$\therefore \cos(x + 45^\circ) = \cos(90^\circ - 3x) \quad \checkmark \text{ie } \text{same}$$

$$\sin(45^\circ - x) = \sin 3x$$

$$\text{Q1: } x + 45^\circ = 90^\circ - 3x + k360^\circ$$

$$\text{Q1: } 45^\circ - x = 3x + k360^\circ$$

$$4x = 45^\circ + k360^\circ$$

$$-4x = -45^\circ + k360^\circ$$

$$x = 11,3^\circ + k90^\circ \quad \checkmark \text{ie } (k \in \mathbb{Z})$$

$$x = 11,3^\circ + k90^\circ$$

$$\text{Q4: } x + 45^\circ = 360^\circ - (90^\circ - 3x) + k360^\circ$$

$$\text{Q2: } 45^\circ - x = 180^\circ - 3x + k360^\circ$$

$$-2x = 225^\circ + k360^\circ$$

$$2x = 135^\circ + k360^\circ$$

$$x = -112,5^\circ + k180^\circ \quad \checkmark \text{ie } (k \in \mathbb{Z})$$

$$x = 67,5^\circ + k180^\circ$$

$$k=0: x = 11,3^\circ$$

$$k=0: x = 11,3^\circ \text{ OR } 67,5^\circ$$

$$k=1: x = 88,8^\circ \text{ OR } 101,3^\circ$$

$$k=1: x = 101,3^\circ$$

- (c) Determine the values of x for which $g(x) > f(x)$, for $x \in [0^\circ; 180^\circ]$.

(2)

$$11,3^\circ < x < 67,5^\circ \text{ OR } 101,3^\circ < x \leq 180^\circ$$

(a) Consider the identity: $\frac{\cos \theta - \cos 2\theta + 2}{3 \sin \theta - \sin 2\theta} = \frac{\cos \theta + 1}{\sin \theta}$

(i) Prove the identity.

(5)

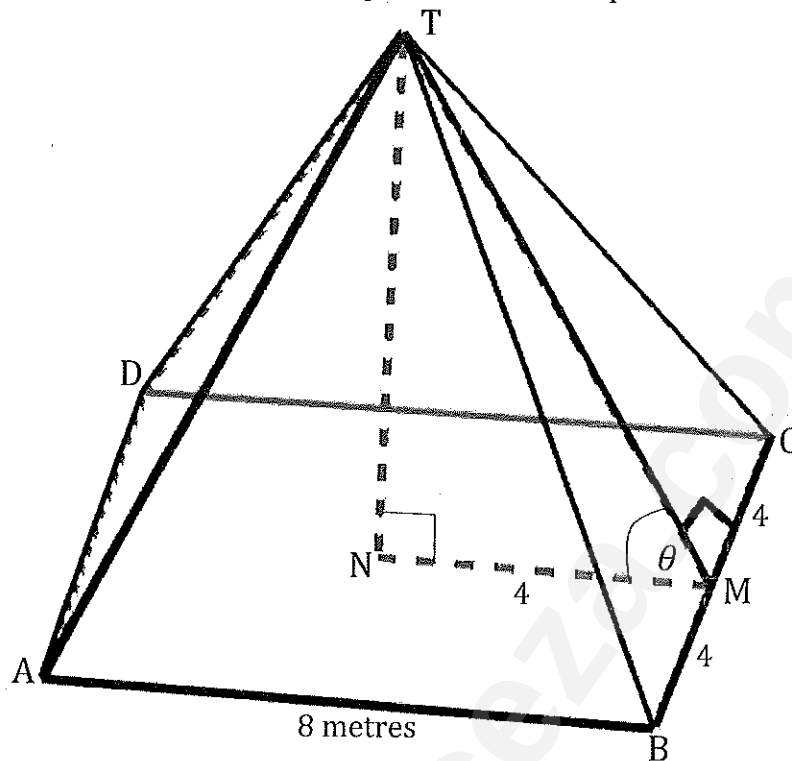
$$\begin{aligned}
 \text{LHS: } & \frac{\cos \theta - (2\cos^2 \theta - 1) + 2}{3 \sin \theta - 2 \sin \theta \cos \theta} \quad \checkmark m(\text{double l.}) \\
 & = \frac{-2\cos^2 \theta + \cos \theta + 3}{\sin \theta (3 - 2\cos \theta)} \\
 & = \frac{(3 - 2\cos \theta)(1 + \cos \theta)}{\sin \theta (3 - 2\cos \theta)} \quad \checkmark m(\text{factorize}) \\
 & \quad \checkmark m(\text{cancel}) \\
 & = \frac{\cos \theta + 1}{\sin \theta} \\
 & = \text{RHS} \\
 & \quad \longrightarrow
 \end{aligned}$$

(ii) For which value(s) of θ will the identity be undefined?

(3)

$$\begin{aligned}
 \text{undefined if: } & 3 \sin \theta - \sin 2\theta = 0 \quad \checkmark m(\text{idea}) \\
 \text{ie: } & \sin \theta (3 - 2\cos \theta) = 0 \\
 \sin \theta = 0 & \quad \text{OR} \quad \cos \theta = 3/2 \\
 \theta = k \cdot 180^\circ (k \in \mathbb{Z}) & \quad \checkmark m \\
 & \quad \text{No soln} \quad \checkmark m \\
 & \quad \longrightarrow
 \end{aligned}$$

- (b) In the diagram below, TABCD is a right pyramid with a square base side of 8 metres. M is the midpoint of BC and TN is the height of the pyramid. Each of the triangular faces makes an angle of θ with the square base.



- (i) Express TM in terms of θ .

(2)

$$\text{In } \triangle TMN: \cos \theta = \frac{4}{TM} \quad \checkmark \text{m (trig ratio)}$$

$$\therefore TM = \frac{4}{\cos \theta} \quad \checkmark \text{A}$$

- (ii) If the total surface area of the pyramid is equal to 256 square metres, determine the value of θ , correct to one decimal digit.

(4)

$$TSA = (8 \times 8) + 4 \left(\frac{1}{2} (8 \times TM) \right)$$

$$256 = 64 + 16 \left(\frac{4}{\cos \theta} \right) \quad \checkmark \text{m (sub)}$$

$$192 = \frac{64}{\cos \theta}$$

$$\therefore \cos \theta = \frac{64}{192}$$

$$\theta = 70,5^\circ \quad \checkmark \text{A}$$

- (c) (i) Show that $2 \cos(x - 60^\circ) = \cos x + \sqrt{3} \sin x$ (2)

$$\text{LHS: } 2 [\cos x \cdot \cos 60^\circ + \sin x \cdot \sin 60^\circ] \quad \checkmark \text{ (expand)}$$

$$= 2 \cos x \left(\frac{1}{2}\right) + 2 \sin x \left(\frac{\sqrt{3}}{2}\right) \quad \checkmark \text{ (simplify)}$$

$$= \cos x + \sqrt{3} \sin x$$

$$= \text{RHS.}$$

- (ii) Determine the minimum value of $\cos x + \sqrt{3} \sin x$, and the corresponding value(s) of x if $0^\circ \leq x \leq 360^\circ$. (3)

$$2 \cos(x - 60^\circ) \text{ has a minimum value of } -2$$

$$\text{at } x = 180^\circ + 60^\circ \quad (\text{shifted } 60^\circ \text{ to the right})$$

$$\underline{x = 240^\circ} \quad \checkmark \text{ (include)}$$

[19]

Green garden services mow lawn for customers. To calculate the charges for their work, they measure the approximate area, x , (in m^2) of a random sample of 12 of their customers' lawns and the time, y , in minutes that it takes them to mow the lawns.

Their results are shown in the table:

Area (x) (m^2)	360	120	845	602	1190	530	245	486	350	1005	320	250
Time (y) (minutes)	50	28	130	75	120	95	55	70	48	110	55	60

- (a) Use your calculator to determine the equation of the least squares regression line. Give your answers correct to 4 decimal digits. (3)

$$a = 28,1412 \quad \checkmark^A \quad \therefore y = 28,1412 + 0,0886x$$

$$b = 0,0886 \quad \checkmark^A$$

- (b) Calculate the value of r , the correlation coefficient for the data, correct to 4 decimal places. (1)

$$r = 0,9135 \quad \checkmark^A$$

- (c) Given that the garden service charge a flat call-out fee of R 150, as well as R 50 per half hour (or part thereof), estimate the charge for mowing a customer's lawn that has an area of 560m^2 . (For example: 100 minutes would be taken as 2 hours) (3)

$$y = 28,1412 + 0,0886(560) \quad \checkmark^A \quad \text{OR using the calculator}$$

$$= 77,7572 \quad \checkmark^A \quad y = 77,7447$$

$$\therefore \text{Fee} = R150 + 3(50)$$

$$= R300 \quad \checkmark^A$$

- (d) The local high school wants Green garden services to mow their rugby field, which is rectangular, 100m long and 70m wide. Should they use the regression equation found in (a) to calculate the time it would take to mow this area? Give a reason for your answer. (2)

$$7000 \text{ m}^2 \quad \therefore y = 28,1412 + 0,0886(7000)$$

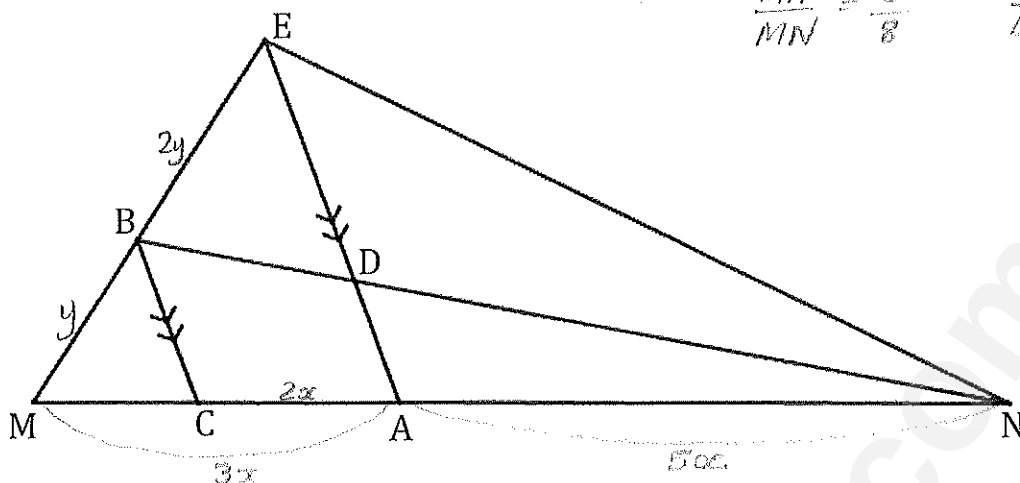
$$= 648,3412 \approx 11 \text{ hrs}$$

No, it would take too long and be too expensive.

(Point also an outlier so given cost structure not good in this scenario)

In the diagram (not drawn to scale), BC is parallel to EA. $MA = \frac{3}{8}MN$ and $2MB = BE$.

$$\frac{MA}{MN} = \frac{3}{8} \quad \frac{MB}{BE} = \frac{1}{2}$$



(a) Find the value of BD:DN.

(3)

In $\triangle MEA$: $\frac{CA}{MA} = \frac{BE}{EM}$ $\checkmark$ (prop. int. thm, $BC \parallel EA$)

$$\therefore CA = \frac{2y}{3y} \times 30x$$

$$CA = 20x$$

In $\triangle BCN$: $\frac{BD}{DN} = \frac{CA}{AN}$ $\checkmark$ (prop. int. thm, $AD \parallel BC$)

$$= \frac{20x}{50x}$$

$$= \frac{2}{5} \checkmark$$

(b) Determine the $\frac{\text{Area } \triangle BMC}{\text{Area } \triangle BMN}$

(2)

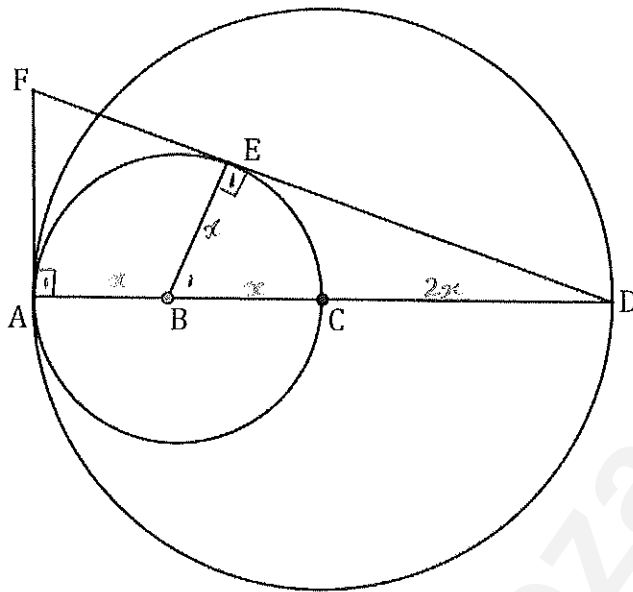
$$= \frac{MC}{MN} \checkmark \text{ (same ht. from } \hat{B} \text{)}$$

$$= \frac{x}{8x}$$

$$= \frac{1}{8} \checkmark$$

In the diagram, AD is a diameter of the larger circle with centre C and AC is the diameter of the smaller circle with centre B.

FD is a tangent to the smaller circle at E while FA is a tangent to both circles at A.



(a) Prove that $\triangle DEB \parallel \triangle DAF$.

(3)

In $\triangle DEB$ and $\triangle DAF$

$$1. \hat{E}_1 = \hat{A}_1 = 90^\circ \quad (\text{radii} \perp \text{tangents})$$

$$2. \hat{D} = \hat{D} \quad (\text{common})$$

$$3. \hat{B}_1 = \hat{F} \quad (\text{L sum of } \triangle)$$

$$\therefore \triangle DEB \parallel \triangle DAF \quad (AAA)$$

(b) Hence, show that $DF = 3 AF$.

(3)

$$\frac{DE}{DA} = \frac{DB}{DF} = \frac{EB}{AF} \quad (\text{radii}) \quad (\text{sides in prop.})$$

$$\text{Let } AB = BC = BE = x \quad (\text{radii of small circle})$$

$$\therefore CD = 2x \quad (\text{radius of big circle})$$

$$\therefore DB = 3x$$

$$\text{So, } \frac{3x}{DF} = \frac{x}{AF} \quad (\text{subst})$$

$$\therefore \frac{DF}{AF} = 3 \quad \text{So, } DF = 3AF$$

- (c) If it is further given that the diameter of the larger circle is 8 cm, determine the area of $\triangle DEB$.

$$4x = 8 \text{ cm}$$

$$\therefore x = 2 \text{ (radius small circle)}$$

$$\text{So, Area } \triangle DEB = \frac{1}{2} (EB \times ED)$$

$$= \frac{1}{2} (2 \times 4\sqrt{2})$$

$$= 4\sqrt{2} \text{ u}^2$$

$$= \underline{5,7 \text{ u}^2} \quad \checkmark \text{ CA}$$

$$\text{In } \triangle EBD$$

$$ED^2 = 6^2 - 2^2 \text{ (Pythag)}$$

$$ED = 4\sqrt{2}$$

(4)

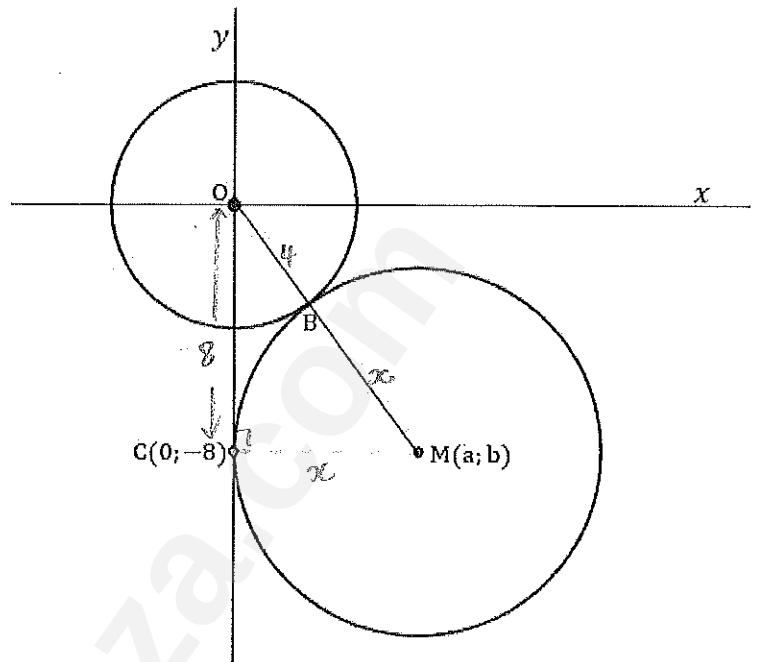
[10]

Two circles with centre $O(0;0)$ and $M(a;b)$ touch externally at B . The equation of the smaller circle with centre O is $x^2 + y^2 = 16$.

Circle centre M touches the y -axis at $C(0; -8)$.

Determine the co-ordinates of M .

(Hint: Let $CM = x$)



$$\angle OCM = 90^\circ \quad (\text{rad} \perp \text{tang})$$

Let CM be x

$$\text{then } OB = x \quad (\text{radii}) \quad \text{and } OB = 4$$

In $\triangle OCM$

$$CM^2 + OC^2 = OM^2 \quad (\text{Pythag})$$

$$x^2 + 8^2 = (x + 4)^2$$

$$x^2 + 64 = x^2 + 8x + 16$$

$$8x = 48$$

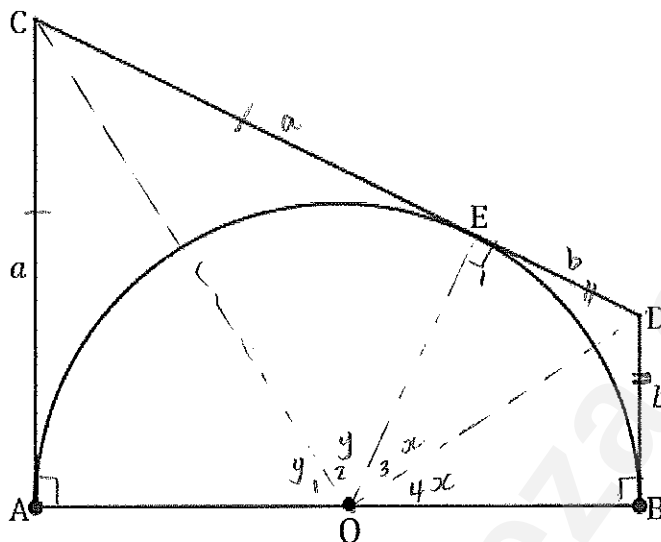
$$x = 6 \quad \checkmark$$

$$\therefore M(6; -8)$$

QUESTION 14

In the diagram below, AC, BD and CD are tangents to the circle O with diameter AB = 4cm. CD touches the circle at point E.

If $AC = a$ and $BD = b$, prove that $ab = 4$.



Constr: Draw CO, OE and OD

$$\hat{E} = 90^\circ \quad (\text{rad} \perp \text{tang})$$

and $CE = CA = a$

$$DE = DB = b \quad (\text{tangs from same pt})$$

$\therefore$ CAOE and EDBO are kites

$$\therefore \hat{O}_1 = \hat{O}_2 = y \quad \text{and} \quad \hat{O}_3 = \hat{O}_4 = x$$

$$2x + 2y = 180^\circ \quad (\text{L's on str. line})$$

$$\therefore x + y = 90^\circ$$

$$\therefore OE^2 = ab \quad (\perp \text{ from rt } \angle \text{ to hyp thm})$$

$$\therefore ab = 2^2 = 4.$$

setting up
a proof
for $\hat{O}_2 + \hat{O}_3 = 90^\circ$

[4]



Total for section B: 73

eduresourceza.com