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St Anne's Diocesan College
CORE MATHEMATICS: PAPER I
TRIAL EXAMINATION

Form 6
 Time: 3 hours

August 2019
 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 18 pages and a formulae sheet. Please check that your paper is complete.
2. The last page is blank and may be used for extra working if required.
3. Read the questions carefully.
4. Answer all the questions.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to **one decimal digit** where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your best interest to write legibly and to present your work neatly.
9. Write down your Exam Number and /Name:

Exam Number:

Model Answer

Name:

FOR OFFICIAL USE ONLY

Question	1	2	3	4	5	6	7	8	9	10	Total
Topic	Algebra	Functions	Finance	Sequence	Sequences	Calculus	Calculus	Functions	Calculus	Sequences	%
Max Mark	25	26	19	8	15	12	19	12	9	5	150
Your Mark											

SECTION A

QUESTION 1

- (a) Solve for x in each of the following, show all necessary working, and leave answers in surd form where necessary:

(1) $2x^2 = x$ (3)

$$2x^2 - x = 0$$

$$x(2x - 1) = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{1}{2}$$

(2) $(5x + 2)(x^2 - 5) = 0$ (3)

$$x = -\frac{2}{5} \quad \text{or} \quad x^2 = 5$$

$$x = \pm \sqrt{5}$$

(3) $(x - 2)^3 + 64 = 0$ (2)

$$(x - 2)^3 = -64 \quad \text{or} \quad (x - 2)(x^2 - 4x + 4) + 64 = 0$$

$$x - 2 = \sqrt[3]{-64}$$

$$x^3 - 6x^2 + 12x + 56 = 0$$

$$x = 2 + (-4)$$

$$f(-2) = 0 \quad \therefore (x + 2) \text{ is a factor}$$

$$= -2$$

$$\therefore (x + 2)(x^2 - 8x + 28) = 0$$

$$\therefore x + 2 = 0 \rightarrow x = -2$$

(4) $x + \sqrt{x - 2} = 4$ (4)

$$\sqrt{x - 2} = 4 - x$$

$$x - 2 = (4 - x)^2$$

$$x - 2 = 16 - 8x + x^2$$

$$0 = x^2 - 9x + 18$$

$$= (x - 6)(x - 3)$$

$$\therefore x \neq 6 \quad \text{or} \quad x = 3$$

Check: $x = 3$ is the only soln.

$$(5) \quad 1 = (0,5)^{x^2-1}$$

(3)

$$(0,5)^0 = (0,5)^{x^2-1}$$

$$\therefore 0 = x^2 - 1 \quad \text{or} \quad 0 = (x-1)(x+1)$$

$$x^2 = 1$$

$$x = \pm 1$$

$$x = \pm 1$$

(b) If $x^2 < 25$ and $x < 0$, find x .

(4)

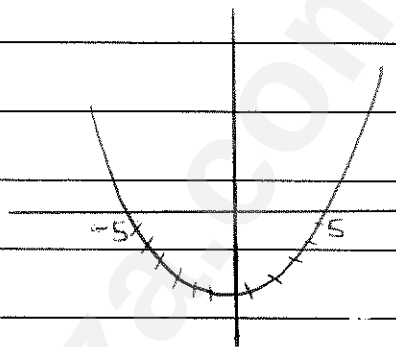
$$x^2 - 25 < 0$$

$$(x-5)(x+5) < 0$$

$$\text{Soln: } -5 < x < 5$$

$$\text{but } x < 0$$

$$\therefore -5 < x < 0$$



(c) (1) Show, by completing the square, that if m is any real number, $m^2 - 4m + 8$ is always positive.

(3)

$$m^2 - 4m + 8$$

$$= m^2 - 4m + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 + 8$$

$$= (m-2)^2 - 4 + 8$$

$$= (m-2)^2 + 4$$

$$(m-2)^2 \geq 0$$

$$\therefore (m-2)^2 + 4 \geq 4 \quad \therefore m^2 - 4m + 8 \text{ is always } \oplus$$

(2) Hence, or otherwise, show that the equation $x^2 - mx + m = 2$ has two real and unequal roots.

(3)

$$a=1 \quad b=-m \quad c=m-2$$

$$b^2 - 4ac = m^2 - 4(1)(m-2)$$

$$= m^2 - 4m + 8$$

$$\therefore b^2 - 4ac > 0 \quad , \text{ proved in (1) above}$$

$$\therefore x^2 - mx + m = 2 \text{ has two real, unequal roots.}$$

[25]

QUESTION 2

(a) Given the function: $f(x) = -(x+1)^2 + 4$

- (1) Sketch the graph of $f(x)$ showing all intercepts with the axes, turning point and another point using symmetry. (4)

Shape: $\cap$

y-int: $(0; 3)$

x-ints: $0 = -(x+1)^2 + 4$

$$(x+1)^2 = 4$$

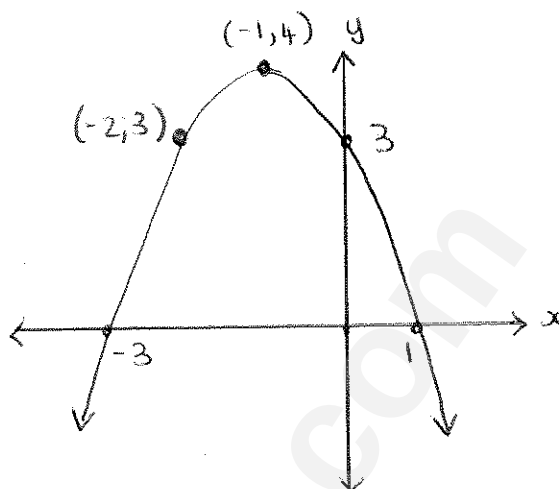
$$x+1 = \pm 2$$

$$x = -1 \pm 2$$

$$= -3 \text{ or } 1$$

$$(-3; 0) \text{ or } (1; 0)$$

T.P: $(-1; 4)$



- (2) Calculate the coordinate of a point on f for which the tangent to f will have a gradient of 3. (4)

$$f(x) = -(x+1)^2 + 4$$

$$= -(x^2 + 2x + 1) + 4$$

$$= -x^2 - 2x + 3$$

$$f'(x) = -2x - 2$$

$$3 = -2x - 2$$

$$2x = -5$$

$$x = -5/2 \quad \therefore f(-5/2) = -(-5/2 + 1)^2 + 4 = 7/4 \rightarrow \left(-\frac{5}{2}, \frac{7}{4}\right)$$

- (3) i. Explain why the inverse of f is not a function. (1)

The inverse is a one-to-many relation (many x 's map to one y).

- ii. Write down the equation of g , the graph of f shifted 1 unit to the right. (1)

$$g(x) = -x^2 + 4$$

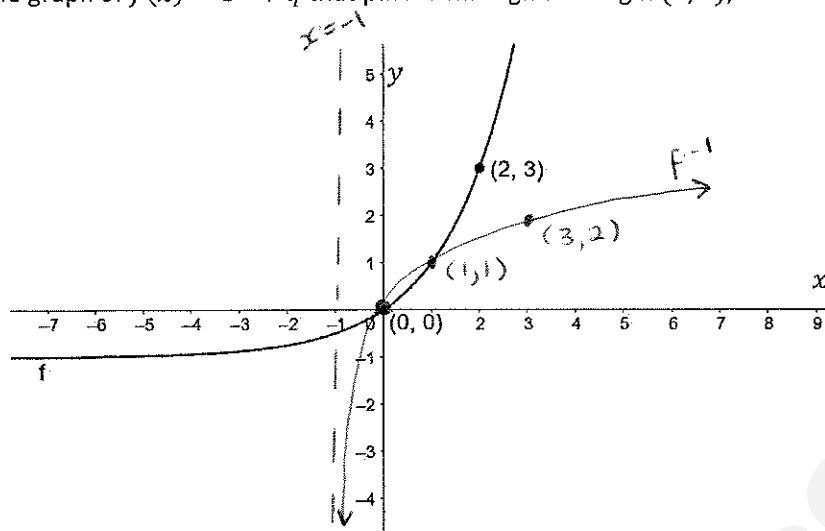
- iii. Write down ONE restriction on the domain of g so that its inverse will be a function. (1)

$$x \geq 0 \quad \text{or} \quad x \leq 0$$

- (4) Determine the values of p such that $f(x) - p$ will always be a negative value. (1)

$$p > 4$$

- .) Consider the graph of $f(x) = b^x + q$ that passes through the origin $(0; 0)$, and the point $(2; 3)$.



- (1) Show that $b = 2$ and $q = -1$, showing all necessary calculations. (3)

$$(0; 0): 0 = b^0 + q$$

$$0 = 1 + q$$

$$q = -1$$

$$(2; 3): 3 = b^2 - 1$$

$$4 = b^2$$

$$b = \pm 2$$

$$\text{But } b > 0 \therefore b = 2$$

- (2) Determine:

i. The equation of the asymptote of $f(x)$.	ii. $f(1)$.	iii. The range of f .
$y = -1$ (1)	$f(1) = 2^{(1)} - 1$ $= 1$ (1)	$y > -1$ (1)

- (3) Determine the equation of $f^{-1}(x)$, the inverse of $f(x)$ in the form $y = \dots$ (3)

$$y = 2^x - 1 \therefore \text{Inverse: } x = 2^y - 1$$

$$x + 1 = 2^y$$

$$y = \log_2(x + 1)$$

- (4) Sketch the graph on the same set of axes as f , above. Show the asymptote, intercepts with the axes and two other points. (3)

- (5) Use the graph to determine for which values of x , $f(x) < f^{-1}(x)$. (2)

$$0 < x < 1$$

QUESTION 3

(a) Mrs Van der Morgy buys a new car for R500 000.

- (1) The car dealership estimates, using the reducing balance method, that the value of the car in 5 years' time will be R320 000.

Calculate the annual rate of depreciation over the 5 years.

(3)

$$320\,000 = 500\,000 (1 - i)^5$$

$$1 - i = 0,9146\dots$$

$$i = 0,0853\dots$$

$$\therefore IR = 8,5\% \text{ p.a.}$$

- (2) Mrs Van der Morgy wants to set up a sinking fund on the day she buys the car, so that she can replace the car when she retires in 5 years' time. The rate of inflation is expected to be 10%.

Mrs Van der Morgy will sell her current car for R320 000 in 5 years' time and use that money as a deposit to put towards buying the new car.

Calculate how much her monthly instalments will be if the bank is prepared to offer her an interest rate of 11,5% p.a compounded monthly.

(5)

Inflation: $A = 500\,000 (1 + 0,1)^5$

$$= R805\,255$$

Sinking Fund Amnt: $R805\,255 - 320\,000 = R485\,255$

$$\therefore 485\,255 = x \frac{\left[\left(1 + \frac{0,115}{12} \right)^{12 \times 5} - 1 \right]}{\frac{0,115}{12}}$$

$$\therefore x = R \underline{6\,021,66}$$

- (b) Determine the annual rate of interest, compounded monthly, needed to yield the same amount of money in three years as an investment paying 10% p.a. simple interest. (4)

$$\begin{aligned} \text{SI: } A &= x(1 + 0,10(3)) & \text{OR } 1 + 0,1(3) &= (1 + \frac{i}{12})^{36} \\ &= 1,3x & \sqrt[36]{1,3} &= 1 + \frac{i}{12} \\ \text{C.I: } 1,3x &= x(1 + \frac{i}{12})^{12 \times 3} & i &= 0,0877 \\ i &= 0,0877 \dots & \text{IR} &= 8,8\% \\ \therefore \text{IR} &= 8,8\% \text{ p.a.} \end{aligned}$$

- (c) Louise wants to buy a townhouse for R1 700 000. She has a deposit of R50 000 and takes out a loan for the balance at a rate of 11,5% p.a. compounded quarterly.

- (1) Show that her quarterly payments will be R 63 804,98 if she wishes to settle the loan in 12 years. (4)

$$\begin{aligned} \text{Loan Amnt} &= 1\,700\,000 - 50\,000 = R\,1\,650\,000 \\ \therefore 1\,650\,000 &= \frac{x \left[1 - \left(1 + \frac{0,115}{4} \right)^{-4 \times 12} \right]}{0,115/4} \end{aligned}$$

$$x = R\,63\,804,98$$

- (2) Louise is short of cash and wants to make her first payment 6 months after she took out the loan. Determine the value of her new quarterly payments. (3)

$$1\,650\,000 \left(1 + \frac{0,115}{4} \right)^1 = \frac{x \left[1 - \left(1 + \frac{0,115}{4} \right)^{-47} \right]}{0,115/4}$$

$$1\,697\,437,50 = \frac{x [0,736101 \dots]}{0,115/4}$$

$$\therefore x = R\,66\,297,02 \rightarrow$$

[19]

QUESTION 4

The sequence $2; p; 16; \dots$ is a quadratic sequence. The sequence of first differences is $5; q; \dots$

- (a) Calculate p and q , show all working.

$$\begin{array}{ccc} 2 & ; & p & ; & 16 \\ & \vee & & \vee & \\ & 5 & & q & \end{array}$$

(3)

$$p - 2 = 5$$

$$p = 7$$

$$16 - q = 7$$

$$q = 9$$

- (b) Determine the n th term of the quadratic sequence above, if it is given that $p = 7$ and $q = 9$.

(5)

$$\begin{array}{ccc} 2 & ; & 7 & ; & 16 \\ & \vee & & \vee & \\ & 5 & & 9 & \\ & & \vee & & \\ & & 4 & & \end{array}$$

$$2a = 4$$

$$3a + b = 5$$

$$a + b + c = 2$$

$$\therefore a = 2$$

$$6 + b = 5$$

$$2 - 1 + c = 2$$

$$b = -1$$

$$c = 1$$

$$\therefore T_n = 2n^2 - n + 1$$

[8]

[Section A Total: 78 marks]

SECTION B

QUESTION 5

(a) $6 + 18 + 54 + \dots$ are the first three terms of a series.

(1) Is this series geometric or arithmetic?

(1)

Geometric

(2) Write down the next term in the series.

(1)

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(3) Determine $S_{11} - T_{11}$.

(3)

$$S_{11} = \frac{6(3^{11}-1)}{3-1}$$

$$\text{OR } S_{11} - T_{11} = S_{10}$$

$$= \frac{6(3^{10}-1)}{3-1}$$

$$= 531\,438$$

$$T_{11} = 6(3)^{11-1}$$

$$= 177\,144$$

$$= 354\,294$$

$$\therefore S_{11} - T_{11} = 531\,438 - 354\,294$$

$$= 177\,144$$

(b) Gugu is training for the 2020 Midmar Mile. She begins her training by swimming 250 metres on the first day. She increases this distance by 100 metres every day. On which day will she first exceed the distance she has to swim in the race? $1 \text{ mile} \approx 1,6 \text{ km}$

(3)

$$\begin{array}{ccc} T_1 & T_2 & T_3 \\ 250 & 350 & 450 \end{array}$$

$$\text{OR } 250 \quad 350 \quad 450 \quad 550$$

$$650 \quad 750 \quad 850 \quad 950$$

$$a = 250 \quad d = 100$$

$$1050 \quad 1150 \quad 1250 \quad 1350$$

$$1450 \quad 1550 \quad 1650$$

$$1600 = 250 + (n-1)(100)$$

$$\therefore 15^{\text{th}} \text{ day}$$

$$1350 = 100n - 100$$

$$1450 = 100n$$

$$\therefore n = 14,5$$

$$\therefore \text{On the } 15^{\text{th}} \text{ day}$$

(c) Determine the value of x if:

$$\sum_{k=1}^{\infty} 9x^k = \sum_{t=1}^{14} (24-3t)$$

(7)

$$\sum_{k=1}^{\infty} 9x^k$$

$$\sum_{t=1}^{14} (24-3t)$$

$$T_1 = 9x$$

$$T_1 = 24-3 = 21$$

$$T_2 = 9x^2$$

$$T_2 = 24-6 = 18$$

$$T_3 = 9x^3$$

$$T_3 = 24-9 = 15$$

$$\therefore GP; a=9x; r=x$$

$$\therefore AP; a=21; d=-3$$

$$S_{\infty} = \frac{9x}{1-x}$$

$$S_{14} = \frac{14}{2} [2(21) + (14-1)(-3)]$$

$$= 21$$

$$\textcircled{\text{OR}} T_{14} = 24-3(14) = -18$$

$$S_{14} = \frac{14}{2} [21 + (-18)]$$

$$= 21$$

$$\therefore \frac{9x}{1-x} = 21$$

$$9x = 21(1-x)$$

$$9x = 21 - 21x$$

$$30x = 21$$

$$x = \frac{7}{10}$$

[15]

QUESTION 6

- (a) Determine $f'(x)$ from first principles if $f(x) = 3x^2 + 1$ (4)

$$\begin{aligned}
 f(x) &= 3x^2 + 1 \quad ; \quad f(x+h) = 3(x+h)^2 + 1 \\
 &= 3(x^2 + 2xh + h^2) + 1 \\
 &= 3x^2 + 6xh + 3h^2 + 1 \\
 f'(x) &= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 1 - 3x^2 - 1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(6x + 3h)}{h} = 6x
 \end{aligned}$$

- (b) Given that $y = \frac{4}{x^2} + 6\sqrt{x} - \frac{9x^2-1}{3x+1}$; $x > 0$, determine $\frac{dy}{dx}$.

Leave your answer with positive, rational exponents. (5)

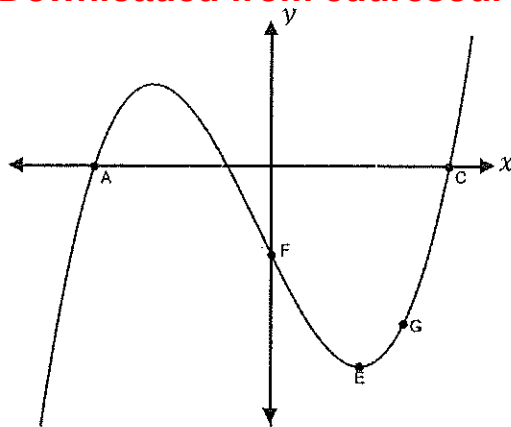
$$\begin{aligned}
 y &= 4 \cdot x^{-2} + 6 \cdot x^{\frac{1}{2}} - \frac{(3x-1)(3x+1)}{(3x+1)} \\
 &= 4x^{-2} + 6x^{\frac{1}{2}} - 3x + 1 \\
 \therefore \frac{dy}{dx} &= -8x^{-3} + 3x^{-\frac{1}{2}} - 3 \\
 &= \frac{-8}{x^3} + \frac{3}{x^{\frac{1}{2}}} - 3
 \end{aligned}$$

- (c) A particle moves along a straight line in such a way that its displacement, s , in metres, from a fixed point after any time, t , in seconds, is given by the formula: $s = -t^2 + 4t + 4$.
What is its velocity after 2 seconds? Explain what your answer means. (3)

$$\begin{aligned}
 V &= \frac{ds}{dt} = -2t + 4 \\
 V @ 2 \text{ secs} &= -2(2) + 4 = 0 \\
 \text{The object has stopped moving}
 \end{aligned}$$

[12]

QUESTION 7



- (a) Given the cubic graph, $f(x) = 2x^3 + x^2 - 8x - 4$.
A and C are x -intercepts, F is the y -intercept and E is a stationary point. G is the point $(1\frac{1}{2}; -7)$ which lies on f .

(1) Determine the coordinates of E.

(4)

$$f'(x) = 6x^2 + 2x - 8$$

$$0 = 6x^2 + 2x - 8$$

$$0 = 3x^2 + x - 4$$

$$= (3x + 4)(x - 1)$$

$$\therefore x = -4/3 \text{ or } x = 1$$

$$f(1) = 2(1)^3 + (1)^2 - 8(1) - 4$$

$$= -9$$

$$\therefore E(1; -9)$$

(2) Show that $(x - 2)$ is a factor of f .

(2)

$$f(2) = 2(2)^3 + (2)^2 - 8(2) - 4$$

$$= 16 + 4 - 16 - 4$$

$$= 0$$

$$\therefore (x - 2) \text{ is a factor.}$$

- (3) Determine the distance AC, showing all necessary working.

(4)

$$f(x) = 2x^3 + x^2 - 8x - 4$$

$$= (x-2)(2x^2 + 5x + 2)$$

$$= (x-2)(2x+1)(x+2)$$

$$\therefore x\text{-ints: } x = 2; -\frac{1}{2}; -2$$

$$\therefore AC = 2 - (-2)$$

$$= 4$$

- (4) Determine the equation of the tangent at F.

(4)

$$\text{At F, } x=0 \quad \therefore f'(x) = 6x^2 + 2x - 8$$

$$f'(0) = -8 = m$$

$$F(0; 4) \quad \therefore y - (-4) = -8(x - 0)$$

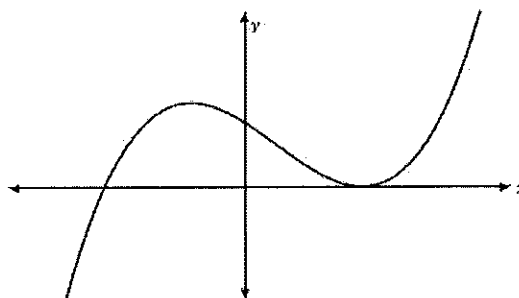
$$y = -8x - 4$$

- (5) If the equation of
- f
- was changed to
- $y = 2(x+3)^3 + (x+3)^2 - 8(x+3) - 4$
- , what would the new coordinate of point G be?

(2)

$$G(1\frac{1}{2}; -7) \rightarrow (-1\frac{1}{2}; -7)$$

- (b) The given figure shows a sketch of part of the curve where
- $f(x) = (2x-5)^2(x+3)$
- .



Given that:

- (1) the curve with equation
- $y = f(x) + q$
- ,
- $x \in \mathbb{R}$
- , passes through the origin, find the value of the constant
- q
- .

(2)

$$y\text{-int: } y = (-5)^2(3) = 75$$

$$\therefore q = -75$$

- (2) the curve with equation
- $y = f(x+p)$
- ,
- $x \in \mathbb{R}$
- , has a local minimum stationary point at the origin, find the value of the constant
- p
- .

(1)

$$p = 5/2$$

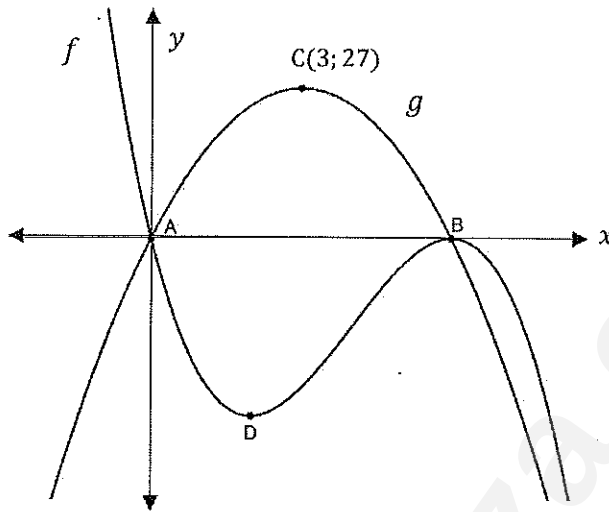
[19]

QUESTION 8

Given the graphs of $f(x) = -x(x-6)^2 = -x^3 + 12x^2 - 36x$ and $g(x) = ax^2 + bx$.

A is a point at the origin and A and B are x -intercepts for both f and g .

D and B are stationary points of f and C (3; 27) is the turning point of g .



- (a) Use the information provided to show that $a = -3$ and $b = 18$.

(4)

$$g(x) = a(x-3)^2 + 27$$

$$\text{or } y = a(x-6)(x)$$

(0;0): $0 = a(0-3)^2 + 27$

$$27 = a(3-6)(3)$$

$$-27 = 9a$$

$$27 = -9a$$

$$a = -3$$

$$-3 = a$$

$$\therefore g(x) = -3(x-3)^2 + 27$$

$$\therefore g(x) = -3(x-6)(x)$$

$$= -3(x^2 - 6x + 9) + 27$$

$$= -3(x^2 - 6x)$$

$$= -3x^2 + 18x - 27 + 27$$

$$= -3x^2 + 18x$$

$$\therefore b = 18$$

$$\therefore b = 18$$

(b) Determine the values of x , where f and g are both concave down.

(4)

 g is concave down for all values of x .

$$f: f(x) = -x^3 + 12x^2 - 36x$$

$$f'(x) = -3x^2 + 24x - 36 \quad \xrightarrow{\text{OR}} \quad -3x^2 + 24x - 36$$

$$f''(x) = -6x + 24 \quad \quad \quad 0 = x^2 - 8x + 12$$

$$0 = -6x + 24 \quad \quad \quad = (x-6)(x-2)$$

$$6x = 24 \quad \quad \quad \therefore x = 6 \text{ or } 2$$

$$x = 4 \quad \quad \quad \text{p.o.i lies halfway}$$

$$\therefore x = \frac{6+2}{2} = 4$$

$$\therefore x > 4$$

$$x > 4$$

(c) Determine the values of x for which

(i) $f(x) \geq 0$ $x < 0$ or $x \geq 6$ (2)

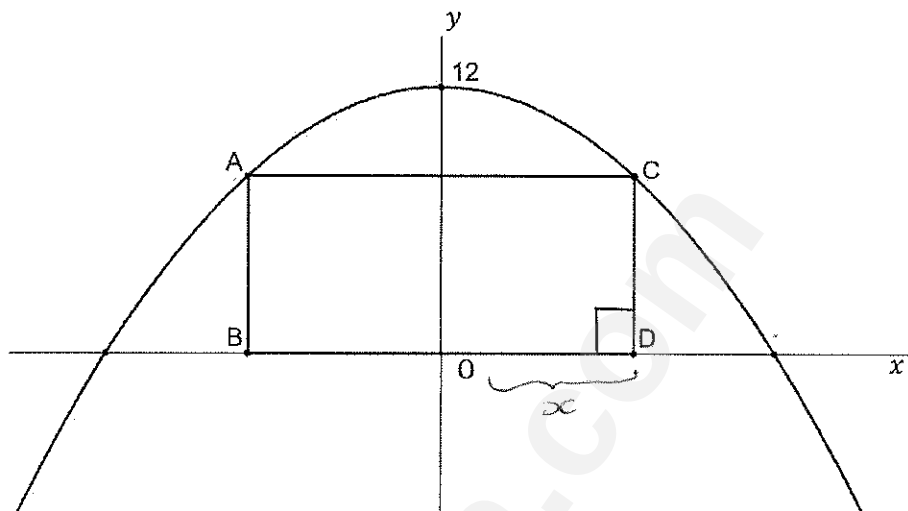
(ii) $\frac{g'(x)}{f(x)} < 0$ $0 < x < 3$

(2)

[12]

QUESTION 9

A rectangle, ABCD, is constructed such that its base lies on the x -axis and the other two vertices on the curve given by $y = 12 - x^2$, in the first and second quadrant.



- (a) Let $OD = x$. Hence, write down the coordinates of A and C in terms of x . can also be visa versa (2)

$$A(-x; 12 - x^2) \quad C(x; 12 - x^2)$$

- (b) Use calculus to show that the maximum possible area for this rectangle is 32 units². (4)

$$\text{length} = 2x \quad ; \quad \text{breadth} = 12 - x^2$$

$$\therefore A = 2x(12 - x^2)$$

$$= 24x - 2x^3$$

$$\frac{dA}{dx} = 24 - 6x^2 = 0$$

$$6x^2 = 24$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\text{But } x > 0 \therefore x = 2$$

$$\therefore \text{Area} = 24(2) - 2(2)^3$$

$$= 32 \text{ units}^2$$

- (c) Hence, or otherwise, calculate the vertices of the rectangle determined in (b). (3)

$$A(-2; 8) \quad C(2; 8)$$

$$B(-2; 0) \quad D(2; 0)$$

[9]

QUESTION 10

Given that $x; y; z; \dots$ form a geometric sequence, show that $\log x; \log y; \log z; \dots$ form an arithmetic sequence.

$$\text{G.P. : } x; y; z$$

$$\therefore r = \frac{y}{x} = \frac{z}{y}$$

$$\log x; \log y; \log z; \dots$$

$$d = \log y - \log x$$

$$= \log \frac{y}{x}$$

$$\text{or } d = \log z - \log y$$

$$= \log \frac{z}{y}$$

$$\text{But } \frac{y}{x} = \frac{z}{y}$$

$$\therefore \log \frac{y}{x} = \log \frac{z}{y}$$

$\therefore$ common difference

$\therefore$ AP

[5]

[Section B Total: 72 marks]

Extra working page.

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