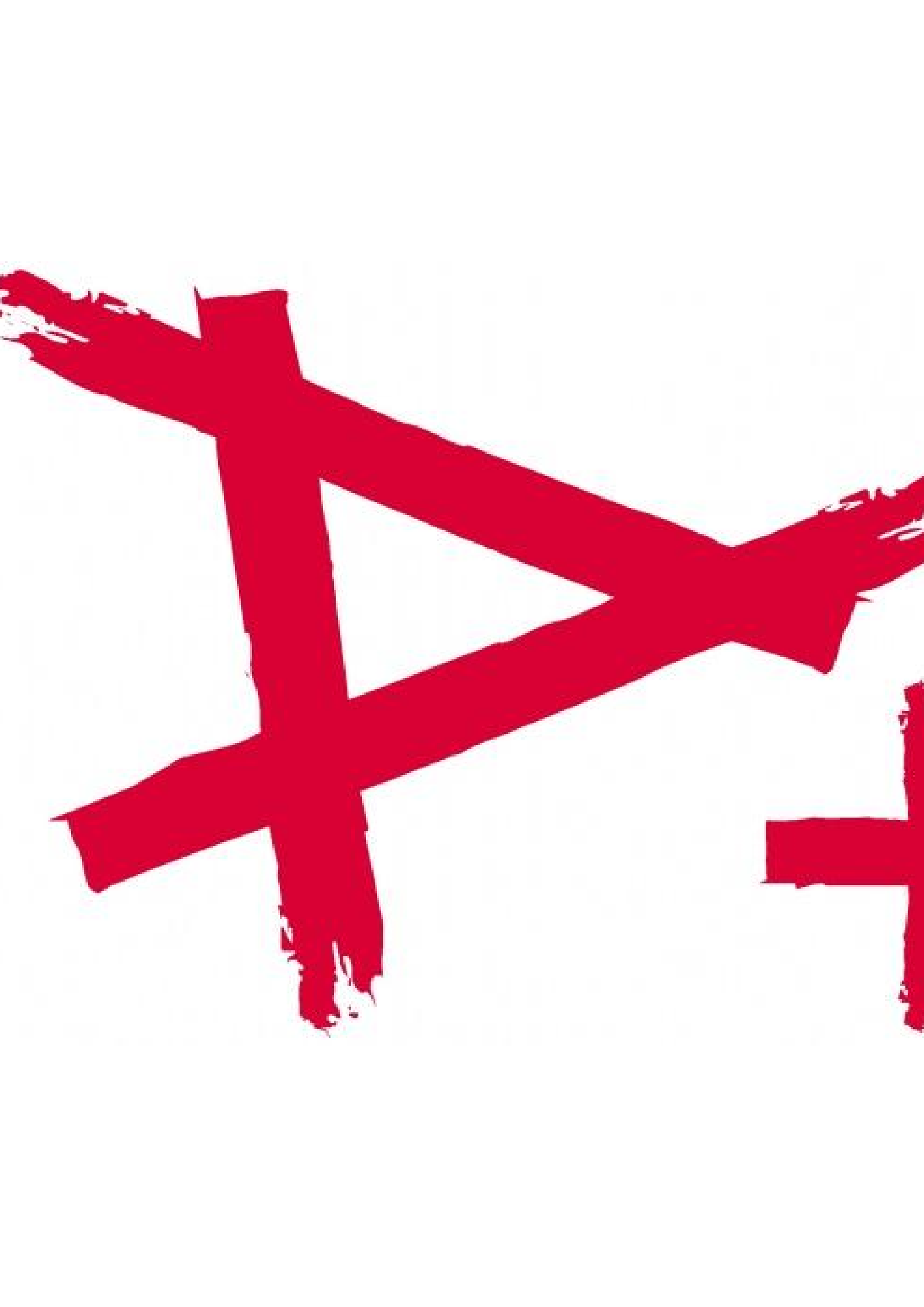


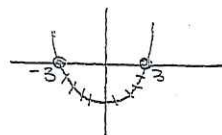


2018 Maths Core Paper 1

Model Answer:

SECTION A.

1a. i. $(x-2)^2 = 1$ or $x^2 - 4x + 4 = 1$
 $x - 2 = \pm 1$ take across $x^2 - 4x + 3 = 0$
 $x = 3$ or 1 $(x-3)(x-1) = 0$
 $\therefore x = 3$ or 1



Soln: $-3 \leq x \leq 3$, $x \in \mathbb{R}$. (4)

2. $4x^2 - 3x - 2 = 0$
 $x = \frac{-(-3) \pm \sqrt{(9) - 4(4)(-2)}}{2(4)}$
 $= \frac{3 \pm \sqrt{41}}{8}$
 $= 1.2$ or -0.4

3. $7 + \sqrt{6x+7} = 2x$
 $\sqrt{6x+7} = 2x - 7$
 $6x + 7 = (2x - 7)^2$
 $6x + 7 = 4x^2 - 28x + 49$
 $0 = 4x^2 - 34x + 42$
 $0 = 2x^2 - 17x + 21$
 $0 = (2x - 3)(x - 7)$
 $\therefore x = \frac{3}{2}$ or $x = 7$

Check: $x = 7$ only solution

4. $x^{2/3} - 7 = 9$
 $x^{2/3} = 16$
 $x^2 = (2^4)^3 = 2^{12}$
 $x = \pm 2^6$
 $= \pm 64$

5. $x^2 - 3 \leq 6$
 $x^2 - 9 \leq 0$
 $(x-3)(x+3) \leq 0$

b. $3^x = \frac{1}{81}$ $4^y = 2$

$3^x = 3^{-4}$ $2^{2y} = 2^1$
 $\therefore x = -4$ $2y = 1$
 $\therefore y = \frac{1}{2}$
 $\therefore xy = -4 \cdot \frac{1}{2}$

$= -2$

c. $kx^2 + 5x - 7 = 0$
 $b^2 - 4ac = \text{perf square for rational rts}$
 $b^2 - 4ac = 25 - 4(k)(-7)$
 $= 25 + 28k$

If $k=1$ $\Delta = 53 \neq \text{perf sq.}$
 $k=2$ $\Delta = 81 = \text{perfect sq.}$

$\therefore k=2$ * must make p.s. and be an integer
 [23]

2a. $f(x) = \frac{-2}{x+1} - 2$

1. $x = -1$
 $y = -2$ (2)

2. $0 = \frac{-2}{x+1} - 2$

$2 = \frac{-2}{x+1}$

$2x + 2 = -2$

$2x = -4$

$x = -2$

$\therefore x\text{-int: } (-2, 0)$

$y = \frac{-2}{0+1} - 2$

$= -2 - 2$

$= -4$ $\therefore y\text{-int: } (0, -4)$

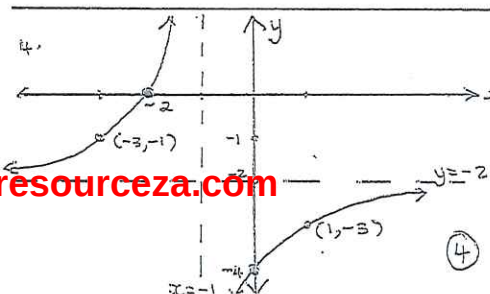
3. $(-1, -2)$ $\leftarrow$ pt where axes of symmetry meet.

$y = -x + c$ or $y = x + c$

$-2 = -(-1) + c$ $(-2) = (-1) + c$

$-3 = c$ $-1 = c$

$\therefore y = -x - 3$ $\therefore y = x - 1$ (3)



5. $(x, y) \rightarrow (x, -y)$ (reflected about x-axis)

$-y = \frac{-2}{x+1} - 2$

$y = \frac{2}{x+1} + 2$

* shifted 3 units rt

$y = \frac{2}{(x-3)+1} + 2$

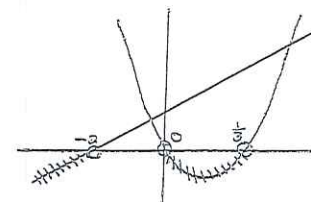
$\therefore g(x) = \frac{2}{x-2} + 2$ (2)

b. 1. $x = -2$ (1)

2. $3x^2 - x = 0$
 $x(3x - 1) = 0$
 $x = 0$ or $x = \frac{1}{3}$ (2)

3. $\frac{3x^2 - x}{x+2} < 0$ ie. $\frac{+}{-} \rightarrow -$ or $\frac{-}{+} \rightarrow -$

from the graph:



$x < -2$ or $0 < x < \frac{1}{3}$, $x \in \mathbb{R}$ (3)

Question 3

3. a1. $1 + ie = \left(1 + \frac{i}{m}\right)^m$

$$1 + ie = \left(1 + \frac{0.095}{4}\right)^4$$

$$ie = 0.0984382791...$$

∴ Eff IR = 9.8% p.a. (2)

2. $42000 = 16600 \left(1 + \frac{0.095}{4}\right)^{4p}$

$$\left(\frac{21}{8}\right) = \left(\frac{89}{800}\right)^{4p}$$

$$\log_{\frac{89}{800}} \left(\frac{21}{8}\right) = 4p$$

$$4.1156... = 4p$$

$$p = 10.2789$$

≈ 10 yrs 3 mo
and a bit*

∴ 10 yrs 4 months (2)

b.1. $105000 = \frac{15}{100} \times \text{selling price}$

$$\therefore S.P = 105000 \times \frac{100}{15}$$

Selling Price = R700 000 (1)

2. PV: loan amnt = 700 000 - 105 000

$$= 595000$$

$$595000 = x \left[1 - \left(1 + \frac{0.0925}{12}\right)^{-180} \right]$$

$$R6123.69 = x \quad (4)$$

Ans: R1502,72 (3/4) / R1767,91 (2/4) / R: 27309,57 (3/4)

c. $FV = \frac{2000 \left[\left(1 + \frac{0.085}{12}\right)^{480} - 1 \right]}{\frac{0.085}{12}} \times \left(1 + \frac{0.085}{12}\right)^{10 \times 12}$

Fr (5yrs only)
≈ R19217778,27 (2/5)

= R18 841 501,44 (5) [15]

Question 4

a. -1; 3; 7; ...

1. $d = 4$ (1)

2. $T_n = 191 = a + (n-1)d$

$$191 = -1 + 4n - 4$$

$$196 = 4n$$

$$49 = n$$

∴ $T_{49} = 191$ must be in (3)

b. $S_n = 4n^2 + 2$

$$S_2 = 4(2)^2 + 2 = 18$$

$$S_1 = 4(1)^2 + 2 = 6$$

$$\therefore S_1 = T_1$$

$$S_2 = T_1 + T_2$$

$$\therefore T_2 = S_2 - S_1$$

$$= 18 - 6$$

$$= 12. (3)$$

c. $T_2 = 25 = ar$
 $T_4 = 16 = ar^3$

$$1. \frac{T_4}{T_2} = \frac{16}{25} = \frac{ar^3}{ar}$$

$$\frac{16}{25} = r^2$$

$$r = \pm \frac{4}{5}$$

But all terms are positive
∴ $r = 4/5$ (3)

2. $S_\infty = \frac{a}{1-r}$

if $r = 4/5$

and $25 = ar$

$$25 = a \left(\frac{4}{5}\right)$$

$$a = \frac{125}{4}$$

$$\therefore S_\infty = \frac{\frac{125}{4}}{1 - \frac{4}{5}}$$

$$= \frac{625}{4} (4)$$

d. 2

$$\begin{array}{cccc} \vee & \vee & \vee & \vee \\ 10 & 14 & 18 & 22 \\ \vee & \vee & \vee & \\ 4 & 4 & 4 & \end{array}$$

$$2a = 4$$

$$a = 2$$

$$3a + b = 10$$

$$b = 10 - 6$$

$$= 4$$

$$a + b + c = 2$$

$$c = 2 - 2 - 4$$

$$= -4$$

$$\therefore T_n = 2n^2 + 4n - 4$$

Section B

Question 5

a. 1. $f(x) = x - x^2$
 $f(x+h) = (x+h) - (x+h)^2$
 $= x+h - (x^2 + 2xh + h^2)$
 $= x+h - x^2 - 2xh - h^2$
 $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
 $f'(x) = \lim_{h \rightarrow 0} \frac{x+h-x^2-2xh-h^2-x+x^2}{h}$
 $= \lim_{h \rightarrow 0} \frac{h-2xh-h^2}{h}$
 $= \lim_{h \rightarrow 0} \frac{h(1-2x-h)}{h}$
 $= 1-2x - (0)$
 $= 1-2x$ (4)

2. $f(x)$ increases
 when $f'(x) > 0$
 $1-2x > 0$
 $-2x > -1$
 $2x < 1$
 $x < \frac{1}{2}$ (2)

b. 1. $\lim_{x \rightarrow 1} \frac{x^3-1}{x-1}$
 $= \lim_{x \rightarrow 1} \frac{(x-1)(x^2+x+1)}{(x-1)}$
 $= \lim_{x \rightarrow 1} x^2+x+1$
 $= (1)^2 + (1) + 1$
 $= 3$ (2)

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2. $y = (3x) - \frac{1}{3x}$
 $= 9x^2 - \frac{1}{3}x^{-1}$
 $\therefore \frac{dy}{dx} = 18x + \frac{1}{3x^2}$ (3)

c. 1. $f(x) = \frac{1}{900}x^3 - \frac{1}{5}x^2 + 9x$
 $f'(x) = \frac{1}{300}x^2 - \frac{2}{5}x + 9$
 $0 = \frac{1}{300}x^2 - \frac{2}{5}x + 9$
 $\times 300: 0 = x^2 - 120x + 2700$
 $= (x-30)(x-90)$
 $\therefore x = 30 \text{ or } x = 90$
 $f(30) = \frac{1}{900}(30)^3 - \frac{1}{5}(30)^2 + 9(30)$
 $= 120$
 $\therefore A \text{ is } (30, 120)$ (4)

2. $f(150) = \frac{1}{900}(150)^3 - \frac{1}{5}(150)^2 + 9(150)$
 $= 600$
 $\therefore A(30, 120)$
 $C(150, 600)$
 Ave Grad = $\frac{f(150) - f(30)}{150 - 30}$
 $= \frac{600 - 120}{150 - 30}$
 $= \frac{480}{120}$
 $= 4$ (3)

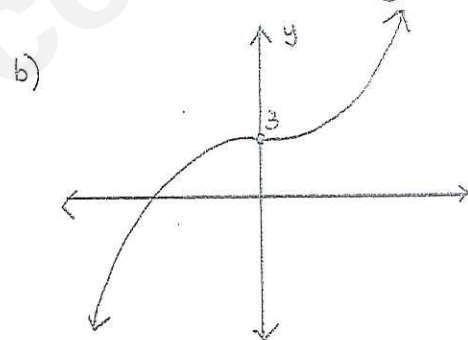
3. $f''(x) = \frac{1}{150}x - \frac{2}{5} = 0$

$\times 150: x - 60 = 0$
 $x = 60$

OR mid-pt of SPs: $\frac{30+90}{2} = 60$ (2)

Question 6

a. $f'(x) > 0$
 $\therefore m$ is positive; or at $x=0$
 $\therefore f(x)$ is increasing (2)



Question 7.

a. Minimum value (1)

b. domain of $f^{-1}: x \geq -3, x \in \mathbb{R}$
 range of $f: y \geq -3, y \in \mathbb{R}$ (1)

c. $f: \text{T.P. } (2; -3)$ other pt $(0; 1)$
 $y = a(x-2)^2 - 3$

$1 = a(0-2)^2 - 3$
 $4 = a \cdot 4$
 $1 = a$

$\therefore y = (x-2)^2 - 3$ (3)

Question 8

a. $x = -\frac{b}{2a} = -\frac{2}{2(-1)} = 1$

$g(1) = -(1)^2 + 2(1) + 3 = 4$
 $\therefore f(1; 4)$ (4)

b. 1. $H(2, 3)$ symmetrical to G

$J(2; y_J)$ $GH \perp HJ$

$y_J = \log_5 2$
 $= 0, 6309 \dots$

$\therefore HJ = 3 - 0, 63$
 $= 2, 4 \text{ units}$ (4)

2. Area of kite

$= \frac{1}{2}(d_1 \times d_2)$

$= \frac{1}{2} GH \times FA$

$= \frac{1}{2} \cdot 2 \times 4$

$= 4 \text{ unit}^2$ (3)

OR
 $2 \times (\frac{1}{2} \text{ base} \times \text{ht}). [\Delta GFA \text{ or } \Delta FHR]$
 $= 2 \times (\frac{1}{2} \cdot FA \times 1)$
 $= 2 \times \frac{1}{2} \cdot (4) \times 1$
 $= 4 \text{ u}^2$

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e. 1. $f(x) = 2$

$\log_3 x = 2$

$3^2 = x$

$x = 9$

(2)

2. $f(x) \leq 2$

$\log_3 x \leq 2$ ie $y \leq 2$

$0 < x \leq 9$

(2)

3. $g'(x) \cdot g(x) > 0$

$-1 < x < 1$ or $x > 3$

(2)

d. 1. $f(x) = \log_3 x$

$y = \log_3 x$

$f^{-1}: x = \log_3 y$

$3^x = y$

$h(x) = 3^x$

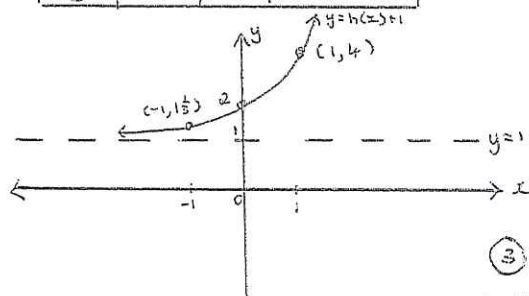
(2)

2. $y = h(x) + 1$

$y = 3^x + 1$

horizontal asymptote: $y = 1$

x	-1	0	1
y	$\frac{1}{3}$	2	4



(3)

[20]

Question 9

a. $g(x) = 5x$ $\therefore m = 5$

tangent grad = 5

$f(x) = x^3 - 7x^2 + 36$

$f'(x) = 3x^2 - 14x$

$5 = 3x^2 - 14x$

$0 = 3x^2 - 14x - 5$

$= (3x + 1)(x - 5)$

$\therefore x = -\frac{1}{3}$ or $x = 5$

(4)

b. $CD = y_f - y_g$

$CD = x^3 - 7x^2 + 36 - 5x$

$CD = x^3 - 7x^2 - 5x + 36$



$\therefore \frac{d(CD)}{dx} = 3x^2 - 14x - 5 = 0$

$0 = (3x + 1)(x - 5)$

$x = -\frac{1}{3}$ or 5

max length occurs @ $x = -\frac{1}{3}$

$\therefore CD = (-\frac{1}{3})^3 - 7(-\frac{1}{3})^2 - 5(-\frac{1}{3}) + 36$

$= 36.851...$

$= 36.9$ units.

(6)

c. $-14.81 < k < 36$

(2)

[12]

Question 10

a. $AC = x$

$AC + BC = 20$

$BC = 20 - x$

$AC \perp BC$ (given)

$\therefore \text{Area } \triangle ABC = \frac{1}{2} \text{ base} \times \text{ht}$

$= \frac{1}{2} (20 - x) \times x$

$A(x) = (10 - \frac{x}{2})x$

$= 10x - \frac{x^2}{2}$

(2)

b. $A(x) = 10x - \frac{x^2}{2}$

$A'(x) = 10 - x = 0$

$10 = x$

$\therefore \text{Perimeter} = AC + BC + AB$

$= x + (20 - x) + AB$

In $\triangle ABC$ $AB^2 = AC^2 + BC^2$ (Pythag)

$= 10^2 + 10^2$

$= 200$

$\therefore AB = \sqrt{200}$

$= 10\sqrt{2}$

$\therefore \text{Perim} = 10 + 10 + 10\sqrt{2}$

$= 20 + 10\sqrt{2}$ units

Question 11

$f(x+y) = f(x) \times f(y)$

$f(1) = 5$

$\sum_{x=1}^n f(x) = 780$

a) $f(2) = f(1+1)$

$= f(1) \times f(1)$

$= 5 \times 5$

$= 25$

(2)

b) $\sum_{x=1}^n f(x) = f(1) + f(2) + \dots + f(n) = 780$

$= 5 + 25 + \dots$

$f(1) = 5$

$f(2) = 25$

$f(3) = f(2+1) = f(2) \times f(1)$

$= 25 \times 5$

$= 125$

$f(4) = f(3+1) = f(3) \times f(1)$

$= 125 \times 5$

or $f(2+2)$

$= f(2) \times f(2)$

$= 25 \times 25 = 625$

So, $5 + 25 + 125 + 625 + \dots$

$\therefore n = 4$

(2)

or

$a = 5$ $r = 5$

$S_n = 780 = \frac{a(r^n - 1)}{r - 1}$

$780 = \frac{5(5^n - 1)}{4}$

$625 = 5^n$ $\therefore n = 4$