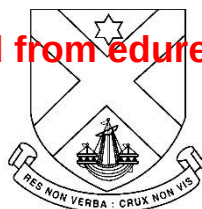




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Teacher: MORGAN, THOMP, VDM

Name:



St Anne's Diocesan College
CORE MATHEMATICS: PAPER I
TRIAL EXAMINATION

Form 6
Time: 3 hours

August 2018
150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 9 pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to **one decimal digit** where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your best interest to write legibly and to present your work neatly.
9. **Write down your name and highlight your teacher's name in the top left corner.**

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Question	1	2	3	4	5	6	7	8	9	10	11	Total
Topic	Algebra	Functions	Finance	Sequence	Calculus	Calculus	Functions	Function	Calculus	Calculus	Notation	
Max Mark	23	20	15	19	20	5	5	20	12	7	4	150
Your Mark												

SECTION A

QUESTION 1

(a) Solve for x in each of the following, show all necessary working:

(1) $(x - 2)^2 = 1$ (3)

(2) $4x^2 - 3x - 2 = 0$ (2)

(3) $7 + \sqrt{6x + 7} = 2x$ (4)

(4) $x^{\frac{2}{3}} - 7 = 9$ (3)

(5) $x^2 - 3 \leq 6$ (4)

(b) Determine the value of xy if $3^x = \frac{1}{81}$ and $4^y = 2$. (4)

(c) Given the equation $kx^2 + 5x - 7 = 0$; write down one integral value of k , ($k > 0$), for which the roots will be rational. (3)

[23]

QUESTION 2

(a) Given $f(x) = \frac{-2}{x+1} - 2$

(1) Write down the equations of the asymptotes of f . (2)

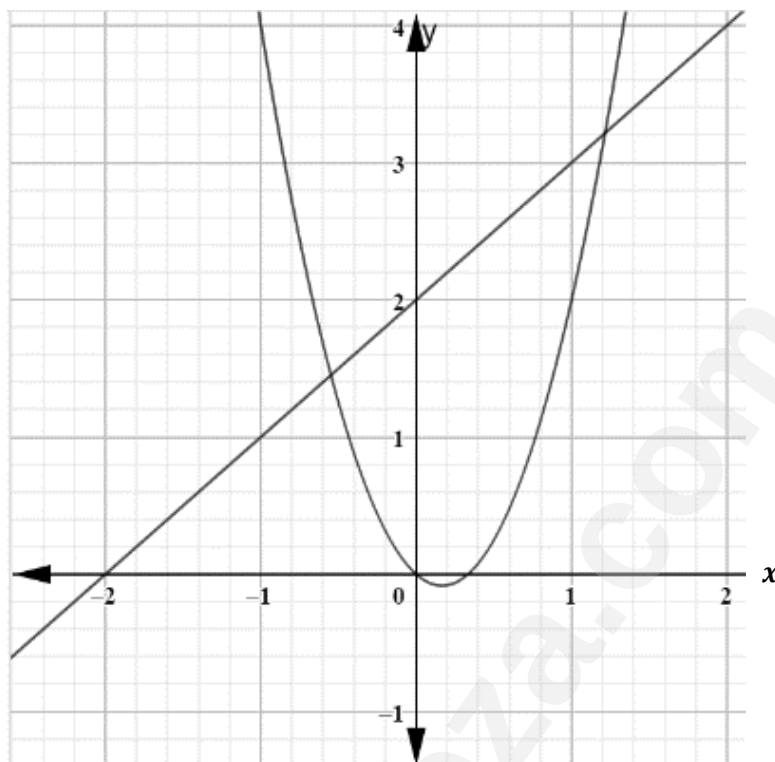
(2) Determine the x and y intercepts of f . (3)

(3) Find the equation(s) of the axis of symmetry of f . (3)

(4) Sketch the graph, showing all asymptotes, and intercepts with the axes and at least one other point on each arm. (4)

(5) Write down the equation of $g(x)$, which is the graph form when f is reflected about the x -axis and shifted 3 units right. (2)

- (b) The graphs of $y = 3x^2 - x$ and $y = x + 2$ are drawn on the same system of axes below.



If $f(x) = \sqrt{\frac{3x^2 - x}{x + 2}}$, determine for which value(s) of x :

- (1) $f(x)$ is undefined (1)
- (2) $f(x) = 0$ (2)
- (3) $f(x)$ is non-real (3)

[20]

QUESTION 3

- (a) Mr Naidoo invests R 16 000 for p years. He earns interest at an interest rate of 9,5% p.a. compounded quarterly. At the end of p years his investment is worth R 42 000.
- (1) Calculate the effective annual interest rate of Mr Naidoo's investment. (2)
 - (2) Determine the value of p in years and months (round off to the nearest month) (3)

- (b) Sashi's parents make an investment and buy a flat near Wits University so that she can stay there next year while studying.

They pay a deposit of R 105 000, which is equivalent to 15% of the selling price of the flat. They take out a loan to cover the outstanding amount they will need to buy the flat. The bank charges them an interest rate of 9,25% per annum compounded monthly. The loan is to be repaid within 15 years.

(1) Determine the selling price of the flat. (1)

(2) They start repaying the loan one month after the loan is granted. Determine how much money Sashi's parents will be paying on a monthly basis i.e. calculate their monthly instalments. (4)

- (c) Sayden saves R 2000 at the end of each month and invests it in an account earning 8,5% p.a compounded monthly. Sayden stops making payments at the end of 40 years of paying these monthly instalments of R 2000. He leaves the money in the account to continue earning interest.

Calculate how much money Sayden will have in this account 50 years after he started his savings plan. (5)
[15]

QUESTION 4

- (a) Given the arithmetic sequence: $-1; 3; 7; \dots$

(1) Determine d , the common difference. (1)

(2) If the n^{th} term of the sequence is equal to 191, determine the value of n . (3)

- (b) $S_n = 4n^2 + 2$ represents the sum of the first n terms of a particular series. Determine the second term of the series. (3)

- (c) The second and fourth terms of a convergent geometric series are 25 and 16, respectively. If all the terms are **positive**:

(1) Show that $r = \frac{4}{5}$. (3)

(2) Calculate the sum to infinity of this series, leave your answer as a simplified fraction. (4)

- (d) The **arithmetic sequence** $10; 14; 18; 22; \dots$ is the sequence of **first differences** of a **quadratic sequence** with a first term equal to 2. Determine the n^{th} term of the **quadratic** sequence. (5)

[19]

TOTAL FOR SECTION A: 77 MARKS

QUESTION 5

(a) Given $f(x) = x - x^2$,

(1) determine $f'(x)$ from **first principles**. (4)

(2) for which values of x will $f(x)$ be increasing? Justify your answer. (2)

(b) Determine:

(1) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$ (2)

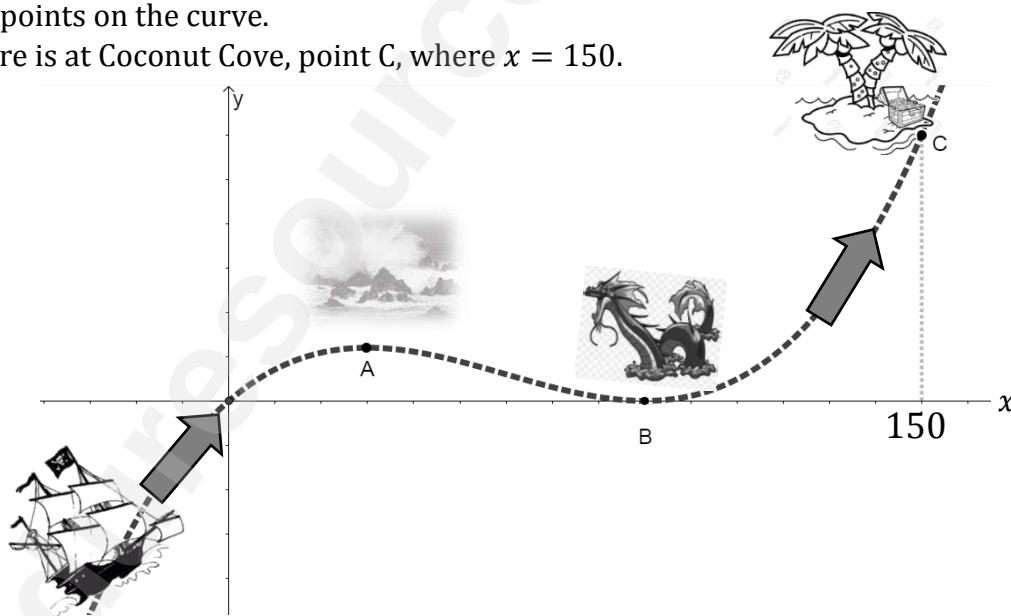
(2) $\frac{dy}{dx}$ for $y = (3x)^2 - \frac{1}{3x}$, and give your answer with **positive** exponents. (3)

(c) Captain Jax Sparrow and a merry band of pirates are searching for treasure. They are aboard the good ship, *The Blue Lady*.

The map below follows the path of a cubic function.

The Rocks of Doom lie at A, and the Sea Monsters Lair is positioned at B, both of which are stationary points on the curve.

The treasure is at Coconut Cove, point C, where $x = 150$.



The Ship follows a path defined by $f(x) = \frac{1}{900}x^3 - \frac{1}{5}x^2 + 9x$

(1) Show that the coordinates of A, the Rocks of Doom, is (30; 120). (4)

(2) Find the average gradient of the curve between the Rocks of Doom, A, and the treasure, C. (3)

(3) For which value of x does Captain Jax stop turning to the right and start turning to the left? (2)

[20]

QUESTION 6

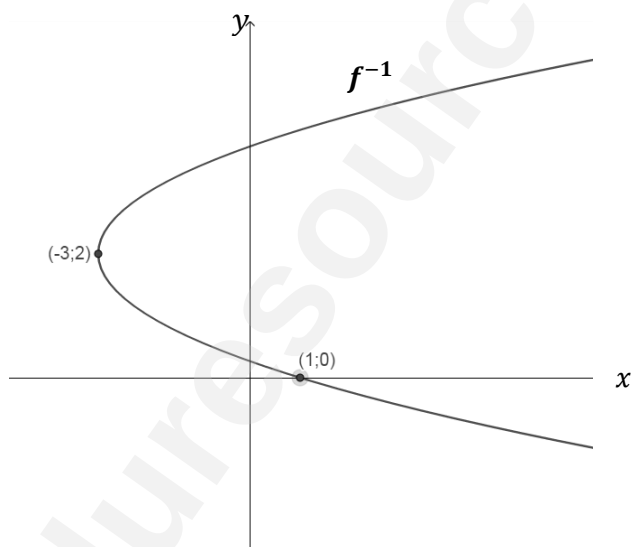
The following table shows the **derivative** of a function f for different values of x :

x	-4	-3	-2	-1	0	1	2	3	4
$f'(x)$	16	9	4	1	0	1	4	9	16

- (a) Comment on whether the graph of f is increasing and/or decreasing?
Give a reason to support your answer. (2)
- (b) Draw a **rough** sketch of a **possible graph of f** , if it is also given that $f(0) = 3$. (3)
[5]

QUESTION 7

The graph of the **inverse of f** is shown to the right.
It passes through a turning point at $(-3; 2)$ and has an x -intercept at $(1; 0)$.



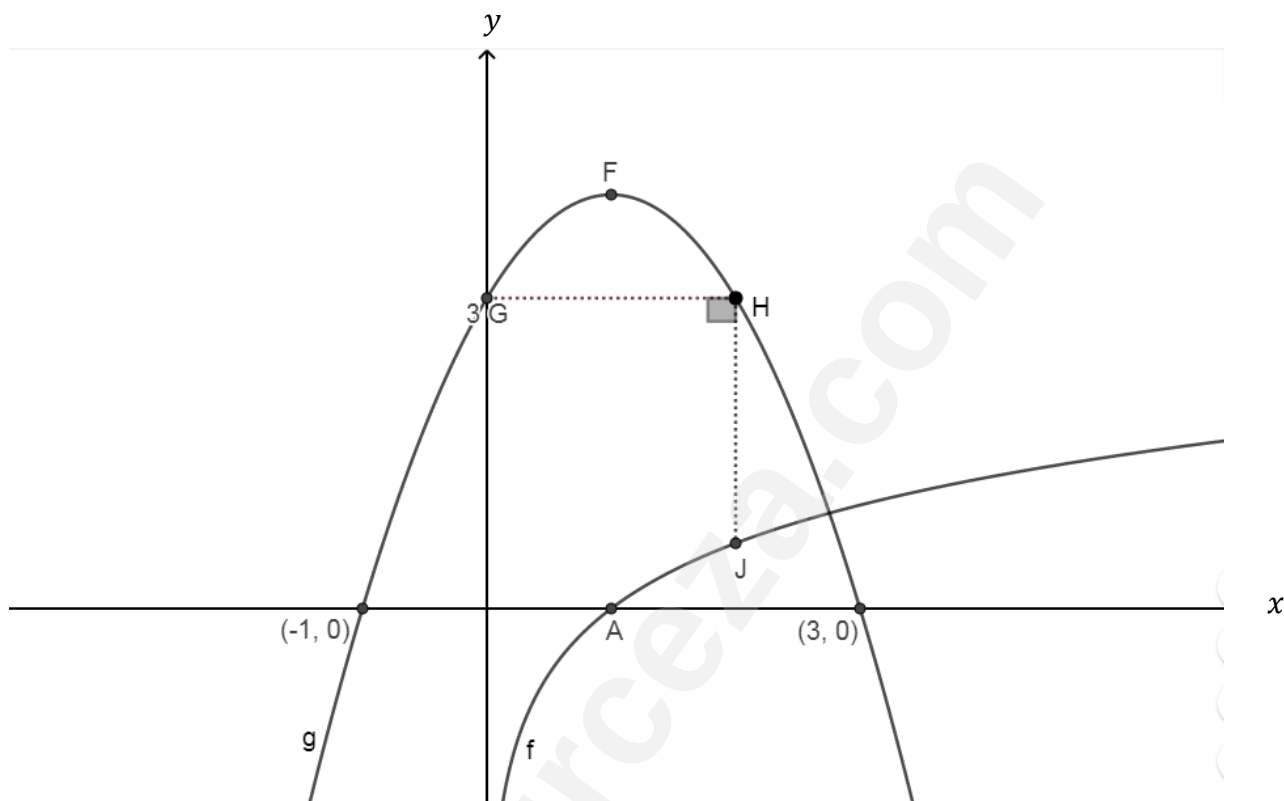
Note: The original graph, f , is not shown.

Determine:

- (a) whether f has a maximum or minimum value? (1)
- (b) Write down the range of f . (1)
- (c) Determine the equation of f in the form $y = a(x + p)^2 + q$. (3)
[5]

QUESTION 8

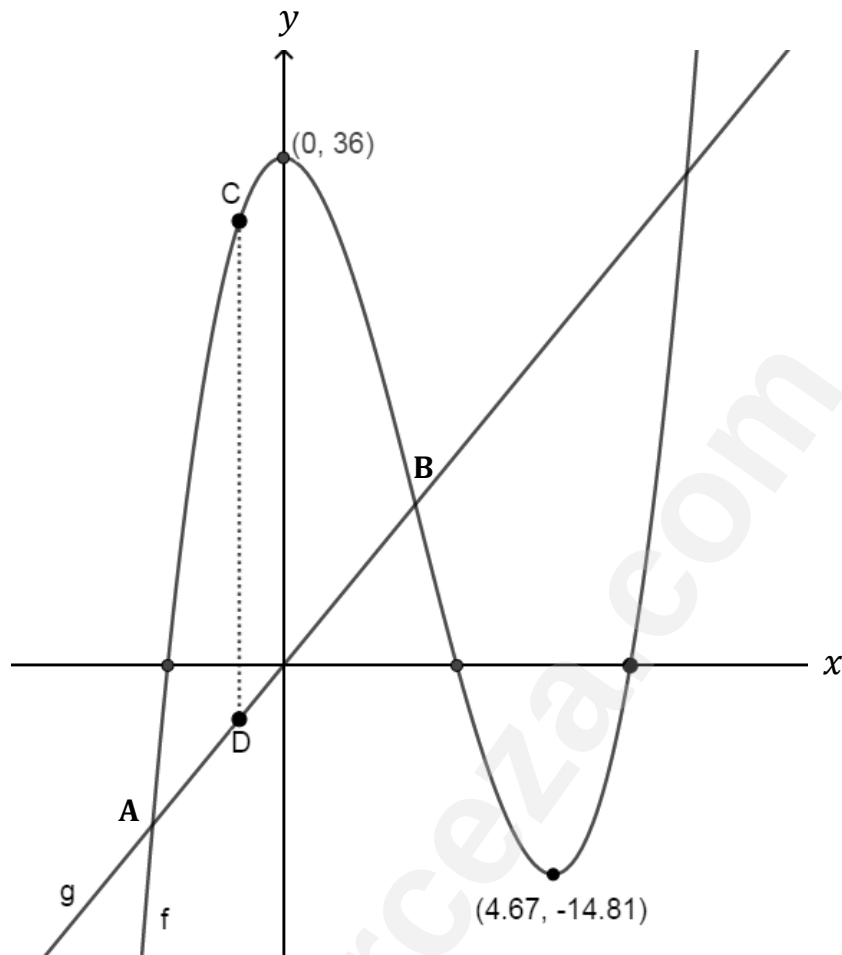
The graphs of $f(x) = \log_3 x$ and $g(x) = -x^2 + 2x + 3$ are drawn below.
 $GH \perp HJ$ and the x and y -intercepts of the parabola are $-1, 3$ and 3 respectively.
 F is the turning point of the parabola and A is the x -intercept of f .



- (a) Show that the coordinates of F , the turning point of the parabola, is $(1; 4)$. (2)
- (b) Determine:
 - (1) the length of HJ . (4)
 - (2) the area of kite, $GFHA$. (3)
- (c) Determine value(s) of x for which:
 - (1) $f(x) = 2$ (2)
 - (2) $f(x) \leq 2$ (2)
 - (3) $g'(x) \cdot g(x) > 0$ (2)
- (d) (1) Give the equation of h , **the inverse of $f(x)$** , in the form $h(x) = \dots\dots\dots$ (2)
- (2) Now sketch the graph of $y = h(x) + 1$.
 Show clearly all intercepts with the axes, asymptotes and at least one other point on the graph. (3)

[20]

QUESTION 9



The graph of $f(x) = x^3 - 7x^2 + 36 = (x - 6)(x - 3)(x + 2)$ and $g(x) = 5x$ is given above.

C is a point on f and D is a point on g such that CD is **perpendicular** to the x -axis.

The stationary points of f are $(0; 36)$ and $(4.67; -14.81)$.

A and B are the points of intersection of graphs f and g .

Determine:

- possible x -value(s) of the point of tangency between the graph of f and a tangent, if it is given that the tangent is also parallel to g . (4)
- the maximum length of line CD for the interval between A and B. (6)
- The value of k if $x^3 - 7x^2 + 36 = k$ has 3 real roots. (2)

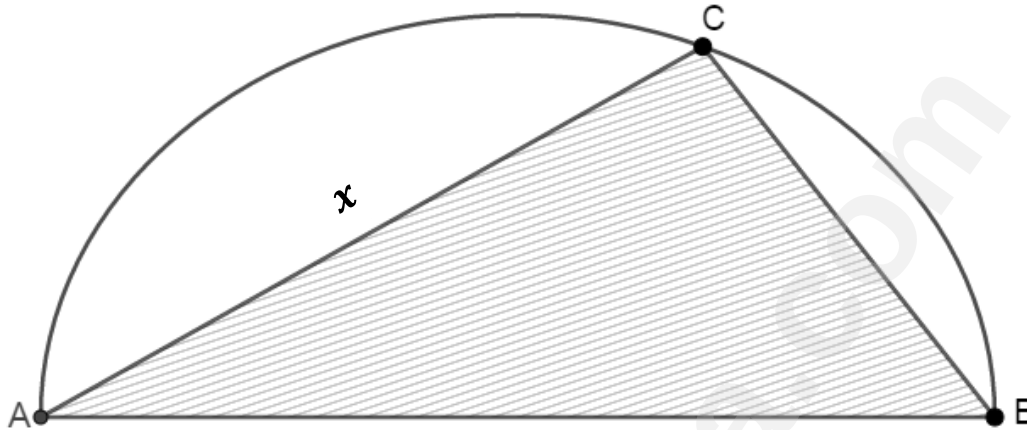
[12]

QUESTION 10

In the figure AB is the diameter of the semi-circle ABC.

$AC \perp BC$ and AC is x units.

The sum of the lengths of the sides AC and CB add up to 20 units.



- (a) Show that the Area of $\triangle ABC$, can be given by the equation:

$$A(x) = 10x - \frac{x^2}{2}, \quad \text{where } x \text{ represents AC.} \quad (2)$$

- (b) Calculate the perimeter of $\triangle ABC$ when $\triangle ABC$ covers a maximum area in the semi-circle. Leave your answer in **simplified surd form**. (5)

[7]

QUESTION 11

Given that f is a function which satisfies $f(x + y) = f(x) \times f(y)$ for all $x; y \in N$.

$$f(1) = 5 \text{ and } \sum_{x=1}^n f(x) = 780.$$

- (a) Calculate the numerical value of $f(2)$. (2)

- (b) Hence, or otherwise, determine the value of n . (2)

[4]

TOTAL FOR SECTION B: 73 MARKS

