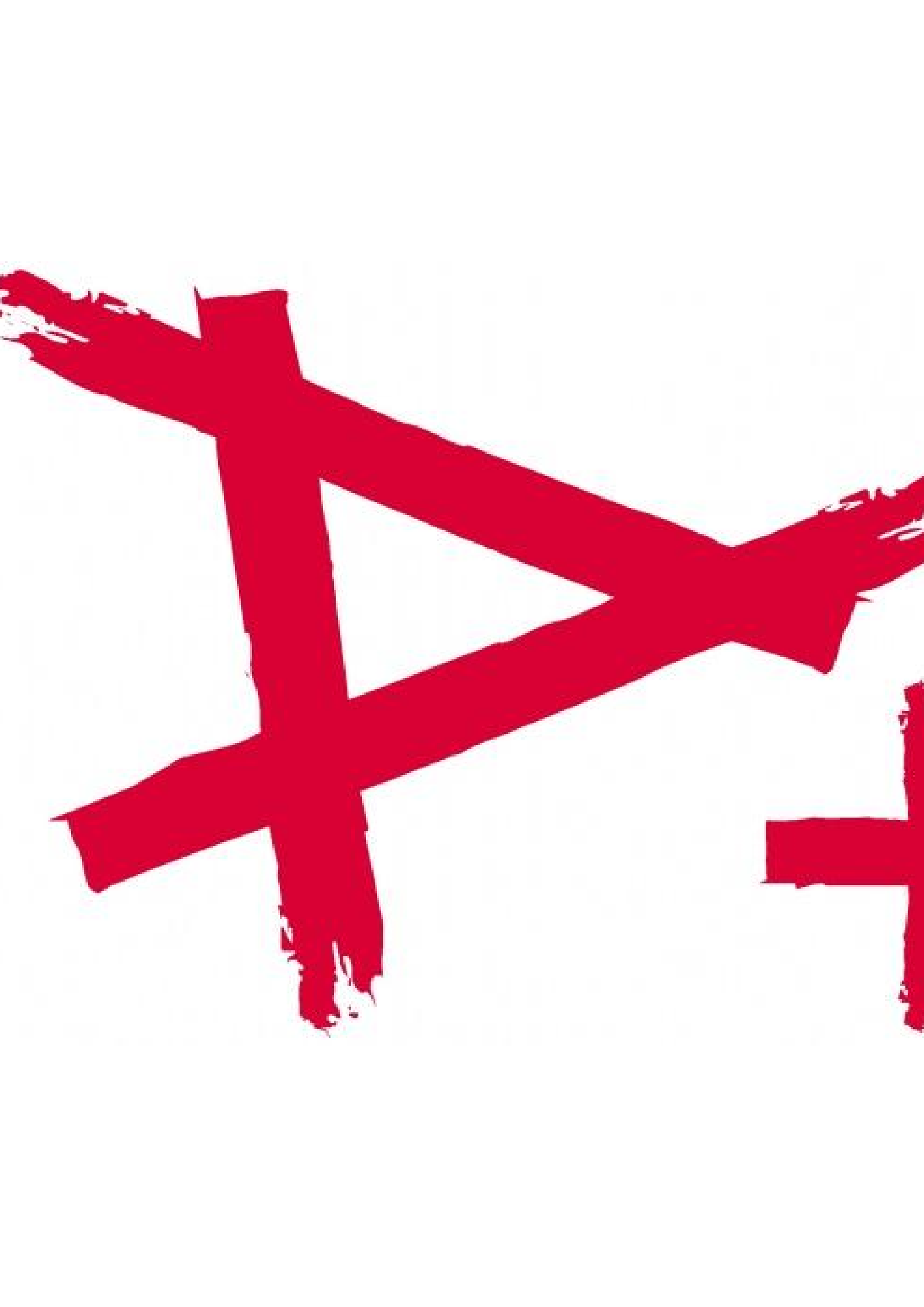
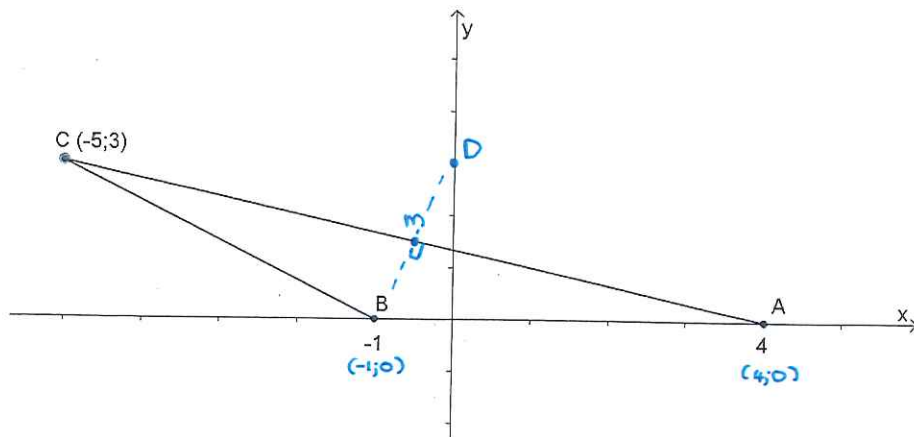




SECTION A
QUESTION 1

Given $\triangle ABC$ with A and B on the x-axis as shown, and C $(-5; 3)$.



- (a) Determine the length of BC.

$$BC^2 = (-5+1)^2 + (3-0)^2$$

$$= 25$$

$$\therefore BC = 5$$

(2)

- (b) (1) Calculate the size of $\hat{ABC}$.

$$m\angle C = \frac{3-0}{-5+1}$$

$$= -\frac{3}{4}$$

$$\therefore \tan \hat{ABC} = -\frac{3}{4}$$

$$\hat{ABC} = 143,1^\circ$$

Key $L = 36,9^\circ$

- (2) Hence, or otherwise, determine the area of $\triangle ABC$.

$$\text{Area} = \frac{1}{2} (BC \times AB) \sin \hat{ABC}$$

$$\text{or } A = \frac{1}{2} \text{ base} \times \text{ht}$$

$$= \frac{1}{2} (5 \times 5) \sin 143,1^\circ$$

$$= \frac{1}{2} (5 \times 3)$$

$$= 7,5 \text{ units}^2$$

$$= 7,5 \text{ units}^2$$

(3)

- (c) Determine M, the mid-point of AC.

$$M = \left(\frac{-5+4}{2}; \frac{3+0}{2} \right) = \left(-\frac{1}{2}; \frac{3}{2} \right)$$

(2)

- (d) If B is the centre of the circle which passes through the points A and C,

- (1) Write down the equation of this circle in the form $(x-a)^2 + (y-b)^2 = r^2$.

$$(x+1)^2 + y^2 = 25$$

(3)

- (2) What relationship will BM have with AC? Give a reason for your answer.

$$BM \perp AC \quad (\text{mid-pt chord thm})$$

(2)

- (e) If BM is extended to meet the point D, $(x; y)$, such that ABCD is a parallelogram,

- (1) Determine the coordinates of D, showing all relevant working and giving reason(s).

$$\frac{x-1}{2} = -\frac{1}{2} \quad \text{and} \quad \frac{y+0}{2} = \frac{3}{2} \quad (\text{diags of parm bisect})$$

$$x-1 = -1$$

$$x = 0$$

$$y = 3$$

$$\therefore D(0; 3)$$

(3)

- (2) ABCD is also one of the quadrilaterals listed below. Highlight/ circle the correct answer and give a reason for your choice.

Rectangle

Kite

Rhombus

Square

Cyclic Quadrilateral

Parm with diagonals that bisect @ 90°

(2)

[20]

QUESTION 2

(a) Complete: $\cos 2\theta = \frac{\cos^2 \theta - \sin^2 \theta}{2\cos^2 \theta - 1}$ or $= \frac{1 - 2\sin^2 \theta}{2\cos^2 \theta - 1}$ (1)

(b) (1) Prove that $\frac{\sin 2\theta}{\cos 2\theta + 1} = \tan \theta$.

LHS = $\frac{\sin 2\theta}{\cos 2\theta + 1}$
 $= \frac{2\sin \theta \cdot \cos \theta}{2\cos^2 \theta - 1 + 1}$
 $= \frac{2\sin \theta \cdot \cos \theta}{2\cos^2 \theta}$
 $= \frac{\sin \theta}{\cos \theta} = \tan \theta = \text{RHS}$ (3)

(2) For which values of θ is $\frac{\sin 2\theta}{\cos 2\theta + 1}$ undefined if $\theta \in [-180^\circ; 180^\circ]$.

$\cos 2\theta + 1 = 0$
 $\cos 2\theta = -1$
 $2\theta = 180^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$
 $\theta = 90^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$
 $\therefore \theta = -90^\circ; 90^\circ$

(3) Hence, solve for θ if $\frac{\sin 2\theta}{\cos 2\theta + 1} = -2$ and $\theta \in [-180^\circ; 180^\circ]$.

$\tan \theta = -2$
 $k.a = 63,4^\circ$
 $Q2: \theta = 116,6^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$
 $\therefore \theta = -63,4^\circ \text{ or } 116,6^\circ$ (3)

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1. If $180^\circ \leq \beta \leq 360^\circ$, determine the value of $\sin \beta$, without the use of a calculator.

$180^\circ \leq \beta \leq 360^\circ$
 $\therefore \beta$ lies in Q3 or Q4 (where $\sin \beta$ will be negative)
 $\cos 2\beta = \frac{1}{9}$
 $\therefore 1 - 2\sin^2 \beta = \frac{1}{9}$
 $1 - \frac{1}{9} = 2\sin^2 \beta$
 $\frac{8}{9} = 2\sin^2 \beta$
 $\frac{4}{9} = \sin^2 \beta$
 $\therefore \sin \beta = \pm \frac{2}{3}$
 But, $180^\circ \leq \beta \leq 360^\circ, \therefore \sin \beta = -\frac{2}{3}$ (3)

(d) Simplify: $\frac{\cos(-\theta) \cdot \sin(\theta - 180^\circ)}{\sin(90^\circ - \theta) \cdot \cos(360^\circ + \theta) \cdot \tan \theta}$
 $= \frac{\cos \theta \cdot (-\sin(180^\circ - \theta))}{\cos \theta \cdot \cos \theta \cdot \tan \theta}$
 $= \frac{-\sin \theta \cdot \cos \theta}{\cos^2 \theta \cdot \frac{\sin \theta}{\cos \theta}} = -1$ (3)

(e) (1) Show that $\cos(A + 45^\circ) = \cos 45^\circ (\cos A - \sin A)$.
 $\cos A \cdot \cos 45^\circ - \sin A \cdot \sin 45^\circ$
 $= \cos A \cdot \cos 45^\circ - \sin A \cdot \cos 45^\circ$
 $= \cos 45^\circ (\cos A - \sin A)$ (3)

(2) Hence, determine the general solution of $\cos A - \sin A = \sqrt{2}$.
 $\cos A - \sin A = \sqrt{2}$
 from (1) $\cos A - \sin A = \frac{\cos(A + 45^\circ)}{\cos 45^\circ}$
 $\therefore \cos(A + 45^\circ) = \cos 45^\circ \cdot \sqrt{2}$
 $\cos(A + 45^\circ) = \frac{1}{\sqrt{2}} \cdot \sqrt{2} = 1$
 $A + 45^\circ = 0^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$
 $A = -45^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$ (4)

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QUESTION 3

A circle with centre B (4; 4) has a tangent AC at point A.

The equation of the tangent is

$y - x = -2$, and C lies on the y-axis.

$$y = x - 2$$

(a) What is the gradient of AC?

$$m = 1$$

(1)

(b) Determine the equation of AB.

$$m_{AB} = -1 \quad (\text{tan} \perp \text{rad})$$

$$(4; 4): y - 4 = -1(x - 4)$$

$$y = -x + 4 + 4$$

$$y = -x + 8$$

(3)

(c) Show that A is the point (5; 3).

$$y = x - 2 \quad \dots \quad (1)$$

$$y = -x + 8 \quad \dots \quad (2)$$

$$(1) \rightarrow (2): x - 2 = -x + 8$$

$$2x = 10$$

$$x = 5 \quad \dots \quad (3)$$

$$(3) \rightarrow (1): y = (5) - 2 = 3 \quad \therefore A(5; 3)$$

(2)

(d) Determine the equation of the circle centre B.

$$(x - 4)^2 + (y - 4)^2 = r^2$$

$$(5 - 4)^2 + (3 - 4)^2 = r^2$$

$$2 = r^2$$

$$\therefore (x - 4)^2 + (y - 4)^2 = 2$$

(3)

(e) If the circle is reflected across the y-axis and the radius is tripled, write down the new circle equation.

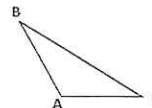
$$(x + 4)^2 + (y - 4)^2 = 18$$

(2)

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QUESTION 4

(a) Complete the following:



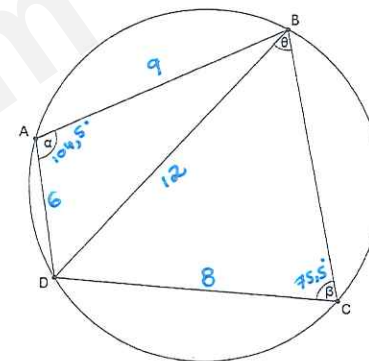
(1) Opposite angles of a cyclic quadrilateral are supplementary (1)

(2) In any $\triangle ABC$: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ or $\frac{a^2 - b^2 - c^2}{-2bc}$ (1)

$$a^2 = b^2 + c^2 - 2bc \cos A$$

(b) ABCD is a cyclic quadrilateral with AD = 6cm, AB = 9cm, BD = 12cm and DC = 8cm.

Diagram NOT drawn to scale.



Answer with reasons, if applicable:

(1) Show that $\alpha = 104,5^\circ$

$$\text{In } \triangle ABD: \cos \alpha = \frac{9^2 + 6^2 - 12^2}{2(9 \times 6)} = -0,25 \quad (\text{obtuse } \angle)$$

$$\alpha = 180 - 75,522 \dots = 104,47 \dots \approx 104,5^\circ$$

(3)

(2) Determine the value of β

$$\beta = 180^\circ - 104,5^\circ = 75,5^\circ \quad (\text{opp } \angle \text{ s of cyclic quad})$$

(2)

(3) Determine the value of θ

$$\text{In } \triangle DBC: \frac{\sin \theta}{8} = \frac{\sin 75,5^\circ}{12}$$

$$\sin \theta = \frac{8 \sin 75,5^\circ}{12} = 0,6454 \dots$$

$$\theta = 40,2^\circ$$

(2)

[9]

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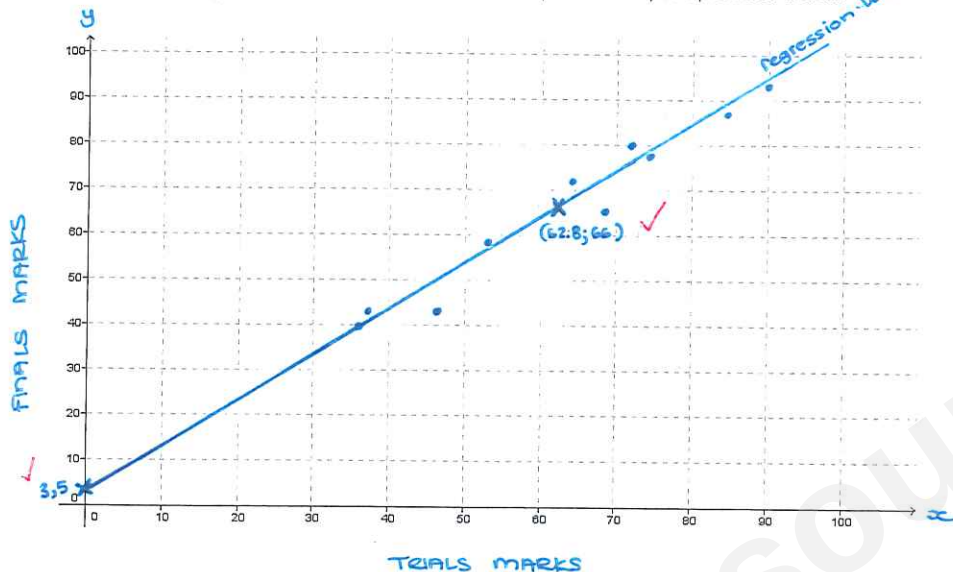
QUESTION 5

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The following data consists of marks obtained in Mathematics by a group of learners in their Trial examinations and in their Final examinations.

Trials % (x)	72	65	47	85	53	37	69	36	90	74
Finals % (y)	80	71	43	87	59	43	65	40	93	79

- (a) Use the data supplied in the table to draw a scatter-plot in the space provided below. (3)



- (b) Calculate the correlation coefficient (to 3 decimal places) for the data above and then comment on the strength of the relationship.

$$r = 0,976$$

Very strong, positive correlation

(2)

- (c) Find the equation for the line of best fit in the form $y = \dots\dots\dots$ (round your variables off to ONE decimal place).

$$a = 3,465\dots \quad b = 0,9957\dots \approx 1,0$$

$$\therefore y = 3,5 + x \quad \text{or} \quad y = x + 3,5$$

$$(\bar{x}; \bar{y}) = (62,8; 66)$$

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- (d) Hence, sketch this regression line from (c) onto your scatter plot. Remember to show the y-intercept and the mean coordinate. $y\text{-int} = 3,5$ $(\bar{x}; \bar{y}) = (62,8; 66)$ (2)

- (e) Thobile scored 55% overall in her Core Maths Trial exams, but was unable to write her Final Maths Exams. Her teacher uses the above regression line to predict an appropriate mark for Thobile to include in her Portfolio Assessment for her Final Maths Assessment. Determine her Final Exam mark, showing all working.

$$y = x + 3,5$$

$$= (55) + 3,5$$

$$= 58,5$$

$$\therefore \text{Final Exam Mark} \approx 58,5\%$$

- (f) What is the name given to the process of predicting Thobile's mark using the regression line.

Interpolation

(1)

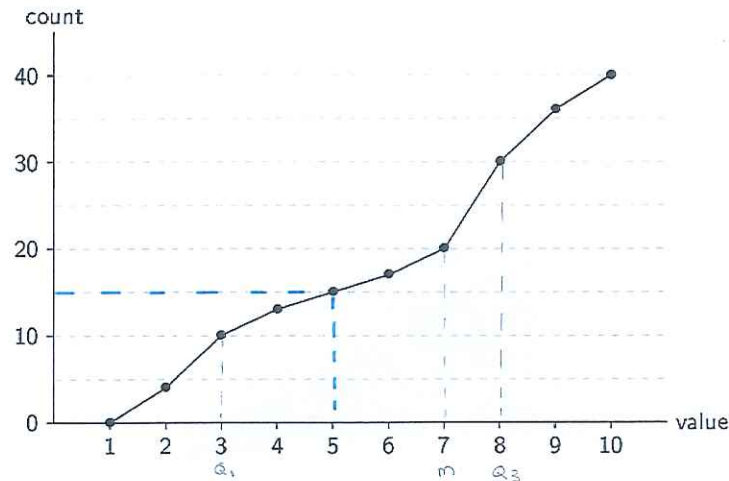
[12]

Total for Section A: 75 marks

Sometimes the questions are complicated and the answers are simple – Dr Seuss

QUESTION 6

- (a) A cumulative frequency graph is plotted below.



Use the graph to answer the following questions:

- (1) determine the five number summary of this data and represent it in a box and whisker diagram in the space provided below.

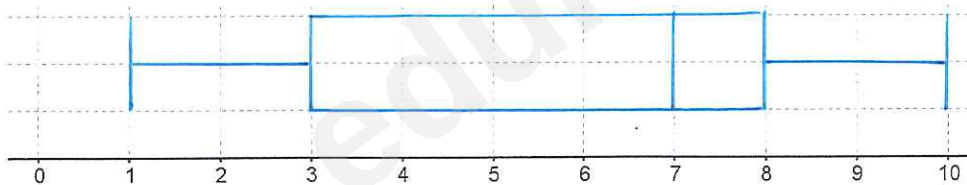
no. in study
↓
min Q1 med Q3 max

40 (1; 3; 7; 8; 10)

✓ Q1 and Q3

✓ med

✓ max/min



- (2) describe the distribution of data.

Skewed to the left OR Negatively skewed ✓ ca (1)

- (3) what percentage of people scored at least a value of 5?

40 - 15 = 25 people ✓ a
∴ $\frac{25}{40} \times 100 = 62,5\%$ or 63% ✓ ca

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- (b) The monthly income of ten sales assistants are listed below:

R 10 000; R 14 500; R 9 500; R 15 000; R 12 000; R 13 800; R 12 250; R 14 000; R 12 500; R 13 000

- (1) Use your calculator to determine:

(i) the mean R12 655 ✓ a (1)

(ii) the standard deviation 1725,2 ✓ a (1)

- (2) What percentage of salaries are less than 1 standard deviation away from the mean?

$(12655 - 1725,2 ; 12655 + 1725,2)$ ✓ m

$= (10929,8 ; 14380,2)$ ✓ a

1 s.d → 60% = 60% of salaries ✓ ca (3)

- (3) In the month of September, the sales assistants are offered a bonus of either R500 added to their pay or 10% of their salary, added to their pay. Which option should they choose. Show all necessary working.

R500 option : Old mean R12 655 New mean R13 155

10% option : New mean R13 920,50

Overall the 10% option works out better for everyone. (2)

Lowest salary = R 9 500 R500 option → R10 000 [14]

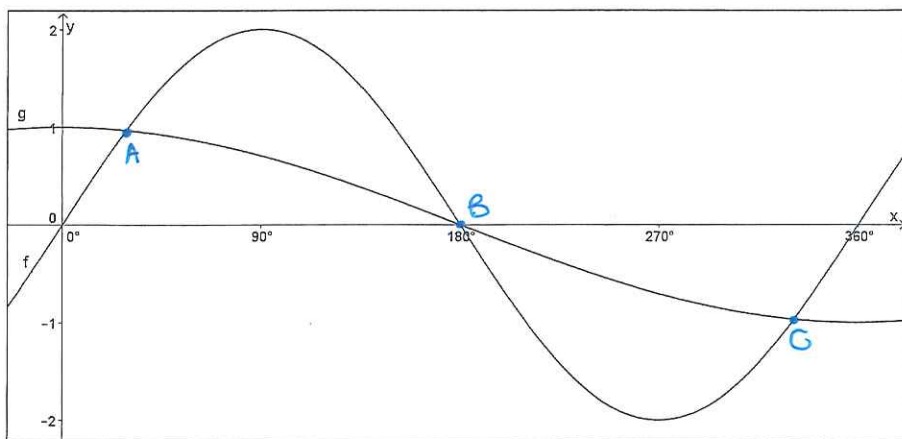
10% option → R10 450

∴ even the lowest salary benefits more with 10%.

✓ some kind of explanation
✓ backed up with stats.

QUESTION 7

The following graphs are drawn below: $f(x) = a \sin x$ and $g(x) = \cos bx$



(a) Why is $a = 2$ and $b = \frac{1}{2}$ for the given graph equations?

The amplitude of f is 2 $\therefore a = 2$ ✓

The period of g is 720° $\therefore b = \frac{360}{720} = \frac{1}{2}$ ✓ (2)

(b) Use the graph to solve $g(360^\circ) - f(90^\circ)$.

$(-1) - (2) = -3$ ✓ (2)

(c) For what x -values in the interval $[0^\circ; 360^\circ]$, will $f(x) \cdot g(x) > 0$?

$0 < x < 360^\circ; x \neq 180^\circ$ or $0 < x < 180^\circ$ or $180^\circ < x < 360^\circ$ ✓ (2)

(d) The class was asked to solve $a \sin x = \cos bx$ for $x \in [0^\circ; 360^\circ]$.

Jenny's answer is copied below.

PLEASE REFER TO THE COLOUR VERSION ON APPENDIX A - A SEPARATE SHEET OF PAPER.

Working $1x = 2 \times \frac{1}{2}x$ $\sin 2A = 2 \sin A \cdot \cos A$ $\div \text{ by } \cos \frac{1}{2}x$	Working $2 \sin x = \cos \frac{1}{2}x$ $2 \sin(2 \times \frac{1}{2}x) = \cos \frac{1}{2}x$ $2 \sin \frac{1}{2}x \cdot \cos \frac{1}{2}x = \cos \frac{1}{2}x$ $\frac{2 \sin \frac{1}{2}x \cdot \cos \frac{1}{2}x}{\cos \frac{1}{2}x} = \frac{\cos \frac{1}{2}x}{\cos \frac{1}{2}x}$	Working $2 \sin \frac{1}{2}x = 1$ $\div \text{ by } 2$ $\sin \frac{1}{2}x = \frac{1}{2}$ $\times \text{ by } 2$ $\sin x = 1$
--	---	---

Diagram showing a sine wave with a peak at $x = 90^\circ$. Soln: $x = 90^\circ$ ✓

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(1) The teacher has circled, in red, 3 non-related mistakes that Jenny has made. Explain briefly why each mistake is mathematically INCORRECT.

1. She has used the double L Rule incorrectly $\rightarrow$ should be $4 \sin \frac{1}{2}x \cdot \cos \frac{1}{2}x$ ✓
2. She can't divide by $\cos \frac{1}{2}x$ as she cancels / divides out an answer for x . ✓
3. If you are multiply the LHS of an equation by 2, you multiply on the outside of the function, not the inside (Δ). (3) ✓

(2) Jenny realised her answer was wrong when she looked at the graph and saw that the two graphs intersected THREE times – not once and not at 90° ! She asks you for some help. ☺
Look at the given graphs and estimate the three x -values of the points of intersection of f and g .

Label them A, B and C on the graph on the previous page. (1)

A: $x \approx 30^\circ$	B: $x \approx 180^\circ$	C: $x \approx 330^\circ$
-------------------------	--------------------------	--------------------------

(3) Now algebraically solve the equation correctly for Jenny, for $x \in [0^\circ; 360^\circ]$.

Round your answers to the nearest whole number where applicable.

(HINT: Jenny's first working line, in BLUE, is correct!)

$$2 \sin x = \cos \frac{1}{2}x$$

$$2 \sin(2 \times \frac{1}{2}x) = \cos \frac{1}{2}x$$

$$2 \cdot 2 \sin \frac{1}{2}x \cdot \cos \frac{1}{2}x = \cos \frac{1}{2}x$$

$$4 \sin \frac{1}{2}x \cdot \cos \frac{1}{2}x - \cos \frac{1}{2}x = 0$$

$$\cos \frac{1}{2}x (4 \sin \frac{1}{2}x - 1) = 0$$

$$\therefore \cos \frac{1}{2}x = 0 \quad \text{or} \quad \sin \frac{1}{2}x = \frac{1}{4}$$

Ref ✓ $\frac{1}{2}x = 90^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$ $\cdot$ key L = $14,5^\circ$ ✓

$$x = 180^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$$

$$Q1: \frac{1}{2}x = 14,5^\circ + k \cdot 360^\circ$$

$$x = 29^\circ + k \cdot 720^\circ$$

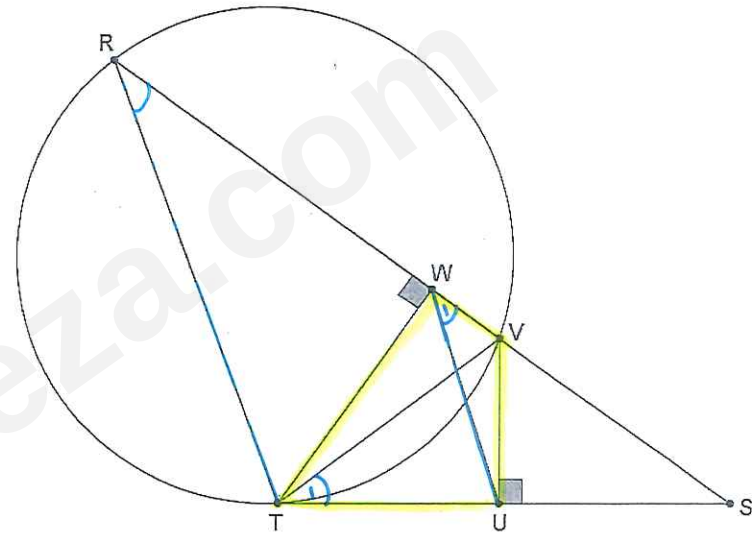
$$Q2: \frac{1}{2}x = 165,5^\circ + k \cdot 360^\circ$$

$$x = 331^\circ + k \cdot 720^\circ$$

$$\therefore x = 29^\circ; 180^\circ \text{ or } 331^\circ \quad \checkmark \quad (6)$$

QUESTION 9

- (a) (1) The Proportion-Intercept Theorem states that the line drawn parallel to one side of a triangle divides the other 2 sides in the same ^{ratio} proportion. (1) ✓
- (2) R, V and T are points on the given circle. W is a point on RV such that $TW \perp RS$. RW produced meets the tangent at T at S. U is a point on the tangent such that $UV \perp TS$.



- (i) Why is TWVU a cyclic quadrilateral?

Ext $\angle =$ to int opp $\angle$ ✓

(1)

- (ii) Prove that $\frac{RW}{WS} = \frac{TU}{US}$. * Need to prove $RT \parallel WU$ first

$$\hat{W}_1 = \hat{T}_1 \quad (\text{Ls in same seg}) \quad \checkmark$$

$$\hat{T}_1 = \hat{R} \quad (\text{tan-chord thm}) \quad \checkmark$$

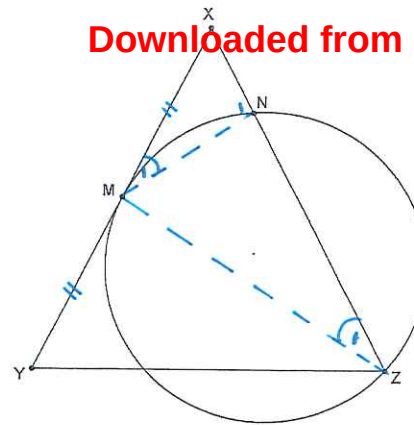
$$\therefore \hat{W}_1 = \hat{R}$$

$$\therefore RT \parallel WU \quad (\text{Corresp Ls} =) \quad \checkmark$$

$$\therefore \frac{RW}{WS} = \frac{TU}{US} \quad (\text{Prop Int Thm, } RT \parallel WU) \quad \checkmark$$

(4)

- (b) XYZ is an isosceles triangle with $XY = XZ$.
M is the mid-point of XY.
XY is a tangent to the circle at M.



- (1) Prove that $\triangle XMN \sim \triangle XZM$. You will need to construct.

Join MN and MZ. In $\triangle XMN$ and $\triangle XZM$

① $\hat{X}$ is common

② $\hat{M}_1 = \hat{Z}_1$ (tan-chord thm)

③ $\hat{N}_1 = \hat{M}_2$ (L sum of Δ)

$\therefore \triangle XMN \sim \triangle XZM$ (AAA) (3)

- (2) Prove that $XZ = 4XN$.

$\frac{XM}{XZ} = \frac{MN}{XN} = \frac{XN}{XM}$ (corresp sides $\sim \Delta$ s)

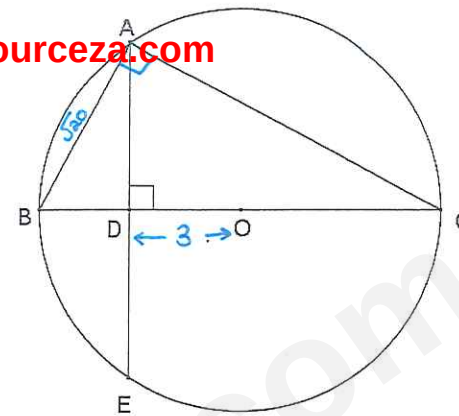
$\therefore \frac{XM^2}{XZ} = XN$ let $xm = p \therefore xz = 2p$

$\therefore \frac{p^2}{2p} = XN$

$p = 2XN$

$XZ = 2p$
 $= 2(2XN)$
 $= 4XN$

(c)



Given circle, centre O, with points A, C, E and B on the circumference. The straight line BDOC is the diameter which is perpendicular to chord ADE.

- (1) Why is $\hat{A} = 90^\circ$? L in a semi-circle ✓ (1)

- (2) Give a reason why $\triangle ABD \sim \triangle CBA$? AAA OR $\perp$ from R+L to hypotenuse thm. (1)

- (3) If $AB = \sqrt{20}$ units and $DO = 3$ units, prove that the radius of the circle is 5 units, showing all relevant working.

$AB^2 = BD \cdot BC$ ($\perp$ from rt L to hyp. thm). ✓ idea.

let BD be x .

$\therefore 20 = x \cdot (2(x+3))$

$20 = x(2x+6)$

$20 = 2x^2 + 6x$

$0 = 2x^2 + 6x - 20$

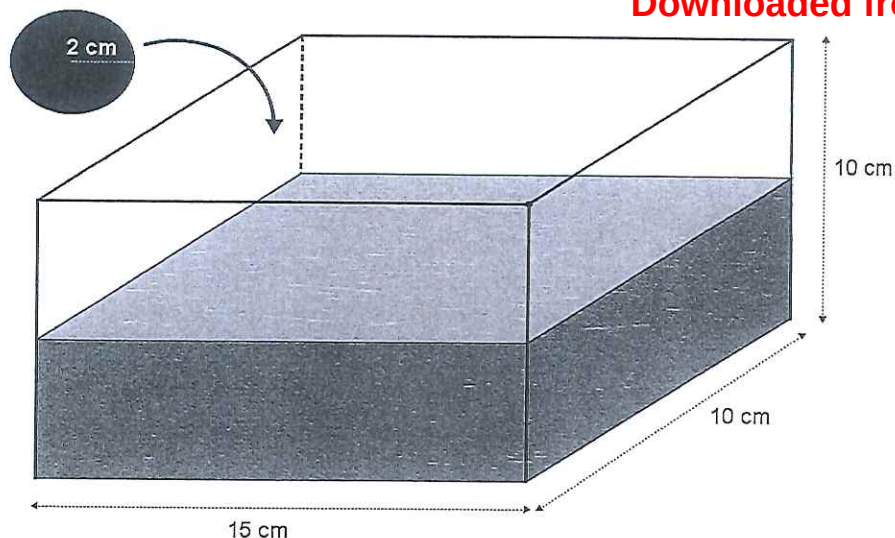
$0 = x^2 + 3x - 10$

$= (x+5)(x-2)$

$\therefore x = -5$ or $x = 2$

$\therefore$ Radius $BO = 2+3 = 5$ units

QUESTION 10



- (a) A rectangular glass vase measures 15 cm by 10 cm by 10 cm as shown in the picture above. The vase is half full with water.

A solid, decorative glass marble, with a radius of 2 cm, is placed into the vase and drops to the bottom.

Useful Formulae:	$V = l \times b \times h$	$V = \frac{4}{3}\pi r^3$
	$S.A = 2lb + 2bh + 2lh$	$S.A = 4\pi r^2$

- (1) Assuming that the marble sinks to the bottom, and stays there, calculate the amount by which the water level in the vase rises.

$$\begin{aligned} \text{Vol marble} &= \frac{4}{3}\pi (2)^3 = 33,5 \text{ cm}^3 \quad \therefore \text{water displaced by } 33,5 \text{ cm}^3 \\ \text{Vol water displaced} &= 33,5 = 15 \times 10 \times h \\ \frac{33,5}{150} &= h \\ \therefore h &= 0,2 \text{ cm} \end{aligned}$$

or

$$\begin{aligned} \text{Vol of water} &= 15 \times 10 \times 5 = 750 \text{ cm}^3 \\ \text{New Vol} &= 783,5 \text{ cm}^3 \\ 783,5 &= 15 \times 10 \times h \\ h &= 5,2 \text{ cm} \\ \therefore 5,2 - 5 &= 0,2 \text{ cm} \quad \text{risen} \end{aligned}$$

- (2) Based on your calculation in (a), use Maths to determine the most number of marbles you could drop into the vase without overflowing any water?

$$5 \text{ cm ht left} \therefore \frac{5}{0,2} = 25 \text{ marbles.}$$

Power of not rounding off!!!

$$5 \div \frac{33,5}{150} = 22,388 \approx 22 \text{ marbles (most accurate answer!)} \quad (2)$$

Both answers accepted if Maths is correct.

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- b) The vase is going to sit on top of a cylindrical pillar.

The surface area of the cylindrical pillar is $6\,283,20 \text{ cm}^2$ and its volume is $31\,416 \text{ cm}^3$, find the ratio of its diameter to its height.

Useful Formulae: $V = \pi r^2 h$ $S.A = 2\pi r^2 + 2\pi r h$

$$SA = 2\pi r h \text{ only}$$

$$\therefore 2\pi r h = 6\,283,20 \quad \text{--- (1)}$$

$$\pi r^2 h = 31\,416 \quad \text{--- (2)}$$

$$\text{(2)} \div \text{(1)}$$

$$\frac{\pi r^2 h}{2\pi r h} = \frac{31\,416}{6\,283,2}$$

$$\frac{r}{2} = 5$$

$$\therefore r = 10 \quad \text{--- (3)}$$

$$\text{(3)} \rightarrow \text{(1)}$$

$$2\pi (10) h = 6\,283,2$$

$$\therefore h = 100,000 \dots$$

$$\therefore \text{diameter : height} = 20 : 100$$

$$= 1 : 5$$

(4)
(using dbl radl). [10]



Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding. — William Paul Thurston

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