

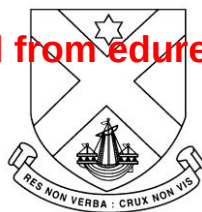


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Exam No:

Teacher: MORGAN, THOMP, VDM

Name:



St Anne's Diocesan College
CORE MATHEMATICS : PAPER I
TRIAL EXAMINATION

Form 6

Time: 3 hours

August 2015

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 9 pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to **one decimal digit** where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your best interest to write legibly and to present your work neatly.
9. **Highlight your teacher's name in the top right corner.**

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Question	1	2	3	4	5	6	7	8	9	10	11	Total
Topic	Algebra	Exp/ surds	Seq & Series	Calculus	Functions	Finance	Functions	Cubic Function	Functions/ Calc	Calculus	Functions/ Calc	
Max Mark	15	5	21	17	16	19	9	16	14	11	7	150
Your Mark												

SECTION A

QUESTION 1

(a) Solve for x in each of the following:

$$(1) (x - 4)(x + 2) = 7 \quad (3)$$

$$(2) 8^{x-5} = 32^{10-2x} \quad (3)$$

$$(3) x(x + 2) < 3 \quad (4)$$

$$(4) \sqrt[3]{3x - 23} = -2 \quad (2)$$

(b) The roots of a quadratic equation are given by $x = \frac{-5 \pm \sqrt{20 + 8k}}{6}$, where $k \in \{-3; -2; -1; 0; 1; 2; 3\}$.

(1) Write down TWO values of k for which the roots will be rational. (2)

(2) Write down ONE value of k for which the roots will be non-real. (1)

[15]

QUESTION 2

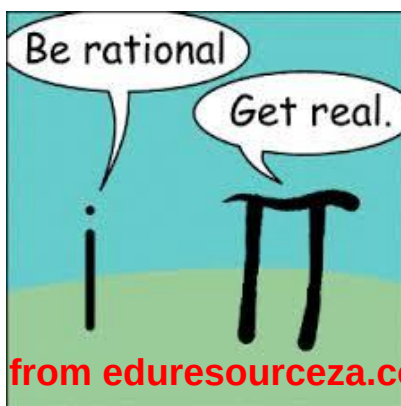
(a) Simplify, **without the use of a calculator**:

$$\sqrt{3} \cdot \sqrt{48} - \frac{4^{x+1}}{2^{2x}} \quad (3)$$

(b) If x is a rational number then $g(x) = 3$.

If x is an irrational number then $g(x) = \frac{5}{4}$

Find the value of $2g(g(\sqrt{2}))$. (2)



[5]

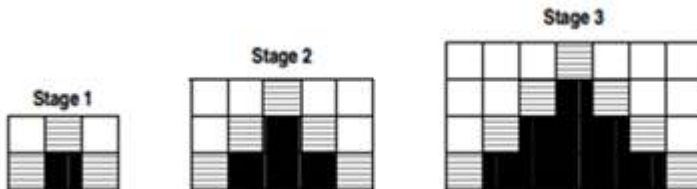
QUESTION 3

- (a) Given the geometric sequence $\frac{3}{8}; \frac{3}{4}; \frac{3}{2}; \dots$

Determine which term has a value of 96.

(3)

- (b) Examine the tiling pattern below:



	Stage 1	Stage 2	Stage 3	Stage 4	Stage n
Number of patterned tiles	3	5	7	9	
Number of black tiles	1	4	9	16	
Number of white tiles	2	6	12	20	$n^2 + n$
Total number of tiles	6	15	28	45	

- (1) Determine:

(i) stage n (ie: the general term) for the number of patterned tiles.

(2)

(ii) stage n for the number of black tiles.

(1)

(iii) stage n for the total number of tiles.

(4)

- (2) At which stage will the total number of tiles be 5151?

(3)

(c) Given: $\sum_{n=4}^{123} (3n - 1) = k$

- (1) Complete: The first five terms of the series are: 11 ; 14 ; 17 ; ____ ; ____

(2)

- (2) Determine the value of k , using a suitable formula.

(3)

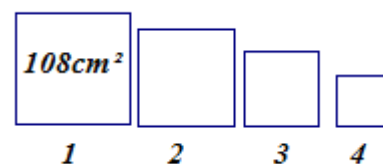
- (d) An infinite number of rectangles are formed.

The area of the first rectangle is 108 cm^2 .

Each subsequent rectangle has an area 20% less than the previous one.

Find the **total possible area** of all

the rectangles added together. Show all working clearly.



(3)

[21]

QUESTION 4

(a) If $f(x) = \frac{x^2}{4}$, determine $f'(x)$ using first principles. (4)

(b) (1) If $P - 3v = 2v^3$, determine $\frac{dP}{dv}$ (3)

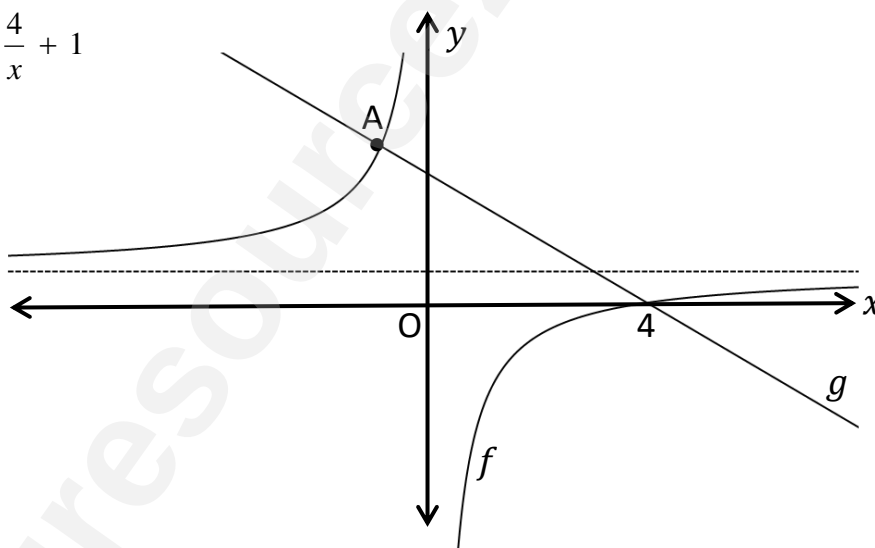
(2) Determine $D_x \left[\frac{x^2 - 4x + 3}{x} \right]$ (3)

(3) If $g(x) = 2\sqrt{x} + \frac{1}{x^3} - \sqrt{2}x$, show that $g'(\sqrt{2}) = -1,3$. (4)

(c) Given $f(x) = x^3 - 3x^2 + kx + 8$ where k is a constant. The graph has a turning point at $x = 1$. Find the value of k , (3)

[17]**QUESTION 5**

(a) Given: $f(x) = -\frac{4}{x} + 1$



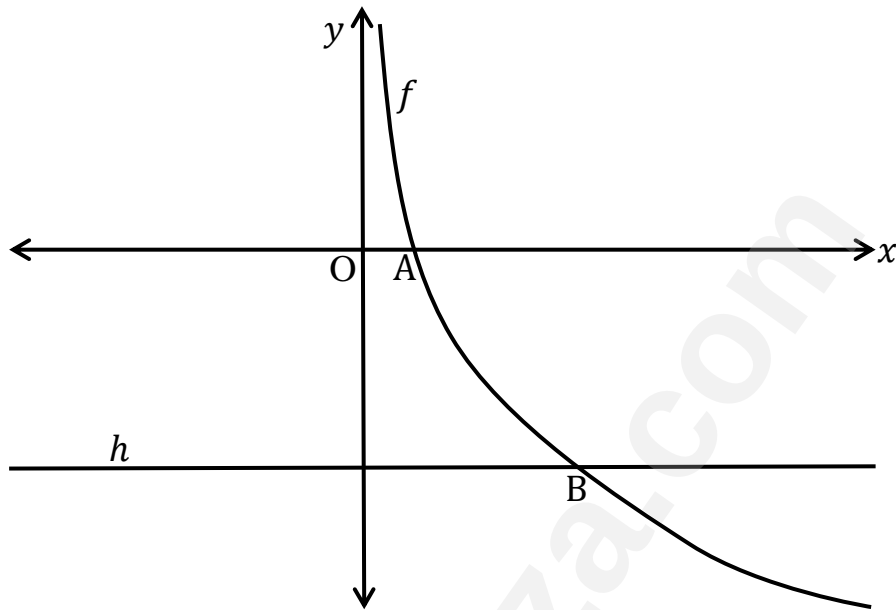
(1) Give the equation of an asymptote of f . (1)

(2) Determine the coordinates of point A, the point where the straight line $g(x) = -x + 4$ intersects the hyperbola. (4)

(3) Give the equation of h if h is the translation of f by two units to the left. (1)

(4) Determine the values of x for which $f(x) \geq g(x)$ (2)

- (b) The graph of $f(x) = \log_{\frac{1}{3}} x$ and $h(x) = -3$ are given below:



- (1) Determine the coordinates of:
- (i) A (1)
- (ii) B (2)
- (2) Use the graph to solve for x if $\log_{\frac{1}{3}} x \geq -3$ (2)
- (3) Determine the equation of $f^{-1}(x)$ in the form $f^{-1}(x) = \dots\dots\dots$ (2)
- (4) Determine the new equation of $f(x)$, if $f(x)$ is reflected about the x -axis. (1)

[16]

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74 marks

"It's important to learn math because someday you might accidentally buy a phone without a calculator."

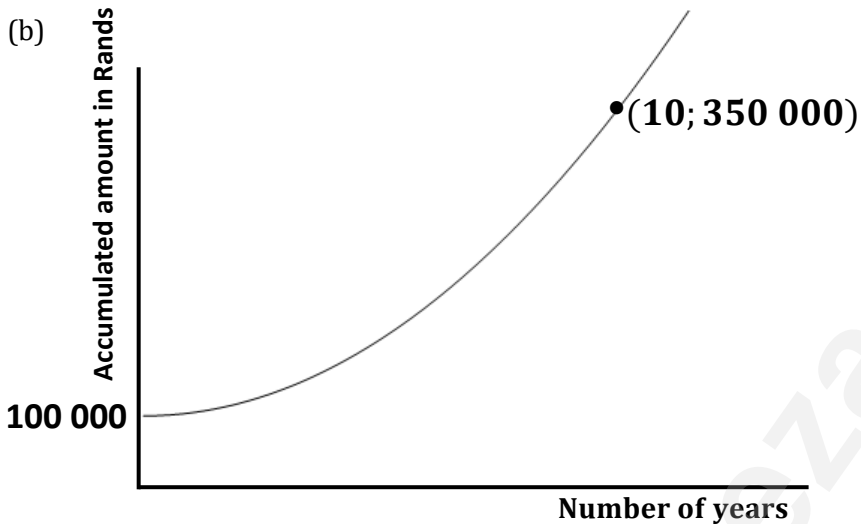
SECTION B

QUESTION 6



- (a) Craig owns a Toyota Yaris which depreciates at a reducing balance rate of 10% p.a. Determine how many years and months it will take for his Toyota to halve in value.

(4)



The graph above shows the growth of a lump sum of money that Sipho invested in *Supreme Bank*.

Determine the annual compound interest rate that he received, as a percentage, using the graph above.

(4)

- (c) Amy buys a flat for R 980 000. She puts down a deposit for R 150 000. She makes the remainder of the payment by means of a loan from the bank at a fixed interest rate of 9% per annum compounded monthly. She agrees to pay back the loan by means of equal monthly payments over 15 years starting one month after she loaned the money.

- (1) Determine the equal monthly payments. (4)
- (2) Assuming Amy's payments are R 8418,41, calculate the outstanding balance of her loan after 7 years (immediately after she makes her 84th payment) (4)
- (3) Calculate the total amount of interest Amy would have paid to the bank once she has fully paid back her loan at the end of 15 years. (3)

[19]

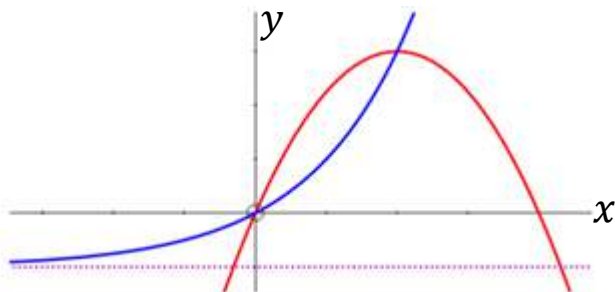


QUESTION 7

In the diagram, the graphs of $f(x) = ax^2 + bx + c$ and $g(x) = p^x + q$ are represented.

$y = -1$ is an asymptote to $g(x)$.

$f(x)$ passes through the origin. The turning point of $f(x)$ is $(2; 3)$ which is also the point of intersection between $g(x)$ and $f(x)$.



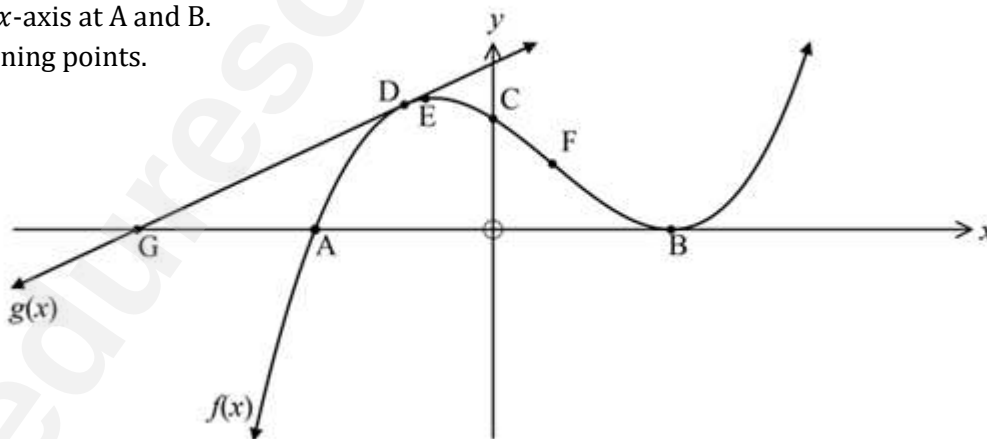
- Determine the equation of $f(x)$ in the form $y = ax^2 + bx + c$ (3)
- Determine the equation of $g(x)$. (3)
- Give the range of $g(x)$. (1)
- Determine the coordinates of the turning point of $f(x)$ if it undergoes a transformation of $f(x + 4) + 5$ (2)

[9]**QUESTION 8**

The sketch given below represents the function f with equation: $f(x) = x^3 - 2x^2 - 4x + 8$

The curve cuts the x -axis at A and B.

B and E are the turning points.



- Determine the coordinates of A, B and E. (7)
- Determine the x -coordinate of the point F, the point of inflection of $f(x)$. (3)
- The straight line g with equation $g(x) = 3x + c$ is a tangent to f at D. Determine the coordinates of G. (6)

[16]

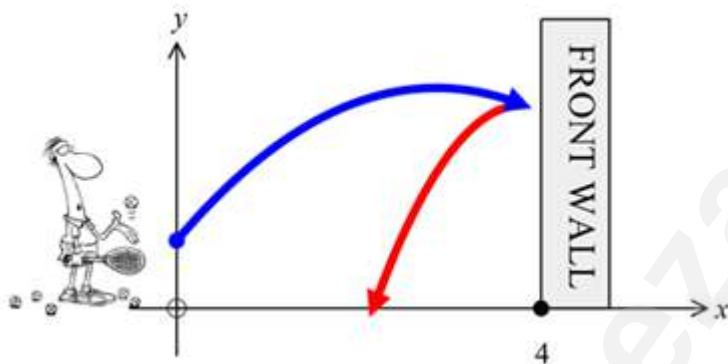
QUESTION 9

- (a) A squash player hits a ball against a wall which is 4 metres away and it rebounds as shown in the diagram below.

The initial path of the ball as it is struck by the player is given by the equation:

$$y = -\frac{1}{4}x^2 + px + 1$$

where y is the height that the ball is above the ground and x is the distance away from the player (in metres).

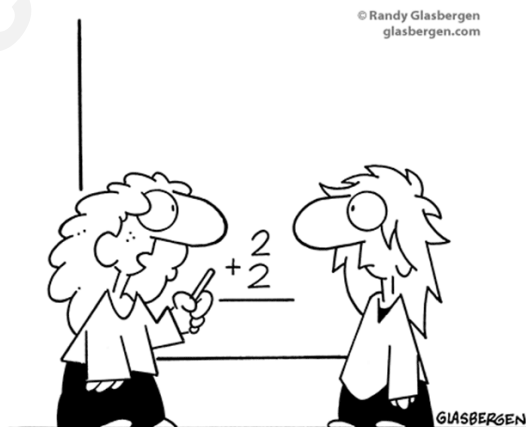


- (1) From what height is the squash ball struck? (1)
- (2) Given that the ball strikes the wall at a **gradient** of $-\frac{1}{2}$, calculate the value of p . (4)
- (3) The ball rebounds off the wall along the curve defined by: $y = -3x^2 + 24x - 45$
Determine the value of x at which the ball hits the ground. (3)

- (b) Given $f(x)$ and $g(x)$ are both parabolas, such that: $f(x) = -g(x)$

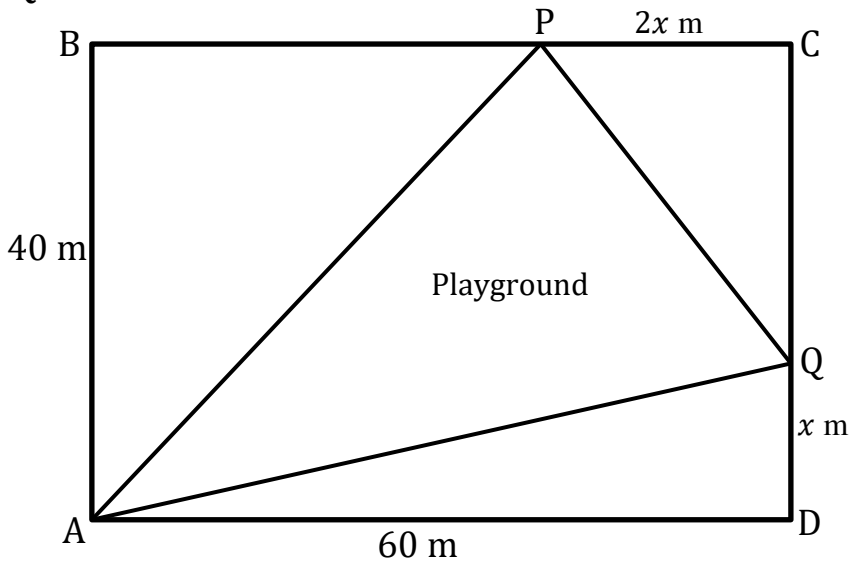
$$f(0) = -3; \quad f(2) = f(6) = 0; \quad f'(a) = 0; \quad f(a) = 5$$

- (1) Draw the sketch graph of $g(x)$. (4)
- (2) If $h'(x) = g(x)$, state the value(s) of x for which $h(x)$ is increasing. (2)



[14]

QUESTION 10



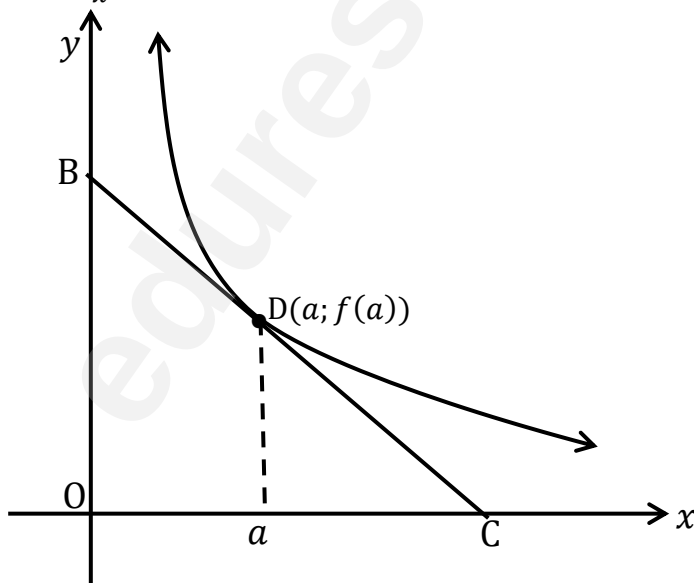
The diagram shows a plan for a rectangular park, $ABCD$, in which $AB = 40$ metres and $AD = 60$ metres. Points P and Q lie on BC and CD respectively and AP , PQ and QA are paths that surround a triangular playground. The length of DQ is x metres and the length of PC is $2x$ metres.

- (a) Write an expression for the lengths BP and CQ , in terms of x . (2)
- (b) Show that the area, m^2 , of the triangular playground APQ is given by $A = x^2 - 30x + 1200$ (5)
- (c) Given that x can vary, find the minimum area of the playground. (4)

[11]

QUESTION 11

Given: $f(x) = \frac{1}{x}$. A tangent, BC , touches f at $x = a$.



- (a) Show that the equation of the tangent at D is $x + a^2y = 2a$ (4)
- (b) Calculate the area of $\triangle OBC$. (3)

76 marks

[7]

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